УДК 625.1«401.7»:625.1.041.1

DR.-ING. ULF GERBER (Dresden University of Technology, Faculty of Traffic and Transportation Sciences «Friedrich List», Institute for Traffic Infrastructure)

A RELATIONSHIP BETWEEN THE SERVICE LIFE OF THE PERMANENT WAY AND THE INHOMOGENEITY OF THE SUBSTRUCTURE

У даній статті неоднорідність складових елементів колії описана так званою дисперсією основи. Результатом буде отримання терміну служби верхньої будови колії як функції дисперсії субструктури. Приклад розрахунку включено для того щоб проілюструвати розрахунки.

В этой статье неоднородность составляющих элементов пути описана так называемой дисперсией основания. Результатом будет получение срока службы верхнего строения пути, как функции дисперсии основания. Пример расчета включен для того чтобы проиллюстрировать результаты расчетов.

The inhomogeneity of the substructure will be described by the so-called variance of the substructure in this paper. The result will be a formulation of the permanent way's service life as a function of the variance of the substructure. An example will be included to illustrate the results of the calculations.

1. Waviness of the permanent way

Due to the natural variation in the mechanical properties of the substructure, waviness occurs in the trackway. This waviness is modeled as a trough in the track, which is characterized by the differential settling Δs and the wave length λ, (fig. 1):

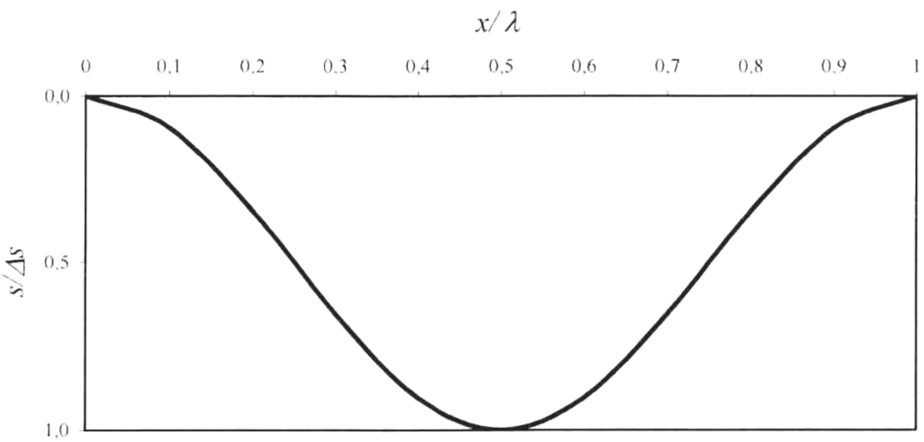

Fig. 1. The permanent way's waviness is modeled as a trough in the track, which is, according to (1), completely characterized by the differential settlement Δs and the length λ

$$s(x) = \frac{\Delta s}{2}\left(1 - \cos\left(2\pi \frac{x}{\lambda}\right)\right). \qquad (1)$$

The differential settlement λ – develops in course of the time T, caused by the varying rate of settlement, $\dot{s}$, of the trackway under the load $F(T)$ (fig. 2). Calculating it requires knowledge of the settling over the time, $s(T)$, which can be modeled by a damper with a viscosity $\eta(s)$ depending on the settling:

$$\dot{s} = \frac{F(T)}{\eta(s)}. \qquad (2)$$

Settling results from a consolidation of the substructure that is tied to an increase in viscosity. To describe the consolidation, the following exponential equation is used:

$$\eta(s) = \eta_0 \cdot e^{K \cdot s}. \qquad (3)$$

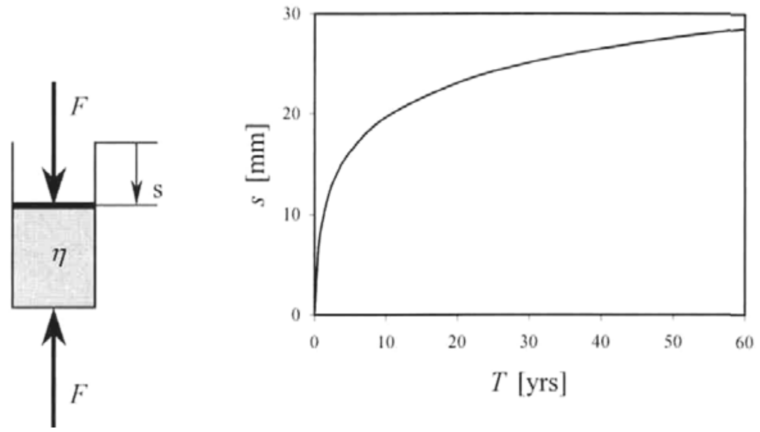

Fig. 2. Settling of the subgrade, s is modeled by damper with a settling-dependent viscosity η.
Adjusting the model to mimic real conditions leads to $s(T)$ the time-dependent settlement of the substructure

Assuming a time-constant load, F_0, substituting (3) into (2) results in the following equation:

$$s(T) = \frac{1}{K} \cdot \ln\left(\left(\frac{K}{\eta_0} F_0 T\right) + 1\right). \qquad (4)$$

Matching (4) to a measured settling curve in accordance with [1. S. 25] leads to the time-dependent settling s [mm] defined for time T [yrs], as shown in fig. 2:

$$s(T) = 5 \cdot \ln(5T + 1). \qquad (5)$$

The differential settling Δs is understood to be a variation from the mean settling *s*. Usually, the mechanical behavior of the substructure is characterized by the deformation modulus *E*. Assuming proportionality between settling and the deformation modulus, the so-called variance of the substructure is defined as follows:

$$Q = \frac{\Delta s}{s} = \frac{\Delta E}{E}. \qquad (6)$$

The (long-term) time-dependent differential settling of the permanent way follows from inserting (6) into (5), (Δs [mm], T [yrs]):

$$\Delta s(T) = 5Q \cdot \ln(5T + 1). \qquad (7)$$

In consideration of (1), the waviness of the permanent way can finally be stated by a place-dependent (*x*) and a (long-term) time-dependent (*T*) differential settlement $\Delta s(x, T)$:

$$\Delta s(x,T) = 2{,}5\boldsymbol{Q} \ln(5T+1) \cdot \left(1\text{-}\cos\left(2\pi \frac{x}{\lambda}\right)\right). \qquad (8)$$

2. Permanent way load model

Due to the waviness of the permanent way, an additional vertical acceleration will occur on a passing vehicle, which leads to an additional load ΔF on the rail track (Fig. 3). The effect of this additional load is usually expressed by the dynamic factor, f_D:

$$f_D = 1 + \frac{\Delta F}{F_0}. \qquad (9)$$

The additional load is also proportional to the differential displacement $z = \bar{z} - \Delta s$ between coach body and permanent way. Therefore, according to (9):

$$f_D = \frac{z}{z_0}. \qquad (10)$$

where z_0 is the differential displacement in the static case.

Of particular interest is the maximum load on the track and, therefore, the maximum differential displacement, $z_{\max}$, which is expressed by the maximum dynamic factor, $f_{D\max}$

$$f_{D\max} = \frac{z_{\max}}{z_0}. \qquad (11)$$

For a rough determination of the maximum dynamic factor as the ratio of the most frequently occurring differential displacement $z_{\max}$ to the static differential displacement z_0, the dynamic character of the oscillation system according to fig. 3 has to be considered.

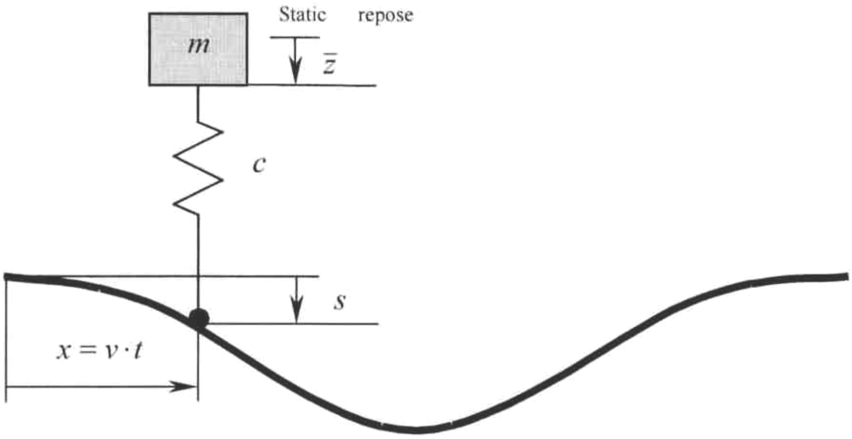

Fig. 3. The load on the permanent way is modeled by a single-mass system.
As a result of the permanent way's waviness, $s(x)$, the mass m is excited to vibration.
The additional load on the permanent way caused by this vibration is proportional to the deflection of the single-mass system: $\Delta F = c(\bar z - s)$

The oscillation is determined to a decisive degree by the frequency ratio η between the applied frequency Ω and the eigenfrequency ω:

$$\eta = \frac{\Omega}{\omega}. \qquad (12)$$

The waviness of the permanent way corresponds to the system's excitation with the applied frequency, Ω. If a vehicle is driving along the rail track (coordinate x) in a short time t with a constant speed v, x is determined by:

$$x = vt. \qquad (13)$$

By inserting this relation into (8), the waviness of the permanent way can be stated both as short-term time-depending (7) and as long-term time-depending (7):

$$\Delta s(t, T) = = 2{,}5Q \ln(5T+1)\left(1-\cos\left(2\pi\frac{v}{\lambda}t\right)\right) = = \frac{\Delta s}{2}\left(1-\cos(\Omega t)\right). \qquad (14)$$

(14) corresponds to the short-term excitation caused by the permanent way's waviness. In consideration of (14), the following dynamic differential equation can be formulated to describe the oscillation system shown in fm. 3 [2, S. 553 f]:

$$m \cdot \ddot{\bar z} + c(\bar z - \Delta s) = 0. \qquad (15)$$

Regarding the definition of the eigenfrequency

$$\omega = \sqrt{\frac{c}{m}} \qquad (16)$$

as well as $z = \bar z - \Delta s$ resp. $\ddot z = \ddot{\bar z} - \Delta \ddot s$ and (14), it follows:

$$\ddot z + \omega^2 z = -\Delta \ddot s = -\frac{\Delta s}{2}\Omega^2 \cdot \cos(\Omega \cdot t). \qquad (18)$$

Further consideration of the definition of the frequency ratio (12) leads to:

$$\ddot z + \omega^2 \cdot z = -\frac{\Delta s}{2}\omega^2 \cdot \eta^2 \cdot \cos(\omega \cdot \eta \cdot t). \qquad (19)$$

We are looking for the most frequently occurring amplitude z_{max}. Only the dip movement of a vehicle with an eigenfrequency of $\omega = 2\pi f = 2\pi \quad [s^{-1}]$ will be considered (this corresponds to a period of 1 s). The solution to the dynamic differential equation (18) with the initial conditions $z(t=0)=0$ and $\dot z(t=0)=0$ leads to the history of the differential displacement z relative to the differential settlement Δs, as shown in fig. 4.

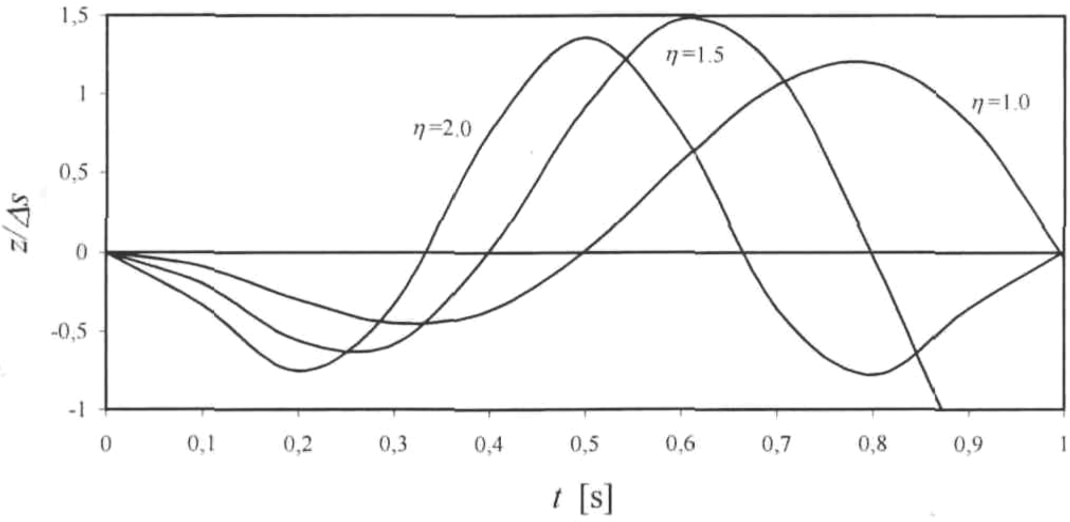

Fig. 4. The numerical solution to the dynamic differential equation (18) leads to the deflection z depending on the time $t(\omega = 2\pi \quad s^{-1})$. The maximum deflection $z_{\max}$ amounts to 150 % of the differential settlement of the settlement bowl Δs and occurs with a frequency ratio of $\eta = 1{,}5$

With the static differential displacement

$$z_0 = \frac{mg}{c} = \frac{g}{\omega^2} \tag{20}$$

and the maximum differential displacement

$$z_{\max} = z_0 + 1{,}5\Delta s \tag{21}$$

follows, for the maximum dynamical factor according to (11):

$$f_{D\max} = 1 + 0{,}006\Delta s\,. \tag{22}$$

(The differential settling Δs needs to be inserted in the dimension [mm]). It occurs with a frequency ratio of $\eta = 1{,}5$.

In consideration of the definitions (12), (13) $\omega = 2\pi \ [s^{-1}]$, the frequency ratio η can be stated as follows, dependent on the speed of travel v [m/s], the wave length of the permanent way λ [m] and the differential settlement Δs [mm]:

$$\eta = \frac{v}{\lambda}\,. \tag{23}$$

Regarding the critical frequency ratio, it appears that the critical wave length of the permanent way's waviness λ_{krit} [m] corresponds to 19 % of the travel speed v [km/h]. With a travel speed of 80/120/160 km/h, the critical wave length of the permanent way therefore amounts to 15/22/30 m.

Knowing the maximum dynamic factor, the maximum trackway load is also known. It depends solely on the long-term time-dependent differential settlement of the permanent way. The place-dependence of the differential settlement does not influence the maximum trackway load.

Considering (21) and (7), the maximum dynamic factor can finally be expressed dependent on the permanent way's lifetime T [yrs] and the variance of the substructure Q:

$$f_{D\max} = 1 + 0{,}006\big(5Q\ln(5T+1)\big)\,. \tag{24}$$

3. Service life of the permanent way

As a result of the famous AASHO – Street test, the 4th power rule was derived. This rule implies that the service life of a permanent way L [yrs] behaves inversely proportional to its load. In the present contemplation, the trackway load is expressed with the aid of the dynamic factor. The service life of a perfectly even permanent way (dynamic factor 1) amounts to L_0, for dynamic factors >1 the service life will decrease to $L < L_0$. With regard to the 4th power rule, an average maximum dynamic factor $\overline{f}_D$ can be stated as:

$$\left(\frac{L}{L_0}\right) = \frac{1}{\overline{f}^{\,4}_{D\max}}\,. \tag{25}$$

The average maximum dynamic factor must consider the rise of the current maximum dynamic factor that increases with the time according to (23). Therefore, the average maximum dynamic factor is interpreted as the arithmetic mean of the current maximum dynamic factors over the whole service life of the permanent way:

$$\bar{f}_{D\max} = \frac{\sum_{T=0}^{L}\left[1+0,006\left(5Q\ln(i+1)\right)\right]}{L}. \quad (26)$$

L is the service life of the permanent way [yrs], T is the time since the commissioning of this trackway.

With regard to (24) and (25), the relative service life can be successively estimated as a function of the variance of the substructure. It can be replaced by a function of best fits with the form (fig. 5):

$$\left(\frac{L}{L_0}\right) = 0,1Q^2 - 0,45Q + 1. \quad (27)$$

With equation (26) a relation has been found which describes the relative service life of a permanent way solely by the variance of its substructure.

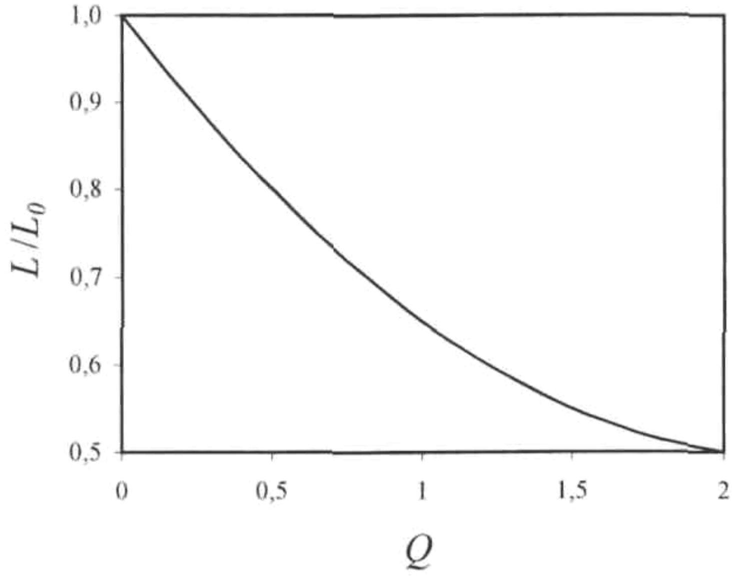

Fig. 5. In consideration of the 4th power rule, the relative service life L/L_0 of the permanent way can be described as a function of the variance of the substructure Q

4. Example

A practical example will clarify the influence of the variance of the substructure on the service life.

The quality of the substructure is usually judged by an experimental determination of its deformation modulus E in certain intervals. According to (6), the substructures quality is now quantified as the variance of the substructure. The variance of the substructure Q_i in the section i corresponds to the quotient of the difference and the mean value of two neighboring deformation modules E_i und E_{i+1} at the place x_i:

$$Q_i = \frac{\left|E_i - E_{i+1}\right|}{\left(\frac{E_i + E_{i+1}}{2}\right)}. \quad (27)$$

The influence of the substructure's quality on the service life of the permanent way is described by means of two track sections of the line Opole - Wroclaw. Both the track section between kilometer 114–115 [3, S. 61, Rys. 5) before the modernization of the substructure and the track section between kilometer 142–146 [3, S. 63, Tab. 1) after the modernization of this section are inspected. The result of the modernization of the substructure is shown in fig. 6.

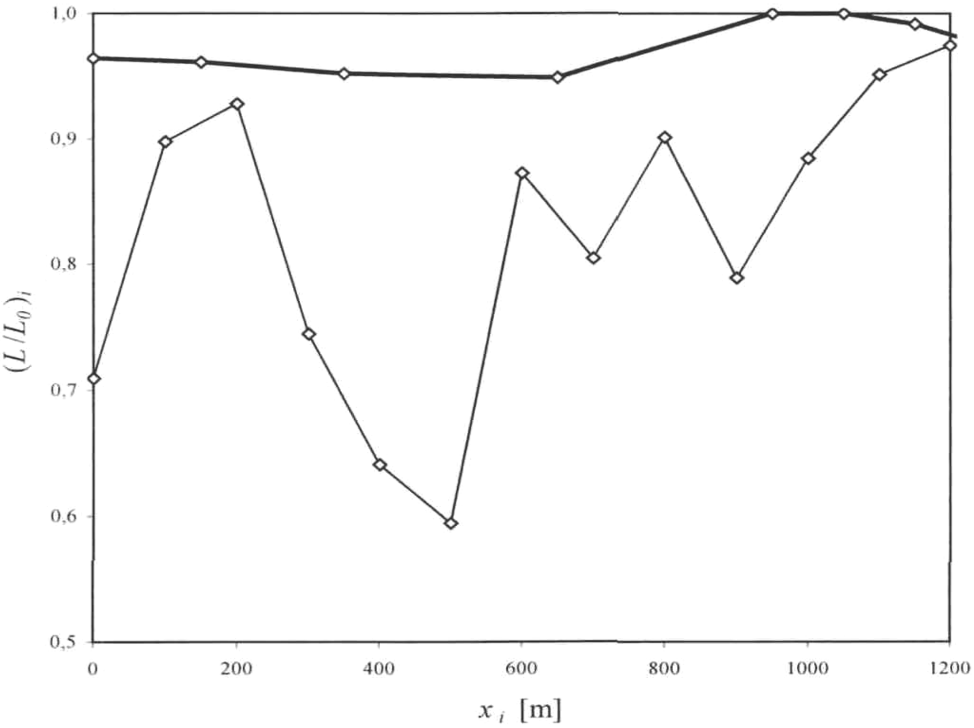

Fig. 6. The relative service life of the permanent way on old substructure (lower line) and on modernized substructure (upper line) along a track section of the line Opole–Wroclaw

Summary

The substructures homogeneity causes a waviness of the permanent way. When traveling on an uneven surface, the inertia of the vehicles produces an additional load that reduces the service life of the trackway.

The parameter for the inhomogeneity of the substructure was described in this paper. The practical calculation example was done for the line Opole – Wroclaw to show the results of the calculation.

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