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DISCRETE MATHEMATICAL MODEL OF HYPERSINGULAR INTEGRAL EQUATION ON A SYSTEM OF INTERVALS

We consider a hypersingular integral equation on a system of intervals, which is reduced to a system of hypersingular integral equations on the standard interval $(-1,1)$. The discretization of this system is carried out on the basis of the method of discrete singularities. The unique solvability of the discrete problem is proved and an estimate of the rate of convergence of the solution of this problem to the exact solution of the system of hypersingular integral equations is given.

Key words: hypersingular integral equation, method of discrete singularities.

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ДИСКРЕТНА МАТЕМАТИЧНА МОДЕЛЬ ГІПЕРСІНГУЛЯРНОГО ІНТЕГРАЛЬНОГО РІВНЯННЯ НА СИСТЕМІ ІНТЕРВАЛІВ

Розглянуто гіперсингулярне інтегральне рівняння на системі інтервалів, яке наведене до системи гіперсингулярних інтегральних рівнянь на стандартному інтервалі $(-1,1)$. Проведена дискретизація цієї системи на основі методу дискретних особливостей. Доведено однозначна розв'язність дискретної задачі і дана оцінка швидкості збіжності рішення дискретної задачі до точного рішення системи гіперсингулярних інтегральних рівнянь.

Ключові слова: гіперсингулярне інтегральне рівняння, метод дискретних особливостей.

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ДИСКРЕТНАЯ МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ГИПЕРСИНГУЛЯРНОГО ИНТЕГРАЛЬНОГО УРАВНЕНИЯ НА СИСТЕМЕ ИНТЕРВАЛОВ

Рассмотрено гиперсингулярное интегральное уравнение на системе интервалов, которое приведено к системе гиперсингулярных интегральных уравнений на стандартном интервале $(-1,1)$. Проведена дискретизация этой системы на основе метода дискретных особенностей. Доказана однозначная разрешимость дискретной задачи и дана оценка скорости сходимости решения этой задачи к точному решению системы гиперсингулярных интегральных уравнений.

Ключевые слова: гиперсингулярное интегральное уравнение, метод дискретных особенностей.

Introduction. When studying a mathematical model of a gyrotron with several resonant cavities of different width and depth one faces the necessity for numerical solving a hypersingular integral equation on a system of intervals. In the paper the numerical method of discrete singularities [1] for solving such equations is introduced and substantiated.

Problem setting. In the paper the following hypersingular integral equation (HSIE) on a system of intervals is considered:

$$\frac{1}{\pi}\int_L \frac{F(\xi)}{(x-\xi)^2}d\xi+\frac{a}{\pi}\int_L \frac{F(\xi)}{x-\xi}d\xi+\frac{b}{\pi}\int_L F(\xi)\ln|x-\xi|d\xi+\frac{1}{\pi}\int_L Q(x,\xi)F(\xi)d\xi=g(x), \tag{1}$$

for the unknown functions $F(\xi),\ \xi\in L$, where

$$L=\bigcup_{q=1}^{m}(\alpha_q,\beta_q),\quad -\infty<\alpha_1<\beta_1<\cdots<\alpha_m<\beta_m<+\infty;$$

the right-hand parts $g(x)\in C_{\overline{L}}^{1,\alpha},\ \alpha>0$, where $C_{\overline{L}}^{1,\alpha}$ stands for the class of functions on $\overline{L}$ such that their first derivative satisfies *the Hölder condition with a positive exponent*; the function of two variables $Q(x,\xi),\ x\in\overline{L},\ \xi\in\overline{L}$ belongs to the same class $C_{\overline{L}}^{1,\alpha},\ \alpha>0$ in each variable uniformly with respect to the other variable; a,b are given constants. The first integral in (1) is to be understood in the sense of *the Hadamard finite part*, the second one – in the sense of *the Cauchy principal value*. Equation (1) is assumed to admit a unique solution.

We are looking for the solution $F(\xi),\ \xi\in L$ from the class of functions such that their restriction to the intervals (α_j,β_j): $F_j(\xi)=F(\xi),\ \alpha_j<\xi<\beta_j,\ j=\overline{1,m}$, admits the representation

$$F_j(\xi)=v_j(\xi)\sqrt{(\xi-\alpha_j)(\beta_j-\xi)},\ \alpha_j<\xi<\beta_j,$$

where $v_j(\xi),\ \xi\in[\alpha_j,\beta_j]$ are smooth functions. We introduce the following notations for the restrictions of the functions $g(x)$ and $Q(x,\xi)$:

$$g_i(x)=g(x),\ \alpha_i<x<\beta_i,\quad i=\overline{1,m};$$
$$Q_{ij}(x,\xi)=Q(x,\xi),\ \alpha_i<x<\beta_i,\ \alpha_j<\xi<\beta_j,\quad i,j=\overline{1,m}.$$

Apparently, equation (1) is equivalent to the following system of HSIE:

$$\frac{1}{\pi}\int_{\alpha_i}^{\beta_i}\frac{v_i(\xi)}{(x-\xi)^2}\sqrt{(\xi-\alpha_i)(\beta_i-\xi)}d\xi+\frac{a}{\pi}\int_{\alpha_i}^{\beta_i}\frac{v_i(\xi)}{x-\xi}\sqrt{(\xi-\alpha_i)(\beta_i-\xi)}d\xi+$$
$$+\frac{b}{\pi}\int_{\alpha_i}^{\beta_i}\ln|x-\xi|v_i(\xi)\sqrt{(\xi-\alpha_i)(\beta_i-\xi)}d\xi+$$
$$+\frac{1}{\pi}\sum_{j=1}^{m}\int_{\alpha_i}^{\beta_i}\left[Q_{ij}(x,\xi)+(1-\delta_{ij})\left(\frac{1}{(x-\xi)^2}+\frac{a}{x-\xi}+b\ln|x-\xi|\right)\right]v_j(\xi)\sqrt{(\xi-\alpha_i)(\beta_i-\xi)}d\xi=g_i(x),$$
$$x\in(\alpha_i,\beta_i),\ i=\overline{1,m}.$$

We denote $\varphi_k(t)=\frac{1}{2}\left[(\beta_k-\alpha_k)t+\alpha_k+\beta_k\right]$ and substitute

$$x=\varphi_i(t_0),\ x\in(\alpha_i,\beta_i),\ -1<t_0<1;\quad \xi=\varphi_j(t),\ \xi\in(\alpha_j,\beta_j),\ -1<t<1,$$

in the system of equations obtained. Then setting $u_j(t)=v_j(\varphi_j(t)),\ f_i(t_0)=g_i(\varphi_i(t_0))$ we arrive at the system of HSIE on the standard interval $(-1,1)$:

$$\frac{1}{\pi}\int_{-1}^{1}\frac{u_i(t)}{(t_0-t)^2}\sqrt{1-t^2}dt+\frac{a_i}{\pi}\int_{-1}^{1}\frac{u_i(t)}{t_0-t}\sqrt{1-t^2}dt+\frac{b_i}{\pi}\int_{-1}^{1}\ln|t_0-t|u_i(t)\sqrt{1-t^2}dt+$$
$$+\frac{1}{\pi}\sum_{j=1}^{m}\int_{-1}^{1}K_{ij}(t_0,t)u_j(t)\sqrt{1-t^2}dt=f_i(t_0),\ |t_0|<1,\ i=\overline{1,m}. \tag{2}$$

The notations here are $a_i=\left(\frac{\beta_i-\alpha_i}{2}\right)a,\quad b_i=\left(\frac{\beta_i-\alpha_i}{2}\right)^2 b,$

$$K_{ij}(t_0,t)=\begin{cases}Q_{ii}(\varphi_i(t_0),\varphi_i(t))\left(\frac{\beta_i-\alpha_i}{2}\right)^2+b_i\ln\left(\frac{\beta_i-\alpha_i}{2}\right),\quad (j=i),\\ \left[Q_{ij}(\varphi_i(t_0),\varphi_j(t))+\frac{1}{(\varphi_i(t_0)-\varphi_j(t))^2}\right]\left(\frac{\beta_j-\alpha_j}{2}\right)^2+\\ \quad+\frac{\alpha_j(\beta_j-\alpha_j)}{2(\varphi_i(t_0)-\varphi_j(t))}+b_j\ln|\varphi_i(t_0)-\varphi_j(t)|,\quad j\neq i.\end{cases}$$

The functions $f_i(t_0)$ are from $C^{1,\alpha}_{[-1,1]},\ \alpha>0$, and $K_{ij}(t_0,t)$ belong to $C^{1,\alpha}_{[-1,1]},\ \alpha>0$ in each variable uniformly with respect to the other one.

The next step is to determine the unknown functions $u_i(t),\ i=\overline{1,m}$ from system (2).

Main operators. We introduce the following operators (see [3]):

$$(Au_i)(t_0)\equiv\frac{1}{\pi}\int_{-1}^{1}\frac{u_i(t)}{(t_0-t)^2}\sqrt{1-t^2}dt,$$

$$(\Gamma^{-1}u_i)(t_0)\equiv\frac{1}{\pi}\int_{-1}^{1}\frac{u_i(t)}{t_0-t}\sqrt{1-t^2}dt,$$

$$(Bu_i)(t_0)\equiv\frac{1}{\pi}\int_{-1}^{1}\ln|t_0-t|u_i(t)\sqrt{1-t^2}dt,$$

$$(K_{ij}u_j)(t_0)\equiv\frac{1}{\pi}\int_{-1}^{1}K_{ij}(t_0,t)u_j(t)\sqrt{1-t^2}dt.$$

Let $\vec{u}(t)=\{u_i(t)\}_{i=1}^{m}$ be a vector function. We denote:

$$\left(\bar{A}\vec{u}\right)(t_0)=\left\{(Au_i)(t_0)\right\}_{i=1}^{m},$$

$$\left(\bar{a}\,\bar{\Gamma}^{-1}\vec{u}\right)(t_0)=\left\{a_i\left(\Gamma^{-1}u_i\right)(t_0)\right\}_{i=1}^{m},$$

$$\left(\bar{b}\,\bar{B}\vec{u}\right)(t_0)=\left\{b_i\,(Bu_i)(t_0)\right\}_{i=1}^{m},$$

$$\left(\bar{K}\vec{u}\right)(t_0)=\left\{\sum_{j=1}^{m}\left(K_{ij}u_j\right)(t_0)\right\}_{i=1}^{m},$$

$$\vec{f}(t_0)=\left\{f_i(t_0)\right\}_{i=1}^{m}.$$

Using these notations, system (2) can be represented in the form of an operator equation:

$$\bar{A}\vec{u}+\bar{a}\bar{\Gamma}^{-1}\vec{u}+\bar{b}\bar{B}\vec{u}+\bar{K}\vec{u}=\vec{f}. \qquad (3)$$

Equation (3) admits a unique solution.

It is proved in [3] that the following relations hold:

$$A:U_{n-1}(t)\to nU_{n-1}(t_0);$$

$$\Gamma^{-1}:U_{n-1}(t)\to T_n(t_0),$$

where $T_n(t)$ is *the first kind Chebyshev polynomial* of degree n and $U_{n-1}(t)$ is *the second kind Chebyshev polynomial* of degree $n-1$.

Since the operator B maps polynomials of degree $n-2$ into polynomials of degree n, we conclude that the operator $A+a\Gamma^{-1}+bB$ maps polynomials into polynomials.

We introduce the Hilbert spaces in which the above operators are defined.

Let L^I be a Hilbert functional space endowed with the scalar product:

$$(u,v)^I=\int_{-1}^{1}u(t)\bar{v}(t)\sqrt{1-t^2}\,dt+\int_{-1}^{1}\left(u(t)\sqrt{1-t^2}\right)^{\prime}\left(\bar{v}(t)\sqrt{1-t^2}\right)^{\prime}dt;$$

and let L^{II} be a Hilbert functional space with the scalar product defined by the formula:

$$(u,v)^{II}=\int_{-1}^{1}u(t)\bar{v}(t)\sqrt{1-t^2}\,dt.$$

We denote by Π_{n-2}^{I} and Π_{n-2}^{II} the subspaces of L^I and L^{II} consisting of the polynomials of the degree less than or equal to $n-2$.

Let H^I and H^{II} be the spaces of the vector functions $\vec{u}(t)=\{u_i(t)\}_{i=1}^{m}$ endowed respectively with the scalar products

$$(u,v)^I=\sum_{i=1}^{m}(u_i,v_i)^I,\quad (u,v)^{II}=\sum_{i=1}^{m}(u_i,v_i)^{II}.$$

Apparently, the operators $\bar{A}$ and $\bar{\Gamma}^{-1}$ are completely continuous in the pair of spaces $\left(\mathrm{H}^I,\ \mathrm{H}^{II}\right)$, hence, the operator $\bar{A}+\bar{a}\bar{\Gamma}^{-1}$ is also completely continuous in $\left(\mathrm{H}^I,\ \mathrm{H}^{II}\right)$. This remark and the fact that equation (3) admits a unique solution imply that the operator

$$\bar{A}+\bar{a}\bar{\Gamma}^{-1}+\bar{b}\bar{B}+\bar{K}$$

is continuously invertible in $\left(\mathrm{H}^I,\ \mathrm{H}^{II}\right)$.

Regularization and discretezation. Let $(P_{n-2}v)(t)$ be *the Lagrange interpolation polynomial* for the function $v(t)$ with the interpolation nodes $t_j^{(n)}=\cos\frac{j}{n}\pi,\quad j=\overline{1,n-1}$ that are the roots of the second kind Chebyshev polynomial $U_{n-1}(t)$.

Denote

$$\vec{u}_{\bar{n}}(t)=\left\{u_{in_i-2}(t)\right\}_{i=1}^{m}\equiv\left\{\left(P_{n_i-2}u_i\right)(t)\right\}_{i=1}^{m},\quad \vec{f}_{\bar{n}}(t_0)=\left\{\left(P_{n_i-2}f_i\right)(t_0)\right\}_{i=1}^{m}\quad \left(\bar{n}=(n_1,n_2,\ldots,n_m)\right).$$

Let $u_{n-2}(t)$ be a polynomial of degree $n-2$. The operator A maps $u_{n-2}(t)$ into a polynomial of degree $n-2$. We regularize the operators Γ^{-1} and B so that the regularized operators take $u_{n-2}(t)$ into a polynomial of degree $n-2$ as well. To this end, following the ideas of [1, 3], we set

$$\left(\Gamma_{n-2}^{-1}u_{n-2}\right)(t_0)=\frac{1}{\pi}\int_{-1}^{1}u_{n-2}(t)\left(\frac{1}{t_0-t}-U_{n-2}(t)T_{n-1}(t_0)\right)\sqrt{1-t^2}\,dt,$$

$$\left(B_{n-2}u_{n-2}\right)(t_0)=\frac{1}{\pi}\int_{-1}^{1}\left(\ln|t_0-t|+\frac{2T_{n-1}(t)T_{n-1}(t_0)}{n-1}+\frac{2T_n(t)T_n(t_0)}{n}\right)u_{n-2}(t)\sqrt{1-t^2}\,dt.$$

The operators Γ_{n-2}^{-1} and B_{n-2} defined by the above formulas are the regularizations of the operators Γ^{-1} and B required.

Next we introduce the notations:

$$\left(\bar{a}\,\bar{\Gamma}_{\bar{n}}^{-1}\vec{u}_{\bar{n}}\right)(t_0)=\left\{a_i\left(\Gamma_{n_i-2}^{-1}u_{in_i-2}\right)(t_0)\right\}_{i=1}^{m};$$

$$\left(\bar{b}\,\bar{B}_{\bar{n}}\,\vec{u}_{\bar{n}}\right)(t_0)=\left\{b_i\left(B_{n_i-2}u_{in_i-2}\right)(t_0)\right\}_{i=1}^{m};$$

$$\left(\bar{K}_{\bar{n}}\vec{u}_{\bar{n}}\right)(t_0)=\left\{\sum_{j=1}^{m}\left(K_{ijn_in_j}u_{jn_j-2}\right)(t_0)\right\}_{i=1}^{m};$$

where

$$\left(K_{ijn_in_j}u_{jn_j-2}\right)(t_0)=\frac{1}{\pi}\int_{-1}^{1}\left(P_{n_i-2_{t_0}}P_{n_j-2_t}K_{ij}\right)(t_0,t)\,u_{jn_j-2}(t)\sqrt{1-t^2}\,dt.$$

The approximate solution $\vec{u}_{\bar{n}}(t)$ to equation (3) is determined by solving the following operator equation:

$$\bar{A}\vec{u}_{\bar{n}}+\bar{a}\bar{\Gamma}_{\bar{n}}^{-1}\vec{u}_{\bar{n}}+\bar{b}\bar{B}_{\bar{n}}\vec{u}_{\bar{n}}+\bar{K}_{\bar{n}}\vec{u}_{\bar{n}}=\vec{f}_{\bar{n}}\,, \tag{4}$$

which is equivalent to the system of HSIE:

$$\frac{1}{\pi}\int_{-1}^{1}\frac{u_i(t)}{(t_0-t)^2}\sqrt{1-t^2}\,dt+\frac{a_i}{\pi}\int_{-1}^{1}u_{in_i-2}(t)\left(\frac{1}{t_0-t}-U_{n_i-2}(t)T_{n_i-1}(t_0)\right)\sqrt{1-t^2}\,dt+$$

$$+\frac{b_i}{\pi}\int_{-1}^{1}\left(\ln|t_0-t|+\frac{2T_{n_i-1}(t)T_{n_j-1}(t_0)}{n_i-1}+\frac{2T_{n_i}(t)T_{n_j}(t_0)}{n_i}\right)u_{in_i-2}(t)\sqrt{1-t^2}\,dt+$$

$$+\frac{1}{\pi}\sum_{j=1}^{m}\int_{-1}^{1}\left(P_{n_i-2_{t_0}}P_{n_j-2_t}K_{ij}\right)(t_0,t)\,u_{jn_j-2}(t)\sqrt{1-t^2}\,dt=\left(P_{n_i-2}f_i\right)(t_0),\quad |t_0|<1, i=\overline{1,m}.$$

We apply the method of discrete singularities to the latter system. Substituting the values $t_0\in\left\{t_{r_i}^{n_i}\right\}$, $r_i=\overline{1,n_i-1}$ $\left(i=\overline{1,m}\right)$ instead of t_0 into the $i-$th equation we arrive at a system of

$$\sum_{i=1}^{m}n_i-m$$

equations. Using the *interpolation type exact quadrature formulas* for the integrals (see [2]), we obtain a system of linear algebraic equations for the values of the functions $u_{in_i-2}(t)$ at the interpolation nodes:

$$\sum_{k_i=1}^{n_i}a_{r_ik_i}^{(1)}u_{in_i-2}\left(t_{k_i}^{n_i}\right)+\sum_{k_i=1}^{n_i}a_{r_ik_i}^{(2)}u_{in_i-2}\left(t_{k_i}^{n_i}\right)+\sum_{k_i=1}^{n_i}a_{r_ik_i}^{(3)}u_{in_i-2}\left(t_{k_i}^{n_i}\right)+$$

$$+\sum_{j=1}^{m}\sum_{k_j=1}^{n_j-1} a_{r_i k_j}^{(4)} u_{j n_j-2}\left(t_{k_j}^{n_j}\right) f\left(t_{r_i}^{n_i}\right), \quad r_i=\overline{1, n_i-1}, \ i=\overline{1, m}. \tag{5}$$

In (5) the notations $a_{r_i k_i}^{(q)}$, $q=1, 2, 3$ stand for

$$a_{r_i k_i}^{(1)}=\begin{cases}\dfrac{\left(1-\left(t_{k_i}^{n_i}\right)^2\right)\left((-1)^{r_i+k_i+1}+1\right)}{n_i\left(t_{r_i}^{n_i}-t_{k_i}^{n_i}\right)^2}, & k_i \neq r_i, \\ -\dfrac{n_i}{2}, & k_i=r_i,\end{cases}$$

$$a_{r_i k_i}^{(2)}=\begin{cases}\dfrac{a_i}{n_i}\left(\dfrac{\left((-1)^{r_i+k_i+1}+1\right)}{t_{r_i}^{n_i}-t_{k_i}^{n_i}}-2(-1)^{r_i+k_i+1} t_{r_i}^{n_i}\right)\left(1-\left(t_{r_i}^{n_i}\right)^2\right), & k_i \neq r_i, \\ 0, & k_i=r_i,\end{cases}$$

$$a_{r_i k_i}^{(3)}=\frac{b_i\left(1-\left(t_{k_i}^{n_i}\right)^2\right)}{n_i}\left(\frac{2(-1)^{r_i+k_i} t_{r_i}^{n_i} t_{k_i}^{n_i}}{n_i-1}+\frac{(-1)^{r_i+k_i}}{n_i}-\ln 2-2\sum_{p=1}^{n_i-1}\frac{T_p\left(t_{k_i}^{n_i}\right) T_p\left(t_{r_i}^{n_i}\right)}{p}\right),$$

$$a_{r_i k_j}^{(4)}=\frac{1}{n_i} K_{r_i k_j}\left(t_{r_i}^{n_i}, t_{k_j}^{n_j}\right)\left(1-\left(t_{k_j}^{n_j}\right)^2\right).$$

Proof of unique solvability for system of linear algebraic equations (SLAE). System (5) admits a unique solution if and only if equation (4) does. So we prove the unique solvability of operator equation (4) here. The argument is based on the following fact which is due to [4].

Let X and Y be normed linear spaces, and denote $\tilde{X} \subset X$, $\tilde{Y} \subset Y$ their finite dimensional subspaces of the same dimension. Consider the two equations:

the exact one

$$Kx=y\left(x \in X, \ y \in Y\right)$$

and the approximate one

$$\tilde{K}\tilde{x}=\tilde{y}\left(\tilde{x} \in \tilde{X}, \ \tilde{y} \in \tilde{Y}\right),$$

where K and $\tilde{K}$ are linear operators, such that $K: X \to Y$, $\tilde{K}: \tilde{X} \to \tilde{Y}$.

Theorem 1. *Assume that*

a) the operator K is continuously invertible in the pair of spaces (X, Y);

b) $p=\left\|K^{-1}\right\|_{Y \to X}\left\|K-\tilde{K}\right\|_{\tilde{X} \to \tilde{Y}}<1.$

Then for any right-hand part $\tilde{y} \in \tilde{Y}$ the approximate equation admits a unique solution. Moreover, if $x^* \in X$ is an exact solution of the equation $Kx=y$ and $\delta=\left\|y-\tilde{y}\right\|_Y$, then

$$\left\|x^*-\tilde{x}^*\right\|_X \leq \left\|K^{-1}\right\|_{Y \to X}(1-p)^{-1}\left[\delta-p\|y\|_Y\right].$$

The proof of ***Theorem 1*** uses the following inequalities (see [1]):

$$\left\|\vec{f}-\vec{f}_{\bar{n}}\right\|_{H^{II}} \leq \frac{F}{n^{1+\alpha}} \text{ for } n=\min\{n_1, n_2, ..., n_m\}>3; \quad \left\|\Gamma^{-1}-\Gamma_{n_i-2}^{-1}\right\|_{\Pi_{n_i-2}^{I} \to L^{II}} \leq \frac{C_i^{(1)}}{n_i};$$

$$\left\|B-B_{n_i-2}\right\|_{\Pi_{n_i-2}^{I} \to L^{II}} \leq \frac{C_i^{(2)}}{n_i^2}; \quad \left\|K_{ij}-K_{ij n_i n_j}\right\|_{\Pi_{n_i-2}^{I} \to L^{II}} \leq \frac{C\left(K_{ij}\right)}{n_i^{1+\alpha}}, \ n_i>3, \tag{6}$$

where $F, C_i^{(1)}, C_i^{(2)}, C(K_{ij}), i, j = \overline{1, m}$ – are constants independent of $\bar{n}$.

Inequalities (6) result into the following estimate:

$$\left\|\left(\bar{A}+\bar{a}\bar{\Gamma}_{\bar{n}}^{-1}+\bar{b}\bar{B}_{\bar{n}}+\bar{K}_{\bar{n}}\right)\vec{u}_{\bar{n}}-\left(\bar{A}+\bar{a}\bar{\Gamma}^{-1}+\bar{b}\bar{B}+\bar{K}\right)\vec{u}_{\bar{n}}\right\|_{\mathrm{H}^{II}} \le \left(\frac{C^{(1)}}{n}+\frac{C^{(2)}}{n^2}+\frac{C(K)}{n^{1+\alpha}}\right)\left\|\vec{u}_{\bar{n}}\right\|_{\mathrm{H}^{I}} \le \frac{D}{n}\left\|\vec{u}_{\bar{n}}\right\|_{\mathrm{H}^{I}}, \qquad (7)$$

where $n = \min\{n_1, n_2, ..., n_m\}$;

$$C^{(1)} = \max_{1\le i\le m}\{a_i\}\max_{1\le i\le m}\left\{C_i^{(1)}\right\}; \quad C^{(2)} = \max_{1\le i\le m}\{b_i\}\max_{1\le i\le m}\left\{C_i^{(2)}\right\}; \quad C(K) = \max_{1\le i,\, j\le m}\left\{C(K_{ij})\right\}.$$

Estimates (6), (7) imply the following theorem:

Theorem 2. *For all* $\bar{n}$ *such that*

$$\bar{n} \ge D\left\|\left(\bar{A}+\bar{a}\bar{\Gamma}^{-1}+\bar{b}\bar{B}+\bar{K}\right)^{-1}\right\|_{\mathrm{H}^{II}\to\mathrm{H}^{I}},$$

equation (4) possesses a unique solution. For $n \to \infty$ *the following estimate of the rate of convergence of the approximate solution to the exact one holds:*

$$\left\|\vec{u}_{\bar{n}}-\vec{u}\right\|_{\mathrm{H}^{I}} = \underline{O}\left(\frac{1}{n}\right).$$

Conclusions. The numerical method of discrete singularities is applied for constructing a system of linear algebraic equations approximating the system of hypersingular integral equations on the standard interval $(-1, 1)$, which, in turn, is equaivalent to the hypersingular integral equation on a set of intervals. In case the kernel of the regular part and the right-hand part of this hypersingular integral equation satisfy some smoothness assumptions, the system of linear algebraic equations constructed admits a unique solution. Moreover, the rate of convergence of the approximate solution to the exact one is estimated.

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