УДК 539.12.01

N. V. Kolomoyets, V. I. Demchik, A. V. Gulov

Oles Honchar Dnipropetrovsk National University

INTRODUCING EXTERNAL CHROMOMAGNETIC FIELDS IN THE SU(3) LATTICE GLUODYNAMICS

В SU(3) ґратковій теорії поля за допомогою «підкручених» граничних умов спільно вводяться нейтральні хромомагнітні поля, що відповідають третьому та восьмому генераторам групи. У даній роботі вони беруться спрямованними уздовж одного напрямку. Розглядається ґратка $N_s^3 \times N_t$ з гіпертороідальною топологією. Постійне хромомагнітне поле спрямоване вздовж осі z. На ґратці воно вводиться за допомогою «підкручених» граничних умов, які визначаються SU(3)-матрицею Ω. Ця матриця містить потоки зовнішніх полів. Розгляд обмежено випадком, коли Ω генерується тільки третьою та восьмою матрицями Гелл-Манна. Метою цієї роботи є знаходження матриці перетворення Ω та встановлення перетворення граткових змінних U_μ під дією «підкручених» граничних умов. Поля U_μ інваріантні відносно елементів центру групи SU(3). У ґратковому моделюванні найчастіше використовується одноплакеточна дія Вільсона. Дія Вільсона є інваріантною відносно центру групи SU(3) внаслідок інваріантності полів відносно центру групи. У загальному випадку, в присутності зовнішнього поля дія не буде калібрувально інваріантною, тому що зовнішнє поле призводить до появи виділеного напрямку. В роботі отримана матриця перетворення «підкручених» граничних умов і знайдено перетворення калібрувального поля на ґратці в присутності зовнішніх хромомагнітних полів.

Ключові слова: ґраткова калібрувальна теорія поля, хромомагнітне поле, SU(3)-глюодинаміка.

В SU(3) решёточной теории поля с помощью «подкрученных» граничных условий совместно вводятся нейтральные хромомагнитные поля, соответствующие третьему и восьмому генераторам группы. В данной работе они берутся сонаправленными. Рассматривается решётка $N_s^3 \times N_t$ с гипертороидальной топологией. Постоянное хромомагнитное поле направлено вдоль оси z. На решётке оно вводится через «подкрученные» граничные условия, определяемые SU(3)-матрицей Ω. Эта матрица содержит потоки внешних полей. Рассмотрение ограничено случаем, когда Ω генерируется только третьей и восьмой матрицами Гелл-Манна. Целью этой работы является найти матрицу преобразования Ω и показать, как преобразуются решеточные переменные U_μ под действием «подкрученных» граничных условий. Поля U_μ инвариантны относительно элементов центра группы SU(3). В решеточном моделировании чаще всего используется одноплакеточное действие Вильсона. Действие Вильсона является инвариантным относительно центра группы SU(3) вследствие инвариантности полей относительно центра группы. В общем случае, в присутствии внешнего поля действие не будет калибровочно инвариантно, так как внешнее поле приводит к появлению выделенного направления. В работе получена матрица преобразования «подкрученных» граничных условий и найдено преобразование калибровочного поля на решетке в присутствии внешних хромомагнитных полей.

Ключевые слова: решёточная калибровочная теория поля, хромомагнитное поле, SU(3)-глюодинамика.

External neutral chromomagnetic fields, which correspond to the third and eight gauge group generators, are introduced in pure SU(3) lattice gauge theory through the twisted boundary conditions. In this paper they are co-directed. A $N_s^3 \times N_t$ lattice with the hypertorus topology is considered. A constant chromomagnetic field is directed along the z-axis. On the lattice it is introduced through the twisted boundary conditions with SU(3)-matrix Ω. This matrix contains the external field flux. In this paper we consider the case, when Ω is generated by the third and eight Gell-Mann matrices, only. The aim of this work is to find this matrix Ω and show how the field variables U_μ are transformed under the twisted boundary conditions. Fields U_μ are invariant under the center elements of the group SU(3). In this work the one-plaquette Wilson action is considered. Due to invariance of fields under the center of the SU(3) group, action is invariant under this center too. In general, action isn't gauge invariant

in presence of external field, because external field brings a preferred direction. In this paper the matrix of twisted boundary conditions transformation is obtained. It is shown how the lattice gauge field is transformed in the presence of external chromomagnetic fields.

Key words: lattice gauge theory, chromomagnetic field, SU(3)-gluodynamics.

Introduction

Nowadays quantum phenomena in external fields are intensively studied in the literature (see [3], [5] and references therein). It is established that external magnetic fields may strongly affect the dynamics of QCD phase transitions. In particular, they could decrease the temperature of confinement-deconfinement phase transition. For example, in [1] the effects of chromomagnetic field corresponding to the third Gell-Mann matrix was considered. Chromomagnetic fields also lead to new phenomena such as the chiral magnetic effect [3], etc.

Another important problem related to external fields is the origin of cosmic magnetic fields. In [6] a possible mechanism of magnetic fields creation was proposed – the spontaneous generation of magnetic fields at high temperature. It was found analytically, that in the SU(3) gauge theory the chromomagnetic field corresponding to the third and eight Gell-Mann matrices can be generated spontaneously with strengths $gH \propto g^4T^2$ [7]. In SU(2) gauge theory the similar research has carried out both analytically [6] and on the lattice [2]. However, non-perturbative analysis of simultaneous generation of the chromomagnetic fields (H_3 and H_8) in SU(3) gauge theory has not been performed yet, as well as the temperature dependence of the chromomagnetic field strengths has not been investigated.

In this paper two neutral external chromomagnetic fields corresponding to the third and eight Gell-Mann matrices are introduced on the lattice. These two matrices are taken due to the diagonality, consequently the corresponding gauge transformation does not imply mixing with the color charges. We assume that these two fields are spatially co-directed.

Twisted boundary conditions

External chromomagnetic fields cannot be introduced on a lattice by setting specific initial configuration. They cannot be also an additive term to field values generated randomly. Hence, external fields must be introduced as parameters, which are forcibly controlled in simulations. They can influence configurations on the lattice, but the Monte Carlo procedure must not change them. Minimizing the thermodynamical potential with respect to the external field the spontaneously generated fields can be found.

There are different ways to introduce external field on a lattice. For example, in [1] it was introduced by fixing link variables at the time coordinate t equal 0.

Another way to introduce external chromomagnetic field on a lattice is through the twisted boundary conditions (t.b.c.) [8]. This method was applied in [2] for the SU(2)-gluodynamics; in this work the external field was introduced through t.b.c. The t.b.c. seem to be convenient to research the problem of spontaneous generation of chromomagnetic field in the SU(3) theory.

Let us consider a $N_s^3 \times N_t$ lattice with the hypertorus topology, N_s is the number of the sites in spatial directions and N_t is the number of sites in the temporal direction. The ordinary periodic boundary conditions on this lattice are (Fig.1, a)

$$\begin{aligned} U_\mu(N_s, n_y, n_z, n_t) &= U_\mu(0, n_y, n_z, n_t) \\ U_\mu(n_x, N_s, n_z, n_t) &= U_\mu(n_x, 0, n_z, n_t) \\ U_\mu(n_x, n_y, N_s, n_t) &= U_\mu(n_x, n_y, 0, n_t) \\ U_\mu(n_x, n_y, n_z, N_s) &= U_\mu(n_x, n_y, n_z, 0), \end{aligned} \tag{1}$$

U_μ expresses the gauge field on the lattice,

$$U_\mu(x) = e^{iA_\mu(x)a}, \tag{2}$$

g is the coupling constant, $A_\mu(x)$ is the gauge field potential in the continuum theory, a is the lattice spacing.

Let us introduce a constant field directed along the z-axis. In continuum case the potential $A_\mu = (0, Hx, 0, 0)$ corresponds to the constant magnetic field $\vec{H} = (0,0,H)$. Such a field obeys the following boundary conditions (Fig.1, b):

$$\begin{aligned} U_\mu(N_s, n_y, n_z, n_t) &= U_\mu(0, n_y, n_z, n_t) \\ U_\mu(n_x, N_s, n_z, n_t) &= \Omega U_\mu(n_x, 0, n_z, n_t)\Omega^+ \\ U_\mu(n_x, n_y, N_s, n_t) &= U_\mu(n_x, n_y, 0, n_t) \\ U_\mu(n_x, n_y, n_z, N_s) &= U_\mu(n_x, n_y, n_z, 0), \end{aligned} \tag{3}$$

where Ω is the SU(3)-matrix, containing the external field flux, the cross denotes the Hermitian conjugation. Eq. (3) sets the t.b.c. on a lattice. Generally, Ω is an arbitrary SU(3)-matrix. In this paper we consider the case, when Ω is generated by the third and eight Gell-Mann matrices, only. The aim of this work is to find this matrix Ω and show how the matrix U_μ is transformed under the t.b.c.

To use these boundary conditions all fields must be invariant under the center elements of the group SU(3) [8]. Fields U_μ are invariant under it. In this work the Wilson action [4]

$$S_W = \frac{\beta}{3} \sum_{n\in\Lambda} \sum_{\mu<\nu} \mathrm{Re}\, Tr\left(I - U_{\mu\nu}(n)\right) \tag{4}$$

is considered. In this equation $\beta = 6/g^2$ is the inverse coupling, $U_{\mu\nu}(n) = U_\mu(n)U_\nu(n+\hat{\mu})U_\mu^+(n+\hat{\nu})U_\nu^+(n)$ is a plaquette variable, $\hat{\mu}$ is the unit vector in μ-direction, I is the 3×3 unit matrix; summation is over all sites of a lattice Λ and over all directions. Due to invariance of fields under the center of the SU(3) group, action (4) is invariant under this center too. In general, action (4) isn't gauge invariant in presence of external field, because external field brings a marked direction.

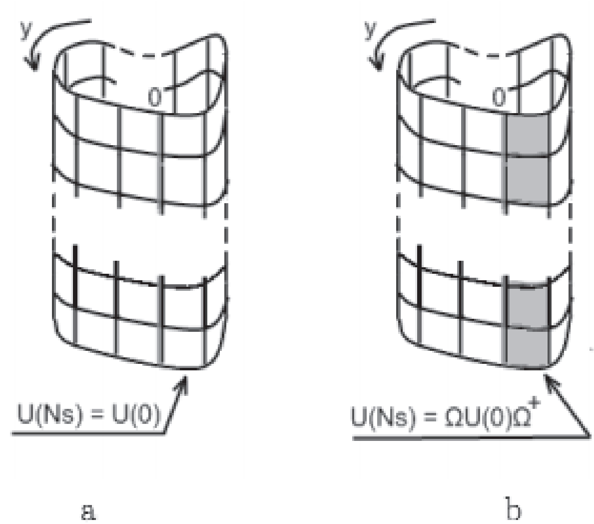

Fig. 1. The common periodic boundary conditions (a) and the t.b.c. (b) in y-direction. For convenience the 2-dimensional section of 4-dimensional lattice is shown. In all directions except for the y-direction the ordinary periodic boundary conditions are always assumed

Since Ω is the SU(3)-matrix corresponding to third and eight SU(3)-generators, it is defined as follows

$$\Omega = e^{i(\phi\lambda_3 + \omega\lambda_8)/2}. \tag{5}$$

The third and eight Gell-Mann matrices have the form

$$\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{6}$$

Consequently,

$$\Omega = \begin{pmatrix} e^{i(\omega+\phi)/2} & 0 & 0 \\ 0 & e^{i(\omega-\phi)} & 0 \\ 0 & 0 & e^{-i\omega} \end{pmatrix}, \tag{7}$$

where the factor $1/\sqrt{3}$ is included in the ω. From [1] it may be seen, that quantities ϕ and ω are the fluxes of fields through a plaquette.

According to Eq. (3), in the points $n_y = N_s$ (the edge links) the lattice gauge field is transformed as

$$U'_\mu = \Omega U_\mu \Omega^+. \tag{8}$$

SU(3) matrices U_μ are parameterized in the form [4]:

$$U_\mu = \begin{pmatrix} \vec{u} \\ \vec{v} \\ \vec{w} \end{pmatrix} = \begin{pmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{pmatrix}, \tag{9}$$

where rows are the components of complex vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$;

$$\vec{w} = \vec{u}^* \times \vec{v}^*, \tag{10}$$

star symbol denotes the complex conjugation. It is seen that there are only twelve independent real components, which form the $\vec{u}$ and $\vec{v}$ vectors, the third matrix row is completely restored according to Eq. (10) by first two rows. The U'_μ matrix has the same form as U_μ matrix. Its components will be denoted by prime symbols.

Multiplying matrices in (8) one obtains

$$U'_\mu = \begin{pmatrix} u_1 & u_2 e^{i\phi} & u_3 e^{i(\phi+3\omega)/2} \\ v_1 e^{-i\phi} & v_2 & v_3 e^{-i(\phi-3\omega)/2} \\ w_1 e^{-i(\phi+3\omega)/2} & w_2 e^{i(\phi-3\omega)/2} & w_3 \end{pmatrix}, \tag{11}$$

or, component-wise,

$\mathrm{Re}\ u'_1 = \mathrm{Re}\ u_1$; $\quad \mathrm{Im}\ u'_1 = \mathrm{Im}\ u_1$;
$\mathrm{Re}\ u'_2 = \mathrm{Re}\ u_2 \cos\phi - \mathrm{Im}\ u_2 \sin\phi$; $\quad \mathrm{Im}\ u'_2 = \mathrm{Im} u_2 \cos\phi + \mathrm{Re} u_2 \sin\phi$;
$\mathrm{Re}\ u'_3 = \mathrm{Re}\ u_3 \cos\xi - \mathrm{Im}\ u_3 \sin\xi$; $\quad \mathrm{Im}\ u'_3 = \mathrm{Im}\ u_3 \cos\xi - \mathrm{Re} u_3 \sin\xi$;

$\mathrm{Re}\ v'_1 = \mathrm{Re}\ v_1 \cos\phi + \mathrm{Im}\ v_1 \sin\phi$; $\quad \mathrm{Im}\ v'_1 = \mathrm{Im} v_1 \cos\phi - \mathrm{Re} v_1 \sin\phi$;
$\mathrm{Re}\ v'_2 = \mathrm{Re}\ v_2$; $\quad \mathrm{Im}\ v'_2 = \mathrm{Im}\ v_2$;
$\mathrm{Re}\ v'_3 = \mathrm{Re}\ v_3 \cos\eta + \mathrm{Im}\ v_3 \sin\eta$; $\quad \mathrm{Im}\ v'_3 = \mathrm{Im} v_3 \cos\eta - \mathrm{Re} v_3 \sin\eta$,

where $\xi = \dfrac{\phi + 3\omega}{2}$, $\eta = \dfrac{\phi - 3\omega}{2}$.

In this form the t.b.c. can be introduced into computational software, which performs lattice simulations.

The fields ϕ and ω are related to the continuous fields H_3 and H_8 by the following relations [1]:

$$a^2 gH_3 = \phi; \qquad a^2 gH_8 = \frac{\omega}{\sqrt{3}}. \tag{12}$$

These relations should be used to obtain the physical values of corresponding fluxes on a lattice.

Conclusions

In this paper the t.b.c. in the lattice SU(3)-gluodynamics were introduced: the gauge transformation matrix which gives the t.b.c. is calculated and it is shown, how the lattice field U_μ is transformed by the t.b.c. All the physical values as, for example, Wilson action (4), are periodic by fluxes ϕ and ω. In general case the period by every argument is 4π. In the slice $\omega = 0$ the period is 2π, whereas on the slice $\phi = 0$ it is $4\pi/3$. The action $S_W(\phi,\omega)$ forms a two-dimensional surface.

As was mentioned above, action $S_W(\phi,\omega)$ is invariant under the center of the SU(3) group. This center consists of three matrices: I, $diag\left(e^{2\pi i/3}, e^{2\pi i/3}, e^{-4\pi i/3}\right)$ and $diag\left(e^{4\pi i/3}, e^{4\pi i/3}, e^{-2\pi i/3}\right)$. The following values of fields are correspond to them: $\omega = 2\pi k$, $\phi = 2\pi(2l-k)$ correspond to the identity matrix, $\omega = 4\pi/3 + 2\pi k$, $\phi = 2\pi(2l-k)$ to the first of the diagonal matrices and $\omega = 2\pi/3 + 2\pi k$, $\phi = 2\pi(2l-k+1)$ to the second one, k and l are integer.

The obtained results can be used in searching for the spontaneous generation of chromomagnetic fields, calculation of magnetic mass in presence of external chromomagnetic field, and also in investigation of fermion dynamics in the external chromomagnetic field background.

Bibliography

1. **Cea P.** Probing Confinement with Chromomagnetic Fields / P. Cea, L. Cosmai // Nuclear Physics - Proceedings Supplements. B. 119. – 2003. – P. 700–702.
2. **Demchik V.** The Spontaneous Generation of Magnetic Fields at High Temperature in SU(2)-gluodynamics on a Lattice / V. Demchik, V. Skalozub// arXiv: hep-lat/0601035v2. – P. 1–10.
3. **Fukushima K.** The Chiral Magnetic Effect /D. E. Kharzeev, H. J. Warringa // Physical Review D. 78. –2008. – P. 074033–074046.
4. **Gattringer C.** Quantum Chromodynamics on the Lattice. An Introductory Presentation / C. C. Gattringer, B. Lang // Quantum Chromodynamics on the Lattice. An Introductory Presentation: Berlin Heidelberg.: Springer. –2010. – 343 p.
5. **Mizher A. J.** Phase diagram of hot QCD in an external magnetic field: possible splitting of deconfinement and chiral transitions / A. J. Mizher, M. N. Chernodub, E. S. Fraga // Physical Review D. 82. – 2010. – P. 105016–105031.
6. **Skalozub V.** Once More on a Colour Ferromagnetic Vacuum State at Finite Temperature / V. Skalozub, M. Bordag // Nuclear Physics. – B 576. – 2000. – P. –430–444.
7. **Skalozub V.V.** On generation of Abelian magnetic fields in SU(3) gluodynamics at high temperature / V. Skalozub, A. V. Strelchenko // The European Physical Journal C. – 2004. – V. 33, № 1. – P. 105–112.
8. **’t Hooft G.** A Property of Electric and Magnetic Flux in Non-Abelian Gauge Theories // Nuclear Pysics. – B 153. – 1979. – P. 141–160.

Надійшла до редколегії 13.07.2012.