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RESEARCH ON POSSIBILITIES TO IMPROVE DIESEL LOCOMOTIVES' MAINTENANCE SYSTEM

Наведені принципи аналізу процесів зносу, а також складання функцій виробки функцій вузлів і агрегатів на основі статистичних даних. Описані види показників надійності та методів їх визначення по пробігу між суміжними відмовами. Наведений алгоритм визначення оптимального пробігу по параметричній і непараметричній надійності. Встановлено, що інтенсивність зносу агрегатів та вузлів тепловозів у великій мірі залежить від умов їх експлуатації і стану залізниці, від геометричних відхилень і нерівностей рейок. Застосування наведеної в статті методики дає можливість різко знизити витрати на технічне обслуговування і ремонт тепловозів, а також підвищити їх експлуатаційну надійність..

Приведены принципы анализа процессов изнашивания, а также составления функций выработки узлов и агрегатов на основе статистических данных. Описаны виды показателей надежности и методов их определения по пробегу между смежными отказами. Приведен алгоритм определения оптимального пробега по параметрической и непараметрической надежности. Установлено, что интенсивность изнашивания агрегатов и узлов тепловозов в большой степени зависит от условий их эксплуатации и состоянии железной дороги, от геометрических отклонений и неровностей рельсов... Применение изложенной в статье методики дает возможность резко снизить расходы на техническое обслуживание и ремонт тепловозов, а также повысить их эксплуатационную надежность.

The analysis of failures of the locomotive's units and parts is presented. The reliability indicators of these parts were determined according to the locomotive run. Numerical and distribution functions describing mileage (run) of the locomotives between overhauls were also defined. The dependence of the flow of failures on mileage distribution between overhauls was determined and the optimal mileage was calculated. Methods of determining mileage based on parametric reliability are presented. These methods help to reduce maintenance and repair costs as well as idle time of locomotives. They also help to increase reliability. The methods suggested in the paper were used in calculating optimal mileage (run) for the locomotives of the company 'Lithuanian Railways'.

The efficient use of traction rolling stock helps to increase the efficiency of transportation and save time, materials, financial resources, etc. Therefore, new ways and methods of increasing the efficiency of rolling stock maintenance and operation should be sought. It is well known that reliability is the main factor determining effective rolling stock operation. We know by experience that the effective use of locomotives is based on the system of scheduled repairs and preventive maintenance because timely repairs and maintenance can ensure reliable rolling stock operation

Now, the efficiency of rolling stock operation is usually assessed by average (mean) values. This method is well suited to our case because operating conditions of locomotives are similar all over the country. The main problem is to determine properly the distance run between overhauls which would help decrease the volume as well as labour input (man / hours) and costs of repairs. This is particularly important for a joint-stock company «Lietuvos geleħinkeliai (Lithuanian railways)» because so far factory repairs of locomotives have been done abroad and their cost, but not quality, has been constantly growing [1, 2]. The present paper considers methods of increasing the efficiency of scheduled repairs and preventive maintenance in Vilnius locomotive depot by investigating the operation of diesel locomotives. A feasibility study aimed to provide sound grounds to repairs currently performed in the depot according to the specified distance run between overhauls was carried out.

Deterioration analysis

The control parameter of a deteriorating part is a continuous random variable. The law of its distribution can be described in terms of distribution density of the parameter [3, 4].

The distribution law is chosen taking into account the analysis of physical processes taking place when parts and units are aging or deteriorating. The serviceability of the parts is determined based on the control parameter chosen. The types of distribution commonly used to sufficiently accurately describe random variables and serviceability of technical devices include normal, exponential, logarithmic and Weibull distribution.

Theoretical and practical analysis [1, 2, 3, 4] shows that random variable of the control parameter

for the fixed run can be adequately described by normal distribution law.

If a random variable is affected by a large number of equally important random factors, then, the distribution of such variables complies with the normal distribution law.

The deterioration of the locomotive parts depends on many random factors, such as the material of which the part is made and its chemical composition; physical properties and quality of the manufactured part; strength characteristics, climatic conditions of the locomotive operation (e.g. temperature, atmospheric pressure, air humidity and dust content; loading modes and their rate, number of starting and braking operations); the time of maximum loading; contamination of rubbing surfaces with abrasive materials (e.g. sand sprinkled between the drive wheels and the rails); the condition of the railroad on which the loading on the locomotive and its intensity depend, and many other factors. It is hardly possible to determine which of the above factors is most important for the process of deterioration of the parts. Under particular conditions, these factors can have actually the same influence on the deterioration of the parts; therefore, the values of the control parameters follow the normal distribution law for the case of fixed mileage. The distribution density will be described as follows:

$$f(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \cdot e^{\frac{(x-m_x)^2}{2\sigma_x^2}}; \qquad (1)$$

where x is the control parameter variable; m_x is mathematical expectation of the control parameter; σ_x is mean square deviation of the control parameter.

The total number of control parameters obtained for the parts with a short operating period after repair can be adequately described by a part of the normal distribution law.

Calculation of the numerical characteristics and distribution functions of mileage between repairs

For predicting the service life or deterioration rate of the considered parts and units the analytical relationship between the numerical characteristics m_x and σ_x and mileage should be established. In general, this relationship may be considered as a non-linear function $y = (a_1, a_2, ..., a_s, l_i)$ of one l_i argument including *S* parameters $a_1, a_2, ..., a_s...$ This function can be approximated by the empirical regression presented in the form of the points (l_i, y_i) i = 1, 2, 3, ..., n. Here, y is assumed to be a parameter of the considered distribution law.

The analysis of the major distribution laws shows that the characteristics of normal, exponential and gamma distribution can be expressed by mathematical expectation of a random value and variance, which, in turn, are the parameters of the normal distribution law.

The parameters of the function y are calculated by the least square method, the conditions of which are expressed as:

$$Z(a_{1}, a_{2}, ..., a_{s}) =$$

$$= \sum_{i=1}^{n} \left[f(a_{1}, a_{2}, ..., a_{s}, l_{i}) - y_{i} \right]^{2} \rightarrow \min. \quad (2)$$

The minimum value of the function can be determined by the method of gradients. The calculation is made by generating a recurrent sequence of values $a_{1k}, a_{2k}, ..., a_{sk}$. This method allows the empirical data to be approximated by any type of relationship. The most suitable relationship criterion is the minimum of residual variance:

$$D_{e} = \frac{l}{n-S-l} \sum_{i=1}^{n} \left[y_{i} - f\left(a_{1}, a_{2}, ..., a_{s}, l_{i}\right) \right]^{2}.$$
 (3)

Suppose, that when approximating the empirical relationship $f_1(l)$ with S_1 parameters, the residual variance is equal to D_{01} . To check up the decrease of variance, while approximating empirical data of another relationship $f_2(l)$ expressed by S_2 parameters, the hypothesis $H_0: D_{01} = D_{02}$ is made. Testing of this hypothesis makes sense if $D_{01} > D_{02}$, and it is performed based on Fisher's criterion [3]:

$$\hat{F} = \frac{D_{01}}{D_{02}} \,. \tag{4}$$

The relationships describing the deteriorating parts $m_x(l)$ and $\sigma_x(l)$ are adequately approximated by linear functions. As shown by theoretical and practical studies [2, 3], the relationships describing the wearing of the parts usually follow the linearity law. Therefore, the values of the numerical characteristics $m_x(l)$ and $\sigma_x(l)$ are adequately approximated by linear functions:

$$y = ax + b . \tag{5}$$

In this case, the conditions of least square difference will be as follows:

$$\sum_{i=1}^{n} \left[y_i - \left(al_i + b\right) \right]^2 \to \min, \tag{6}$$

where $y_i = \begin{cases} m_x, \text{ when approximating } m(l), \\ \sigma_x, \text{ when approximating } \sigma(l). \end{cases}$

Linear function coefficient a, based on least square difference, will be expressed in the following way:

$$a = r_{yl} \frac{\sigma_y}{\sigma_l}; \tag{7}$$

where r_{yl} is correlation coefficient relating to random values y and l; σ_y , σ_l is mean square deviation of y and l, respectively

$$\sigma_l = \sqrt{\frac{l}{N-l}} \sum_{1}^{N} \left(l_i - m_l \right)^2, \qquad (8)$$

where $m_l = \frac{1}{N} \sum_{i=1}^{N} l_i$ is average mileage; $m_y = \frac{1}{N} \sum_{i=1}^{N} y_i$ is mean value of y.

Regression equation coefficient b is as follows:

$$b = m_y - am_l \,. \tag{9}$$

The correlation coefficient σ_{y1} characterizes density of linear relationship between the random values *y* and *l*:

$$\sigma_{yl} = \frac{K_{yl}}{\sigma_y \sigma_l} = \frac{a_u (y_l) - m_y m_l}{\sigma_y \sigma_l}, \qquad (10)$$

where $a_u(y_l l) = \frac{1}{N} \sum_{i=1}^{N} l_i b_i$ is the moment of the second product of the random values y and l.

Calculation of confidence values

To predict deterioration, the relationships m(l) and $\sigma(l)$ are extrapolated to the area of great mileage (run), under the condition that the deterioration of the parts is within the limits of normal operation. This can be achieved by properly choosing the allowable deterioration degree of the part analysed.

Then, by substituting the mileage value l into the expressions (6, 8), we can calculate the numerical values of m(l) and $\sigma(l)$ and to plot distribution density curves. When the mileage (run) lincreases, the probability that the control parameter value will exceed the specified limit also increases. This will be considered as the failure of the part due to deterioration.

Thus, when the mileage l increases, the probability of the part failure Q also increases, while probability of break-free operation of the part P decreases correspondingly.

The mileage (run), at which the probability of break-free operation of the part is equal to the fixed value $\gamma = (1-Q) \cdot 100$ %, is referred to as gamma-percentage mileage (run) between repairs.

The value of the distribution function, when the mileage function value l is fixed, will be obtained in the following way:

$$F(l) = \int_{x_{nom}}^{x_{lei}} f(x) dx,$$
 (11)

where x_{nom} is the nominal value of the control parameter.

When the control parameter (e.g. wheelset deterioration) value is constantly increasing under the normal distribution law, breakage probability for a particular mileage l can be calculated in the following way:

$$F(l) = \frac{1}{\sigma_x(l)\sqrt{2\pi}} \times \int_{x_{lei}}^{m_x + 3\sigma_x} \exp\left\{-\frac{\left[x - m_x(l)^2\right]}{2\sigma_x^2(l)}\right\} dx .$$
(12)

If the control parameter value decreases when *l* is increased (e.g. wheelset thickness), the probability of failure is determined as follows:

$$F(l) = \frac{1}{\sigma_x(l)\sqrt{2\pi}} \times .$$

$$\times \int_{m_x - 3\sigma_x}^{x_{lei}} \exp\left\{-\frac{\left[x - m_x(l)\right]^2}{2\sigma_x^2(l)}\right\} dx . (13)$$

The integral of the formulas (12) and (13) cannot be expressed by elementary functions, therefore it is determined by numerical methods. Given the function of mileage distribution of a deteriorating part, distribution density is found as the first derivative of the function:

$$f(z)F'(l). \tag{14}$$

Statistical density distribution is described by the relationship:

$$f^{*}(l) = \frac{F(L_{i+1}) - F(L_{i})}{\Delta L}.$$
 (15)

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The main parameters of the repaired unit (part) reliability are interrelated by the integral equation as follows:

$$w(l) = f(l) + \int_{0}^{l} w(\tau) f(l-\tau) d\tau.$$
 (16)

Under the law of random value normal distribution, the analytical solution of the above integral equation is possible:

$$w(l) = \sum_{i=1}^{\infty} \left[\frac{1}{\sigma \sqrt{2\pi \cdot i}} \right] \exp \left[\frac{-(1-im)^2}{2\sigma^2 i} \right], \quad (17)$$

where *m* and σ are mathematical expectation and mean square deviation (mileage before failure) of the random value, respectively; *l* is mileage; *i* is the number of failures (from the beginning of observation).

Determining reliability indicators (criteria) based on the distance run between failures

The control parameters' values of the locomotive units and parts allowing us to determine and predict their performance cannot be measured. However, the interval between scheduled repairss should be determined for these units. This interval should be such that, if exceeded, the intensity of failures will considerably increase.

The solution of these problems based on the failure flow variation would allow us to predict the increase of failure rate with the growth of the locomotive mileage and to make a decision about the need for a scheduled repair.

Applying the system of scheduled-preventive maintenance and repairs actually means longevity (durability) test of the parts and units (N, M and L) of a locomotive, when the parts of a particular type N are observed (tested) for a particular period (mileage) L. The obtained mileage in the period between overhauls L_i is a random value because it can deviate from the specified value by ± 10 %. When the mileage increases, the locomotive (or its unit) sent for a scheduled repair will not be observed in operation from the beginning of observation. Thus, the number of the locomotives N(t) selected for testing (observation) is the mileage function, while their testing plan will be as follows:

$$[N, M, L_1, L_2, L_3, ..., L_i, ..., L_N].$$

Suppose, the *i*-th item of the part M_i will fail in the observation period L_i , when the mileage is l_{1i} , l_{2i} ,..., l_{mi} respectively.

Given the information about the failures during the operation, an empirical function can be generated in the absence of data on the applicable law of failure distribution and its numerical characteristics.

The distribution is calculated by using a reconditioning function:

$$\hat{F}(l) = \left[\sum_{i=1}^{N(l)} \frac{n_{i,l}(L_i)}{m_i, L_i}\right] / N(l);$$
(18)

where $n_{i,l}(L_i)$ is the number of the *i*-th item reconditioning (repairs) of the observed part at the mileage L;

i = 1, 2, 3, ..., N(l)); m_i, L_i denote the total reconditioning (repairs) number of the *i*-th item in the period L_i .

Parts and units can work without failure over the whole observation period. In this case, $n_{i,l}(L_i)/m_i, L_i = 0$. By differentiating the empirical distribution function, distribution density functions in the period between failures are calculated:

$$\hat{f}(l) = \frac{\hat{F}(l+\Delta l) - \hat{F}(l)}{\Delta l}.$$
(19)

The parameter of the flow of failures w(l) is related to the mileage (run) density distribution between failures f(l) by the integral equation (16). The analysis of this process shows that the information about the mileage of the unit operation between failures is discontinued on the left side and broken several times on the right side. On the left-hand side, it can be accounted for by the lack of information about the unit failure in the time from the beginning of its consideration to the beginning of its observation. On the right-hand side, the reason is a scheduled repair of the locomotive or writing it off (then, the observation is discontinued). Therefore, reconditioning of each item over a short observation period will not be expressed completely, but rather its particular part will be shown because, at the beginning of the considered period, units (parts) will have different service life (run). By superimposing their reconditioning processes one over another, we will get a generalized reconditioning process characterizing the whole observation period between scheduled repairs.

If the park of locomotives in operation is rather large (>100 items.), a representative sample for determining their reliability and, consequently, reliable indicator of their failure-free operation can be obtained over short observation periods. This may be achieved based on the parameter of the failure flow, calculated by grouping the data on mileage between failures:

$$\hat{w}(l) = \frac{\Delta m}{\sum_{i=1}^{N(l)} \Delta l_i},$$
(20)

where Δm is the number of mechanism failures in the interval Δl ;

N(l) – number of the observed aggregates in the interval Δl ;

 Δl_i – the *i*-th aggregate mileage in the interval Δl ($\Delta l_i \leq \Delta l$).

To determine the number of unit failures in the total mileage grouping intervals, the data on reconditioning of the same type of units (parts) should be provided in terms of run in the considered period between overhauls.

For this purpose, the previous repair moments, which are the starting points of counting the mileage of a particular unit, are conjugated. When the information is provided in this form and the time between overhauls is divided into the intervals, the total mileage of a unit and its failures in every interval are determined. This, in turn, allows us to calculate and draw the failure flow parameter diagram. At the same time, grouping of data may considerably decrease the reliability of the obtained indicators compared to those obtained for directly calculated distribution functions of the mileage before failure. The shift of the values of the failure-free operation indicators will be much greater, when the amount of statistical data is small, and this is typical of highly reliable units.

The empirical failure function can be calculated based on the data collected over short periods of observing the operation of the unit, when the labour input (mileage) at a particular moment of time is different for different units (locomotives) from the beginning of the period analysed:

$$\hat{F}(l) = \left[\sum_{i=1}^{N(l)} \frac{n_{i,l}(L_i)}{m_{i,l}}\right] / N(l), \qquad (21)$$

where L_i is period of collecting the information about the mileage until the *i*-th unit failure.

$$L_{i} = \begin{cases} L_{i} - l_{pri} & \text{when } l_{pri} > 0; \ l_{pi} = L_{i}; \\ l_{pi} & \text{when } l_{pri} = 0; \ l_{pi} \leq L_{i}; \\ l_{pi} - l_{pri} & \text{when } l_{pri} > 0; \ l_{pi} < L_{i} \end{cases}$$

 l_{pri} is labour input (mileage) at the beginning of observation;

 l_{pi} is labour input (mileage) at the end of ob-

servation.

By classifying the data in this way, the information about unit reliability is most effectively used for calculating the distribution function because the data are not grouped. The data on failures obtained in long-term observation may be used for analysing reliability indicators over long operation periods.

By solving the integral equation, the dependence on the parameter of flow of failures may be calculated based on empirical distribution functions. The parameter of flow of failures can also be calculated by grouping the failures in the mileage intervals, e.g. $\Delta l = 50.000$ km. The parameters of flow of failures calculated in two ways show satisfactory agreement [2].

6. Calculating the optimal mileage between repairs

To avoid the locomotive failures due to deterioration of its parts, a system of scheduled-preventive repairs is used. According to it, parts and units should be either reconditioned or replaced if operational parameters approach the admissible limiting values. A scheduled repair should be performed when the mileage *L* is such that the number of failures expressed by the increase of the failure flow parameter m_1 will be larger than the number of failures m_2 in running in over the same period.

To optimize the mileage between repairs as an efficiency function, the parameter of flow of failures as a function of failure-free operation indicator can be used:

$$g(l) = \frac{1}{L} \left[C_n \int_{o}^{L} w(l) dl + C_p \right].$$
 (22)

Optimal mileage between repairs largely depends on the relationship between the costs of scheduled (C_p) and unscheduled (C_n) repairs. The costs of scheduled repairs (C_p) consist of the costs of materials or spare parts (C_1) as well as the costs of labour (C_2) and losses due to the locomotive idle time (C_3) :

$$C_p = C_1 + C_2 + C_3. \tag{23}$$

The costs of unscheduled repairs, beside the above costs (C_1, C_2, C_3) , also include losses C_4 , caused by the locomotive failure on the route, i.e.:

$$C_n = C_1 + C_2 + C_3 + C_4.$$
 (24)

Then, $C_n \ge C_p$, Moreover, $C_n = C_p$ only for the

elements which, when broken, do not cause the delay of the locomotive on the route.

As far as we know, there are no accurate methods of calculating losses brought about by stopping the train now if the locomotive failed between stations. If such methods could be found, they could not properly assess the losses caused by failure C_4 .

However, by using the relative values of C_n and C_p , we can determine the mileage between the scheduled repairs L, to which minimum total costs $g(L_0)$, i.e. optimal mileage between repairs correspond.

Let us note that:

$$K = C_n / C_p \,. \tag{25}$$

Since $C_n \ge C_p$, then $K \ge 1$.

By expressing C_n as K and C_p and substituting (22), we will get:

$$g(L) = C_p \left[K \int_0^L w(l) dl + l \right] / L.$$
 (26)

According to physical meaning, the numerator of the fraction (26) represents the total costs of scheduled and unscheduled repairs. Therefore, the expression in brackets is the total number of the repairs compared, i.e. the value of sets the total number of unscheduled repairs $\int_{0}^{L} w(l) dl$ equal to the number of scheduled repairs equivalent to it costs $K \int_{0}^{L} w(l) dl$.

The relationship

$$S(L) = \left[K\int_{o}^{L} w(l)dl + l\right]/L$$
(27)

is the total relative number of repairs calculated for the unit mileage.

7. Determining the mileage based on parametric and non-parametric reliability

The repair of some units and parts of locomotives should be performed because the limiting value of the control parameter is reached in the aging element or for other reasons unconnected with the process of deterioration. For example, wheelset tyres can be changed because of deterioration or if the fit of wheelset tyre has become not so tight or firm as it should be. It is evident that determining the intervals between repairs for the parts which may have failures of m types ,requiring the same operations for their reconditioning or repair, all the failures should be taken into consideration. The probability of failure-free operation of the unit (part) in this case is determined as the probability of a complicated event, implying that none of m type failures will take place in the considered period of the run:

$$P_{\Sigma}(l) = \prod_{i=1}^{m} P_i(l)$$
, (28)

where $P_i(l)$ is probability that *i* -th type failure will not take place during the run l (probability of *i* -th type failure-free operation);

m is the number of failures, requiring the same repair or reconditioning operations.

The analysis of the data on the locomotive wheelset tyre mileage before failure shows that, when the mileage increases after ER-3 (KR), the value of failure flow parameter also increases because of the wheelset tyre loosening. It means that with the deterioration of the wheelset tyre the number of non-parametric failures also increases. In the case of wheelset tyres, it can be explained by a decrease in their thickness and mass due to deterioration, leading to heating up of wheelset tyres in braking, which facilitates slippage and, therefore, increases the rate of non-parametric failures.

Thus, locomotives can experience two types of failures in operation:

1) parametric failures, caused by deterioration of the locomotive parts, when the control parameters exceed the specified limits;

2) non-parametric failures, including loosening of the fix, breaking, etc. These failures cannot be avoided in operating conditions, but their rate depends on the deterioration level of a part or unit.

The above failures have some common features as follows: each part (or unit) has a control parameter which is randomly changing in operating conditions and is a function of labour input (mileage). When the parameter varies within the specified limits, no parametric failure occurs. However, the increase (or decrease) of the parameter increases the probability of non-parametric failure occurrence. Failure is characterized by a sudden change in the condition of a part or unit. In this case, probability of failure-free operation may be expressed in the following way:

$$P_{\Sigma}(l) = P_p(l) \cdot F_n(l), \qquad (29)$$

where $P_p(l) = 1 - F_p(l)$ is probability to avoid gradual failure of the part during the run (labour input); $F_n(l)$ is distribution function of mileage (run);

 $P_n(l) = 1 - F_n(l)$ is probability to avoid non-parametric failure during the run (labour input);

 $F_n(l)$ is distribution function of labour input (run) until failure occurs.

Based on the total probability of failure-free operation, distribution function $P_{\Sigma}(l)$ of mileage between failures is determined, taking into account parametric and non-parametric reliability of a unit (part). Then, in the course of numerical differentiation (15) with respect to $F_{\Sigma}(l)$, the respective density function $f_{\Sigma}(l)$ is calculated. By solving the integral equation (16) from $f_{\Sigma}(l)$, the dependence of flow of failures on labour input (run) $w_{\Sigma}(l)$, which enters the efficiency function (26), is determined.

The admissible deterioration level of a part depends on the quality of repair and operating conditions. For example, the admissible deterioration level of wheelset tyres should be such that they could be replaced when the mileage (run) allows for slightly higher probability of parametric rather than non-parametric failures, or the probability of both is the same.

The methods described in the present paper were used for developing a rational mileage structure for Lithuanian Railways locomotives' operation between overhauls, taking into account their operational conditions. Based on the current flows of failures and run between overhauls, optimal maintenance and repair volume at minimal cost was determined.

8. Conclusions

1. Deterioration rate of locomotives largely depends on operating conditions.

2. The appropriate classification and processing of the data obtained in the period of Diesel locomotive operation allows to get the reliable criteria of failure-free operation, to assess the effectiveness of measures aimed at achieving higher reliability and to determine optimal time of scheduled-preventive repairs and locomotives mileage.

3. Solving the problem of optimizing mileage between overhauls, the repair costs of parts and units of Diesel locomotives should be determined.

4. To obtain more precise and reliable results, further research should be made according to a complex programme, providing for investigation of intensity of locomotive deterioration as well as geometric railway bed parameters.

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