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SPATIAL STRUCTURE OF THE POLYAKOV LOOP IN EXTERNAL CHROMOMAGNETIC FIELD IN LATTICE SU(2) GLUODYNAMICS

Spatial distribution of Polyakov's loop in 3+1 dimensional SU(2) lattice gauge field theory is investigated in the presence of constant external Abelian chromomagnetic field H at finite temperature. The external field corresponds to the third group generator and is directed opposite x axis. Monte-Carlo simulations are performed on the 2×16^3 lattice at $\beta=3$, and various values of flux of the external field. The flux on the lattice is introduced through the so-called "twisted" boundary conditions. These conditions are the modification of the standard periodic boundary conditions and allow introducing an additional flux of the external field. The computations are performed with graphic processing units, the computer program is written in C++ language using OpenCL. It is discovered that in the presence of the external field the Polyakov loop has a non-trivial periodic spatial structure that is in contrast to a rather uniform distribution in the field absence.

Keywords: lattice gauge field theory, chromomagnetic field, "twisted" boundary conditions, Polyakov loop, SU(2) gluodynamics.

В 3+1 SU(2) решёточной калибровочной теории поля при конечной температуре изучается пространственное распределение петли Полякова в присутствии постоянного внешнего абелева хромомагнитного поля H, соответствующего третьему генератору группы и направленного против оси x. Монте-Карло моделирование проводилось на решётке 2×16^3 при $\beta=3$ и при разных значениях потока внешнего поля, который введен на решётку с помощью так называемых «подкрученных» граничных условий. Эти условия являются модификацией стандартных периодических граничных условий, позволяющей ввести на решётку дополнительный поток внешнего поля. Вычисления производились с использованием видеокарт, компьютерная программа написана на языке C++ с использованием OpenCL. Обнаружено, что при наличии внешнего поля распределение петли Полякова имеет нетривиальную периодическую пространственную структуру, в то время как в отсутствие внешнего поля её распределение скорее однородно.

Ключевые слова: решёточная калибровочная теория поля, хромомагнитное поле, «подкрученные» граничные условия, петля Полякова, SU(2)-глюодинамика.

В 3+1 SU(2) ґратковій калібрувальній теорії поля при скінченній температурі вивчається просторовий розподіл петлі Полякова при наявності постійного зовнішнього абелевого хромомагнітного поля H, відповідного третьому генератору групи і спрямованого проти осі x. Монте-Карло моделювання проводилося на ґратці 2×16^3 при $\beta=3$ і при різних значеннях потока зовнішнього поля, який був введений за допомогою так званих «підкручених» граничних умов. Ці умови є модифікацією стандартних періодичних граничних умов, яка дозволяє ввести на ґратку додатковий потік зовнішнього поля. Розрахунки проводилися за допомогою відеокарт, комп'ютерна програма написана мовою C++ з використанням OpenCL. Виявлено, що за наявності зовнішнього поля розподіл петлі Полякова має нетривіальну періодичну просторову структуру, в той час як за відсутності зовнішнього поля її розподіл скоріш однорідний.

Ключові слова: ґраткова калібрувальна теорія поля, хромомагнітне поле, «підкручені» граничні умови, петля Полякова, SU(2)-глюодинаміка.

Introduction

Interest for studying of quantum phenomena in external magnetic fields is steadily growing. This is stimulated by increasing of experimental data obtained at modern colliders of particles and astrophysics observations (see [1, 2] and Refs therein). Modern experiments demonstrate the importance of accounting for effects related to magnetic fields in different phenomena of high-energy physics. In this regard, it is reasonable to reconsider known quantum effects with taking into account the presence of magnetic fields.

One of such phenomenon is a deconfinement phase transition. The Polyakov loop is the order parameter of it in the SU(N) gauge theories. In continuum limit, it has zero value in confinement and is non-zero in deconfinement phases. The peak of Polyakov loop susceptibility considered as a function of temperature corresponds to the temperature of deconfinement phase transition [3]. The Polyakov loop is sensitive to breaking of Z(N) center subgroup of SU(N) gauge group [3, 4]. It allows for studying the quark-antiquark potential as well as other implicit parameters.

There are several basic approaches to investigate the deconfinement phenomenon. Nowadays, the most popular method is Monte-Carlo (MC) simulations on a lattice. It allows for getting numeric estimates of the quantities studied. In the present paper this method is applied. At zero external fields, the Polyakov loop properties are well investigated in the literature [3-5]. But this is not the case if the field is switched on. Even the influence of the field on the temperature of the phase transition is not settled finally [6].

As shown in literature, the value of the Polyakov loop in SU(2) gauge theory decreases with increasing of the applied external field. This means the increasing of the temperature of deconfinement phase transition with increasing the value of the strength of external field [6]. The opposite behavior is detected in the SU(3) gluodynamics.

The present paper is devoted to investigation of influence of the external Abelian chromomagnetic field on the Polyakov loop. We study the spatial distribution of the Polyakov loop for different values of field strengths at finite temperature. The values of the loop obtained from Monte Carlo simulations are averaged over the plane perpendicular to the external field direction. The distribution of this quantity along the field axis is the main object investigated.

Basic theory

Below, the standard lattice Wilson action for SU(2) lattice gauge theory

$$S_W = \frac{\beta}{2} \sum_{n \in \Lambda} \sum_{\mu<\nu} \mathrm{Re\,Tr}\left(I - U_{\mu\nu}(n)\right) \qquad (1)$$

is used. Here, $U_{\mu\nu}(n) = U_\mu(n)\, U_\nu(n+\hat{\mu})\, U_\mu^{+}(n+\hat{\nu})\, U_\nu^{+}(n)$ is a plaquette variable, $\hat{\mu}$ is the unit vector along μ direction, $\mu = \{1,2,3,4\}$, $\mu = 4$ corresponds to the Euclidian time direction, $\beta = 4/g^2$ is the inverse coupling constant, I is the 2×2 unit matrix and summation is performed over all sites of a lattice Λ and over all directions. Variable U_μ expresses the gauge field on the lattice,

$$U_\mu(x) = e^{i A_\mu(x) a}, \qquad (2)$$

where $A_\mu(x)$ is a gauge field potential in continuum theory, a is a lattice spacing.

To introduce the external field, twisted boundary conditions are used [7]. They read

$$U_\mu(n_x, n_y, N_z, n_t) = \Omega\, U_\mu(n_x, n_y, 0, n_t), \qquad (3)$$

$$\Omega = \begin{cases} diag\left(e^{i\varphi/2}, e^{-i\varphi/2}\right), \mu = 2; \\ I, \mu \neq 2, \end{cases} \tag{4}$$

N_z measures a number of lattice sites in z direction; φ is the flux of the external field. This means that the external constant chromomagnetic field is $\vec{H} = (-H,0,0)$. If $\varphi = 0$, then these boundary conditions restrict to the usual periodic ones. The connection between flux and field strength is the following

$$H = \frac{\varphi}{a^2}. \tag{5}$$

The relevant quantity, the Polyakov loop, is defined as usually [5]:

$$P(\vec{m}) = Tr\left[\prod_{j=0}^{N_t-1} U_4(\vec{m}, j)\right], \tag{6}$$

which is discretized version of its definition in continuous theory,

$$P(\vec{x}) = T\exp\left[ig\oint d\tau\, A_0(\vec{x},\tau)\right], \tag{7}$$

T denotes time ordering. The equation (6) is a trace of the ordered products of all time-directed links corresponding to the space point $\vec{m}$. It gives a closed loop due to the periodic boundary conditions in the time direction.

Simulation results

In this investigation the standard MC lattice simulations are performed. To update a lattice the multi-hit heat-bath algorithm is used (the number of hits 10 is taken). Pseudorandom numbers are produced with RANLUX3 generator.

Production of pseudorandom numbers, updates of the lattice and measurements are performed with graphics processing unit (GPU). Averaging over a configuration is also performed with GPU. Averaging over run is performed with central processing unit (CPU). Computer program is written in C++; the GPU kernels are written in Open Computing Language (Open CL). The trivial parallelization is used: all the GPU procedures are performed in parallel, but there is not parallelization between GPUs.

All calculations are carried out with double precision. The simulations are performed with GPUs of HGPU cluster based on nVidia GeForce GTX 560 Ti, AMD Radeon HD 7970 (Tahiti), HD 6970 (Cypress) and HD 5870 (Cayman).

All the simulations are performed on the 2×16^3 lattice at $\beta = 3$ and flux φ up to 0.15. In this case lattice spacing equals to a = 0.0940246 fm. The relevant quantity is Polyakov loop for every x coordinate. After 300 thermalization sweeps the measured value is obtained as an average over 500 configurations. Nine bulk sweeps are performed to decorrelate configurations used in measurements. There are up to 18500 runs performed in the presence of chromomagnetic field and up to 46000 ones in the absence of it.

Within one sigma accuracy it is obtained that the Polyakov loop in the presence of non-zero chromomagnetic field has some periodic structure. It can be seen that the field brings a decrease of the variance of the loop. Also, we observed a non-monotonic behavior of the mean value of the loop as function of H.

To investigate the shape of the distribution of the measured quantity the standard χ^2 fit method is used. Every data set is fitted by a straight line corresponding to the mean of these data, by a single sine function and by combinations of two sine functions.

To avoid edge effects in fit, the data are periodically extrapolated from both sides. In fact, three periods along x axis were used in the fit. Fit results are presented in Table 1.

The data in columns are the values of minimal χ^2 corresponding to the four values of φ investigated and to the functions tried. In the gray cells the functions used are placed:

- $f_1(x) = M$, M is the mean value of the Polyakov loop over interval of x;
- $f_2(x) = M + a\sin\left(2\pi\frac{x}{X} + x_0\right)$;
- $f_3(x) = M + a_1\sin\left(2\pi\frac{x}{X_1} + x_1\right) + a_2\sin\left(2\pi\frac{x}{X_2} + x_2\right)$;
- $f_4(x) = f_I(x)\theta(b_1 - x) + f_{II}(x)[1-\theta(b_1 - x)]\theta(b_2 - x) + f_I(x)[1-\theta(b_2 - x)]$,

  $f_I(x) = a_1\sin\left(2\pi\frac{x}{X_1} + x_1\right)$, $f_{II}(x) = a_2\sin\left(2\pi\frac{x}{X_2} + x_2\right)$,

where $\theta(x)$ is the Heaviside theta-function, $\theta(0) = 1$.

Table 1

The fit results; φ_0 = 0.000591195

Function	φ			
	0	φ_0	$16\varphi_0$	$256\varphi_0$
f_1	0.1364	11.83	20.31	13.7
f_2	0.08913	8.270	13.42	11.35
f_3	0.06628	5.836	11.13	9.729
f_4	0.07405	2.843	4.616	3.923

The last function means that the period of resulting function is divided by two intervals, and the data in each interval are described by different sine-function; b_1 and b_2 are the points of the connection of these curves. The minimal χ^2 values presented in the Table 1 correspond to the one period of data.

It can be seen from Table 1 that in the field presence the best fit function is the combination of two sine functions in different regions of data. The χ^2 corresponding to this function is in several times less than the one for the case of straight line, so such non-trivial distribution of the Polyakov loop is more preferred than the uniform one. If the external field is absent, the best fit function is superposition of the sine ones. However, for this case all fit functions give almost the same χ^2 because of a high variance, so all of them describe the data almost equally well. The data sets and the corresponding best fits are shown in Fig.1.

The space structure of the Polyakov loop may result in a chromoelectric field in the deconfinement phase. To our knowledge, this interesting phenomenon was not discussed in the literature. It requires further investigations which are out of the scope of the present paper.

Conclusions

In the present paper, a constant chromomagnetic Abelian field is introduced on the lattice through the twisted boundary conditions. The distribution of the Polyakov loop along the field direction is investigated for different values of field flux. It is observed

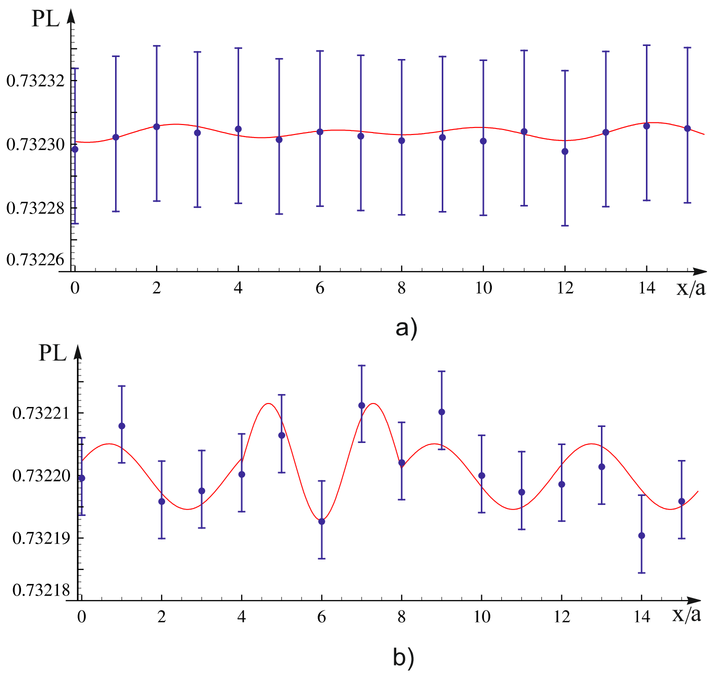

Fig. 1. Data sets for zero and non-zero fluxes and corresponding best fit curves: a) $\varphi = 0$ b) $\varphi = 256\varphi_0$. Error bars correspond to the 68% confidence intervals.

that in the field presence the Polyakov loop has a non-linear structure, within one sigma accuracy. If the external field is zero, such structure is not elucidated. The data fit shows that the distribution of the Polyakov loop is preferably described by a combination of two sine functions in two different intervals of data along the axis investigated. This observation is a signal of interesting new features of the deconfinement phase. These will be investigated separately.

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