UDC 535.14:537.8

S. F. Lyagushyn, Yu. M. Salyuk, A. I. Sokolovsky

Oles Honchar Dnipropetrovsk National University

DYNAMICS OF ELECTROMAGNETIC FIELD AND ITS CORRELATIONS IN A MEDIUM CONSISTING OF TWO-LEVEL EMITTERS

Kinetics of electromagnetic field in a medium of motionless emitters is considered on the basis of reduced description method with using as its parameters the average value of the transversal electromagnetic field, its binary correlations, and energy density of the emitter subsystem. Material equations for such a medium are obtained. Electromagnetic waves existing in it are considered. Equilibrium correlations of the field and their connection with the material equations are investigated. The concept of binary correlation waves existing in a non-uniform case is developed. The description for waves of binary correlations with using the field mode correlations and compact designations are proposed. Four types of correlation modes in the equilibrium medium of emitters are found. Two of them correspond to a damped oscillation process and other two ones correspond to decaying waves. The connection between the energy density of the emitters and field correlations is shown. Modes of the non-equilibrium medium coupled to field correlation modes are investigated near the equilibrium. The phenomenon of creation of field correlations (correlation wave emission) in the process of emitter medium evolution is predicted.

Keywords: medium of emitters, transversal electromagnetic field, correlation modes, decaying waves of correlations, nonequilibrium medium, coupled oscillations.

Рассмотрена кинетика электромагнитного поля в среде, образованной неподвижными двухуровневыми излучателями, на основе методе сокращенного описания с использованием в качестве его параметров средних значений поперечного электромагнитного поля, его бинарных корреляций и плотности энергии подсистемы излучателей. Получены материальные уравнения для такой среды. Рассмотрены электромагнитные волны в ней. Исследованы равновесные корреляции поля и их связь с материальными уравнениями. Развита концепция волн бинарных корреляций, существующих в неоднородном случае. Предложены описание волн бинарных корреляций с помощью корреляций мод поля и компактная форма записи. Найдено четыре типа мод корреляций в равновесной среде из излучателей. Два из них отвечают затухающему колебательному процессу, еще два – затухающим волнам. Показана связь плотности энергии излучателей с корреляциями поля. Изучены моды связанных колебаний неравновесной среды и корреляций поля вблизи равновесия. Предсказано явление возникновения корреляций поля (излучения волн корреляций) при эволюции среды из излучателей.

Ключевые слова: среда из излучателей, поперечное электромагнитное поле, моды корреляций, затухающие волны корреляций, неравновесная среда, связанные колебания.

Розглянуто кінетику електромагнітного поля в середовищі, яке утворене нерухомими дворівневими випромінювачами, на основі методу скороченого опису з використанням у якості його параметрів середніх значень поперечного електромагнітного поля, його бінарних кореляцій і густини енергії підсистеми випромінювачів. Отримано матеріальні рівняння для такого середовища. Розглянуто електромагнітні хвилі в ньому. Досліджено рівноважні кореляції поля та їх зв'язок із матеріальними рівняннями. Розвинуто концепцію хвиль бінарних кореляцій, що існують у неоднорідному випадку. Запропоновано опис хвиль бінарних кореляцій за допомогою кореляцій мод поля та компактну форму запису. Знайдено чотири типи мод кореляцій у рівноважному середовищі з випромінювачів. Знайдено чотири типи мод кореляцій у рівноважному середовищі з випромінювачів. Два з них відповідають згасаючому коливальному процесу, а ще два типи мод кореляції – згасаючим хвилям. Показано зв'язок густини енергії випромінювачів із кореляціями поля. Вивчено зв'язані коливання нерівноважного середовища та кореляцій поля поблизу від рівноваги. Передбачено явище виникнення кореляцій поля (випромінювання хвиль кореляцій) у процесі еволюції середовища з випромінювачів.

Ключові слова: середовище з випромінювачів, поперечне електромагнітне поле, моди кореляцій, згасаючи хвилі кореляцій, нерівноважне середовище, зв'язані коливання.

Introduction

The necessity of taking into account binary field correlations as additional independent variables together with the average field was pointed out for the first time in the paper [1]. This idea was put into life in the paper [2] where the field kinetics in the equilibrium entirely ionized plasma was investigated. In that paper the idea of field correlation waves existing was also expressed. In the paper [3] the kinetics of electromagnetic field in non-equilibrium plasma at the hydrodynamic stage of evolution was studied. In the approximation of a small correlation radius coupled oscillations of field correlations and hydrodynamic variables of plasma were investigated.

In our paper [4] the kinetics of electromagnetic field in a medium consisting of motionless two-level emitters distributed in the space with the density $n(x)$ was developed. The interaction between emitters and field was supposed to be weak (λ is its weakness parameter). The directions of dipole moments of emitters were considered to be equiprobable. Non-equilibrium field state was described by its average value $B_n(x,t) \equiv \xi_{1n}(x,t)$, $E_n(x,t) \equiv \xi_{2n}(x,t)$ and binary correlations $(\xi_{in}^x, \xi_{i'l}^{x'})_t$ defined for arbitrary local quantities $a(x)$, $b(x)$ by the general formula

$$(a^x, b^{x'})_t = \mathrm{Sp}\,\rho(t)\{\hat{a}(x), \hat{b}(x')\}/2 - \mathrm{Sp}\,\rho(t)\hat{a}(x)\mathrm{Sp}\,\rho(t)\hat{b}(x') \tag{1}$$

($\hat{a}(x)$ and $\hat{b}(x)$ are the quantity operators, $\rho(t)$ - a statistical operator of the system). A state of the non-equilibrium system of emitters was described using their energy density $\varepsilon(x,t)$.

The aim of the present paper is to investigate the dynamics of such a system near the equilibrium in terms of the average field and its correlations. Preliminary results of the investigation were presented at the conference SPMTA-2012 [5].

The proposed research is relevant to the investigation of properties of emitted electromagnetic field in a superradiant state described by the Dicke type model. The quantum properties of the field are described by a binary and more complicated correlation functions. Our correlation functions (1) are one-to-one correspondent to ones discussed in quantum optics.

Hereafter the review of the theory developed in our paper [4] is given. The development of the theory of transversal modes of electromagnetic field started in the paper [6] is continued. On such a basis equilibrium correlations of electromagnetic field are calculated. Then the investigation of field correlation modes in the equilibrium medium of emitters is expounded. The last section is devoted to the discussion of coupled electromagnetic field correlations and non-equilibrium emitter medium. Presented here theory is the further development of our research in [6].

Basic equations of the theory

We restrict ourselves with considering transversal field dynamics. Average electromagnetic field obeys the usual Maxwell equations

$$\partial_t B_n(x,t) = -c\,\mathrm{rot}_n\, E(x,t), \qquad \partial_t E_n(x,t) = c\,\mathrm{rot}_n\, B(x,t) - 4\pi J_n(x,t),$$
$$\mathrm{div}\, B(x,t) = 0, \qquad \mathrm{div}\, E(x,t) = 4\pi\rho(x,t) \tag{2}$$

where $J_n(x,t)$ and $\rho(x,t)$ are current and charge densities. The material equation connecting the current density and field with accuracy of order λ^2 takes the form

$$J_n(x,t) = \int d^3x'\sigma(x-x', \varepsilon(x,t))E_n(x',t) + c\int d^3x'\chi(x-x', \varepsilon(x,t))Z_n(x',t) \tag{3}$$

where $Z_n(x) \equiv \mathrm{rot}_n\, B(x)$.

Fourier components $\sigma_k(\varepsilon)$, $\chi_k(\varepsilon)$ of the included kernels $\sigma(x,\varepsilon)$, $\chi(x,\varepsilon)$ (they are supposed to be known functions) are non-equilibrium generalization of the conductivity and magnetic susceptibility of the medium with taking into account spatial dispersion. In the developed theory the quantities $\sigma_k(\varepsilon)$, $\chi_k(\varepsilon)$ are of λ^2 order and do not depend linearly on the energy density ε. Equilibrium values of material coefficients are given by formulas $\sigma_k = \sigma_k(\varepsilon^{eq})$, $\chi_k = \chi_k(\varepsilon^{eq})/[1-4\pi\chi_k(\varepsilon^{eq})]$ since the definition $B_{nk} = \mu_k H_{nk}$, $\mu_k = 1+4\pi\chi_k$ (μ_k is magnetic permeability) is usually applied.

The Maxwell equations can be written more compactly in the form

$$\partial_t \xi_{in}(x,t) = i\sum_{i'}\int dx' c_{in,i'l}(x-x')\xi_{i'l}(x',t) - 4\pi J_{in}(x,t) \tag{4}$$

where we use the notations: $\hat{\xi}_{1n}(x) \equiv \hat{B}_n(x)$, $\hat{\xi}_{2n}(x) \equiv \hat{E}_n(x)$, $J_{1n}(x) \equiv 0$, $J_{2n}(x) \equiv J_n(x)$. Then the material equation (3) takes the form

$$J_{in}(x,t) = \sum_{i'}\int d^3x' \sigma_{in,i'l}(x-x',\varepsilon(x))\xi_{i'l}(x',t)\,. \tag{5}$$

Non-zero matrix components $c_{ii'}(x-x')$, $\sigma_{ii'}(x-x',\varepsilon)$, according to Eqs. (2) and (3), are given by formulas for their Fourier images

$$c_{1n,2l}(k) = -ik_m e_{nml}\,, \qquad c_{2n,1l}(k) = ik_m e_{nml}\,;$$

$$\sigma_{2n,2l}(k,\varepsilon) = \sigma_k(\varepsilon), \qquad \sigma_{2n,1l}(k,\varepsilon) = ice_{nml}k_m\chi_k(\varepsilon)\,. \tag{6}$$

The temporal equation for binary correlations (they are independent variables) has the form

$$\partial_t(\xi^x_{in},\xi^{x'}_{i'l})_t = i\sum_{i''}\int d^3x' c_{in,i''m}(x-x'')(\xi^{x''}_{i''m},\xi^{x'}_{i'n'})_t + i\sum_{i''}\int d^3x' c_{i'l,i''m}(x-x'')(\xi^x_{in},\xi^{x''}_{i''m})_t -$$

$$-4\pi(J^x_{in},\xi^{x'}_{i'l}) - 4\pi(\xi^x_{in},J^{x'}_{i'l}) \tag{7}$$

The material equation for correlation current-field functions is given by the formula

$$(J^x_{in},\xi^{x'}_{i'l})_t = \sum_{i''}\int d^3x'' \sigma_{in,i''m}(x-x'',\varepsilon(x))(\xi^{x''}_{i''m},\xi^{x'}_{i'l})_t + S_{in,i'l}(x-x',n(x)) \tag{8}$$

that matches the Onsager principle. The last term in (8) does not depend on time and is proportional to the emitter quantity density $n(x)$. Note that only the following its components differ from zero

$$S_{2n,1l}(k,n) \equiv e_{nlm}k_m S_k(n), \qquad S_{2n,2l}(k,n) \equiv (\delta_{nl} - \hat{k}_n\hat{k}_l)T_k(n) \tag{9}$$

where $S_k(n)$, $T_k(n)$ are known functions of λ^2 order and $\hat{k}_l \equiv k_l/k$.

The temporal equation for the emitter energy density has the form

$$\partial_t\varepsilon(x,t) = (J^x_n,E^x_n)_t + J_n(x,t)E_n(x,t) + R(n(x)) \tag{10}$$

where the last term correspond to the full dipole emission of the emitters under consideration

$$R(n) = -2\omega^4 nd^2/3\pi c^3\,. \tag{11}$$

Proposed in this section description of the electromagnetic field with using the relevant compact notation for the field, its correlations, and all material equations allows to conduct general investigation of the considered system.

Modes of transversal electromagnetic field in the equilibrium medium of emitters

According to Eqs. (2, 3), the transversal electromagnetic field, obey the equations

$$\partial_t B^t_{nk} = -ic[k,E^t_k]_n\,, \qquad \partial_t E^t_{nk} = i\frac{c}{\mu_k}[k,B^t_k]_k - 4\pi\sigma_k E^t_{nk}\,. \tag{12}$$

The temporal equation for the electric field here has the form

$$\partial_t^2 E_{nk}^t + \partial_t E_{nk}^t 4\pi\sigma_k + E_{nk}^t \omega_k^2 / \mu_k = 0 . \tag{13}$$

Its general solution is given by the formula

$$E_{nk}^t(t) = a_{nk}^t e^{z_{1k} t} + b_{nk}^t e^{z_{2k} t} \tag{14}$$

where a_{nk}^t and b_{nk}^t are vector fields independent of time, z_{1k} and z_{2k} are quantities defined by the formulas

$$z_{1k} = i\Omega_k - \gamma_k , \quad z_{2k} = -i\Omega_k - \gamma_k ; \quad \Omega_k \equiv \sqrt{\omega_k^2 / \mu_k - (2\pi\sigma_k)^2} , \quad \gamma_k = 2\pi\sigma_k \tag{15}$$

($\omega_k \equiv ck$). According to Eqs. (12) and (14), the magnetic field is given by the expression

$$B_{nk}^t(t) = -\frac{ic}{z_{1k}}[k, a_k^t]_n e^{z_{1k} t} - \frac{ic}{z_{2k}}[k, b_k^t]_n e^{z_{2k} t} . \tag{16}$$

Expressions (14) and (16) can be regarded as a set of equations for the functions $a_{nk}^t e^{z_{1k} t}$ and $b_{nk}^t e^{z_{2k} t}$, thus obtaining

$$a_{nk}^t e^{z_{1k} t} = \frac{z_{1k}}{2i\Omega_k} E_{nk}^t(t) + \frac{c}{2i\Omega_k \mu_k} Z_{nk}^t(t) , \quad b_{nk}^t e^{z_{2k} t} = -\frac{z_{2k}}{2i\Omega_k} E_{nk}^t(t) - \frac{c}{2i\Omega_k \mu_k} Z_{nk}^t(t) . \tag{17}$$

Therefore right-hand sides of these relations are modes of the transversal electromagnetic field in the equilibrium medium. For our purposes the functions

$$\zeta_{ink}^t(t) \equiv E_{nk}^t(t) + \frac{c}{z_{ik}\mu_k} Z_{nk}^t(t) , \quad \partial_t \zeta_{ink}^t(t) = z_{1k} \zeta_{ink}^t(t) \tag{18}$$

are convenient for using as modes (later only the transversal field and its correlations are considered, that is why the upper index t is omitted for simplicity). For further consideration it is convenient to introduce compact designations connecting the field modes with the field itself

$$\zeta_{ink} = \sum_{i'} R_{in,i'l}(k) \xi_{i'lk} , \qquad \xi_{ink} = \sum_{i'} R_{in,i'l}^{-1}(k) \zeta_{i'lk} . \tag{19}$$

Appearing here matrixes have, according to (15), the following non-zero matrix elements

$$R_{in,1l}(k) = \frac{ic}{z_{ik}\mu_k} e_{nml} k_m , \quad R_{in,2l}(k) = \delta_{nl} ;$$

$$R_{1n,i'l}^{-1}(k) = (-1)^{i'} \frac{c}{2\Omega_k} e_{nml} k_m , \qquad R_{2n,i'l}^{-1}(k) = (-1)^{i'} i \frac{z_{i'k}}{2\Omega_k} \delta_{nl} . \tag{20}$$

Now we substitute the formula (19) into Eq. (4) with the expression (5) for current and take into account the temporal equation for modes (18). Bearing in mind that field and its modes are arbitrary functions determined by the initial conditions, we come to the identity

$$\sum_{i''} a_{in,i''m}(k, \varepsilon^{eq}) R_{i''m,i'l}^{-1}(k) = R_{in,i'l}^{-1}(k) z_{i'k} \tag{21}$$

where the designation is used

$$a_{in,i'l}(k, \varepsilon) = ic_{in,i'l}(k) - 4\pi\sigma_{in,i'l}(k, \varepsilon) . \tag{22}$$

From the relation (21) via a simple matrix multiplication we obtain the formula

$$\sum_{i''} R_{in,i''m}(k) a_{i''m,i'l}(k, \varepsilon^{eq}) = R_{in,i'l}(k) z_{ik} . \tag{23}$$

These formula shows that $R_{in,i'l}(k)$ is a left eigenvector of the matrix $a_{in,i'l}(k, \varepsilon^{eq})$ determining the evolution of electromagnetic field in an equilibrium medium and corresponds to an eigenvalue z_{ik}. Similarly, $R^{-1}_{in,i'l}(k)$ is a right eigenvector of this matrix and corresponds to the same eigenvalue $z_{i'k}$.

Equilibrium correlations of electromagnetic field

For simplicity let us consider equilibrium binary correlations of the transversal electromagnetic field restricting ourselves with the case of a uniform distribution of emitters in space. Since they do not depend on time, Eq. (7), with taking into account the material equation (8) and the definition (22), gives the relation

$$\sum_{i''}[a_{in,i''m}(k)(\xi^k_{i''m},\xi^{k'}_{i'l})^{eq}+a_{i'l,i''m}(k)(\xi^k_{in},\xi^{k'}_{i''m})^{eq}]=$$
$$=4\pi V[S_{in,i'l}(k)+S_{i'l,in}(k')]\delta_{k',-k} \qquad (24)$$

(we use a periodical boundary conditions, V is a system volume). Now, taking into account the formulas (19) and (23), herefrom we find field mode correlations

$$(z_{ik}+z_{i'k'})(\zeta^k_{in},\zeta^{k'}_{i'l})^{eq}=$$
$$=4\pi V\sum_{i_1n_1i_2n_2}R_{in,i_1n_1}(k)R_{i'l,i_2n_2}(k')\{S_{i_1n_1,i_2n_2}(k)+S_{i_2n_2,i_1n_1}(k')\}\delta_{k',-k}\,. \qquad (25)$$

Actual expressions (9) and (20) for matrixes entering here give the following expressions for the binary correlation functions

$$(\zeta^k_{in},\zeta^{k'}_{i'l})^{eq}=(\zeta^k_{in},\zeta^{-k}_{i'l})^{eq}\delta_{k',-k}\,,$$
$$(\zeta^k_{in},\zeta^{-k}_{i'l})^{eq}=V4\pi(\delta_{nl}-\hat{k}_n\hat{k}_l)\{\frac{2T_k(n)}{z_{ik}+z_{i'k}}+i\frac{\omega_k^2S_k(n)}{c\mu_k z_{ik}z_{i'k}}\}\,. \qquad (26)$$

Here we make allowance that for the system under consideration [1] material coefficients $\sigma_k(\varepsilon)$, $\chi_k(\varepsilon)$ appearing in the material equation (3) and functions $S_k(n)$, $T_k(n)$ appearing in the material equation for correlations (8) are even functions of k_n

$$\sigma_{-k}(\varepsilon)=\sigma_k(\varepsilon)\,,\quad \chi_{-k}(\varepsilon)=\chi_k(\varepsilon)\,,\quad S_{-k}(n)=S_k(n)\,,\quad T_{-k}(n)=T_k(n)\,, \qquad (27)$$

thus providing, in accordance with the formulas (15), the evenness of all the functions that are expressed through them

$$z_{i,-k}=z_{ik}\,,\qquad \Omega_{-k}=\Omega_k\,,\qquad \gamma_{-k}=\gamma_k\,,\qquad \mu_{-k}=\mu_k \qquad (28)$$

(see also (4) in the paragraph ahead). In fact, this result is connected with the rotational invariance of the considered quantities from which their dependence on $|k|$ follows.

Concept of binary correlation waves

In the spatially uniform case the nonequilibrium correlation function $(\xi^x_{in},\xi^{x'}_{i'l})_t$ depends only on the difference of coordinates, therefore their Fourier components possess the property $(\xi^k_{in},\xi^{k'}_{i'l})_t\sim\delta_{k',-k}$. Hence, the spatial non-uniformity of correlations is connected with the dependence of Fourier components $(\xi^{p+k/2}_{in},\xi^{-p+k/2}_{i'l})_t$ on a vector k_l that can be regarded as a wave one and indexes i,n,i',l,p_l should be considered to be component numbers. Obviously, correlation functions of field modes $\mathrm{f}^p_{in,i'l}(k,t)\equiv(\zeta^{p+k/2}_{in},\zeta^{-p+k/2}_{i'l})_t$ can be used instead of correlation modes of field itself. Low-amplitude motions of electromagnetic field correlations can be described by deviations from their equilibrium values

$$\delta(\xi^k_{in},\xi^{k'}_{i'l})_t=(\xi^k_{in},\xi^{k'}_{i'l})_t-(\xi^k_{in},\xi^{k'}_{i'l})^{eq}\,. \qquad (29)$$

According to Eqs. (7) and (8) and the definition (22), in the equilibrium medium of emitters such quantities obey the evolution equation

$$\partial_t \delta(\xi_{in}^k, \xi_{i'l}^{k'})_t = \sum_{i''} [a_{in,i''m}(k, \varepsilon^{eq}) \delta(\xi_{i''m}^k, \xi_{i'l}^{k'})_t + a_{i'l,i''m}(k', \varepsilon^{eq}) \delta(\xi_{in}^k, \xi_{i''m}^{k'})_t] \,. \tag{30}$$

Applying (19) and (23), we find from here an evolution equation for field mode correlations

$$\partial_t \delta(\zeta_{in}^k, \zeta_{i'l}^{k'})_t = (z_{ik} + z_{i'k'}) \delta(\zeta_{in}^k, \zeta_{i'l}^{k'})_t \,, \tag{31}$$

i.e. they are also modes of correlations. The dispersion law for the modes of correlations in an equilibrium medium is evident from the relation

$$\partial_t \delta \mathrm{f}_{in,i'l}^p(k,t) = (z_{i,p+k/2} + z_{i',-p+k/2}) \delta \mathrm{f}_{in,i'l}^p(k,t) \tag{32}$$

where $\delta\, \mathrm{f}_p^{in,i'l}(k,t) \equiv \delta(\zeta_{in}^{p+k/2}, \zeta_{i'l}^{-p+k/2})_t$. (33)

At small wave vectors we have, according to the formulas (15) and (28):

mode	dispersion law
$\delta \mathrm{f}_{1n,1l}^p(k,t)$	$i2\Omega_p - 2\gamma_p + O(k^2)$,
$\delta \mathrm{f}_{2n,2l}^p(k,t)$	$-i2\Omega_p - 2\gamma_p + O(k^2)$,
$\delta \mathrm{f}_{1n,2l}^p(k,t)$	$ic(p)\hat{p}_n k_n - 2\gamma_p + O(k^2)$,
$\delta \mathrm{f}_{2n,1l}^p(k,t)$	$-ic(p)\hat{p}_n k_n - 2\gamma_p + O(k^2)$ (34)

($c(p)\hat{p}_n \equiv \partial\Omega_p / \partial p_n$). Thus, in the limit of small wave vectors, the first two modes of correlations are damped oscillations and the next two modes describe decaying waves with the propagation velocity $\pm c(p)\hat{p}_n$ depending on the direction of the vector p_n .

Coupled oscillations of field correlations and a medium consisting of emitters

Let us consider coupled oscillations of field correlations and medium near the equilibrium. Coupled oscillations of an average field are absent according to (4) and (5) since in equilibrium the average field equals to zero $\xi_{in}^{eq} = 0$. To derive an equation for correlations we proceed from the relations (7), (8), (22), thus obtaining the generalization of Eq. (30)

$$\partial_t \delta(\xi_{in}^k, \xi_{i'l}^{k'})_t = \sum_{i''} [a_{in,i''m}(k, \varepsilon^{eq}) \delta(\xi_{i''m}^k, \xi_{i'l}^{k'})_t + a_{i'l,i''m}(k', \varepsilon^{eq}) \delta(\xi_{in}^k, \xi_{i''m}^{k'})_t] +$$

$$+ \frac{1}{\varepsilon^{eq} V} \sum_{i''} \delta\varepsilon_{k+k'} \sigma_{in,i''m}(-k', \varepsilon^{eq}) (\xi_{i''m}^{-k'}, \xi_{i'l}^{k'})^{eq} +$$

$$+ \frac{1}{\varepsilon^{eq} V} \sum_{i''} \delta\varepsilon_{k+k'} \sigma_{i'l,i''m}(-k, \varepsilon^{eq}) (\xi_{in}^k, \xi_{i''m}^{-k})^{eq} \tag{35}$$

where the proportionality of $\sigma_{in,i'l}(k,\varepsilon)$ to ε is taken into account. Eq. (10) for the density of emitter energy near the equilibrium $\delta\varepsilon(x,t) = \varepsilon(x,t) - \varepsilon^{eq}$ acquires the form taking into account Eq. (5)

$$\partial_t \delta\varepsilon_k = \frac{1}{\varepsilon^{eq} V^2} \delta\varepsilon_k \sum_{ii',k'} \sigma_{in,i'l}(k', \varepsilon^{eq}) (\xi_{i'l}^{k'}, \xi_{in}^{-k'})^{eq} + \frac{1}{V} \sum_{ii',k'} \sigma_{in,i'l}(k', \varepsilon^{eq}) \delta(\xi_{i'l}^{k'}, \xi_{in}^{k-k'}) \,. \tag{36}$$

In terms of mode correlations, these equations, according to the formulas (19) and (23), can be rewritten as follows

$$\partial_t \delta(\zeta_{in}^{k},\zeta_{i'l}^{k'})_t = (z_{ik} + z_{i'k'})\delta(\zeta_{in}^{k},\zeta_{i'l}^{k'})_t +$$

$$+\frac{1}{\varepsilon^{eq}V}\sum_{i_1 i_2 i''}\delta\varepsilon_{k+k'}R_{in,i_1n_1}(k)\sigma_{i_1n_1,i''m}(-k',\varepsilon^{eq})R^{-1}_{i''m,i_2n_2}(-k')(\zeta_{i_2n_2}^{-k'},\zeta_{i'l}^{k'})^{eq} +$$

$$+\frac{1}{\varepsilon^{eq}V}\sum_{i_1 i_2 i''}\delta\varepsilon_{k+k'}R_{i'l,i_1n_1}(k')\sigma_{i_1n_1,i''m}(-k,\varepsilon^{eq})R^{-1}_{i''m,i_2n_2}(-k)(\zeta_{in}^{k},\xi_{i_2n_2}^{-k})^{eq}.$$

$$\partial_t \delta\varepsilon_k = \frac{1}{\varepsilon^{eq}V^2}\delta\varepsilon_k\sum_{i_1 i_2 ii',k'}\sigma_{in,i'l}(k',\varepsilon^{eq})R^{-1}_{i'l,i_1n_1}(k')R^{-1}_{in,i_2n_2}(-k')(\zeta_{i_1n_1}^{k'},\zeta_{i_2n_2}^{-k'})^{eq} +$$

$$+\frac{1}{V}\sum_{i_1 i_2 ii',k'}\sigma_{in,i'l}(k',\varepsilon^{eq})R^{-1}_{i'l,i_1n_1}(k')R^{-1}_{in,i_2n_2}(k-k')\delta(\zeta_{i_1n_1}^{k'},\zeta_{i_2n_2}^{k-k'}). \tag{35}$$

Herefrom with taking into account the definition (33), we obtain the coupled set of equations for correlations of field modes $\delta\,\mathrm{f}_{in,i'l}^{p}(k,t)$ and medium energy density $\delta\varepsilon_k(t)$

$$\partial_t\delta\,\mathrm{f}_{in,i'l}^{p}(k,t) = \{z_{i,p+k/2} + z_{i',-p+k/2}\}\delta\,\mathrm{f}_{in,i'l}^{p}(k,t) + M_{in,i'l}^{p}(k)\delta\,\varepsilon_k(t),$$

$$\partial_t\delta\,\varepsilon_k(t) = \sum_{ii'}\int d^3p N_{in,i'l}^{p}(k)\delta\,\mathrm{f}_{in,i'l}^{p}(k,t) + \nu\,\delta\,\varepsilon_k(t). \tag{36}$$

The equations include coefficients defined by the formulas

$$M_{in,i'l}^{p}(k) = \frac{1}{\varepsilon^{eq}V}\sum_{i_1 i_2 i''}R_{in,i_1n_1}(p+\frac{k}{2})\sigma_{i_1n_1,i''m}(p-\frac{k}{2},\varepsilon^{eq})R^{-1}_{i''m,i_2n_2}(p-\frac{k}{2})\times$$

$$\times(\zeta_{i_2n_2}^{p-k/2},\zeta_{i'l}^{-p+k/2})^{eq} +$$

$$+\frac{1}{\varepsilon^{eq}V}\sum_{i_1 i_2 i''}R_{i'l,i_1n_1}(-p+\frac{k}{2})\sigma_{i_1n_1,i''m}(-p-\frac{k}{2},\varepsilon^{eq})R^{-1}_{i''m,i_2n_2}(-p-\frac{k}{2})\times$$

$$\times(\zeta_{in}^{p+k/2},\zeta_{i_2n_2}^{-p-k/2})^{eq}, \tag{37}$$

$$\nu = \frac{1}{\varepsilon^{eq}V(2\pi)^3}\sum_{i_1 i_2 ii'}\int d^3k'\sigma_{in,i'l}(k',\varepsilon^{eq})R^{-1}_{in,i_1n_1}(-k')R^{-1}_{i'l,i_2n_2}(k')(\zeta_{i_2n_2}^{k'},\zeta_{i_1n_1}^{-k'})^{eq}, \tag{38}$$

$$N_{in,i'l}^{p}(k) = \frac{1}{(2\pi)^3}\sum_{i_1 i_2}\sigma_{i_1n_1,i_2n_2}(p+\frac{k}{2},\varepsilon^{eq})R^{-1}_{i_2n_2,in}(p+\frac{k}{2})R^{-1}_{i_1n_1,i'l}(-p+\frac{k}{2}). \tag{39}$$

Notice that equilibrium correlation functions, according to the expressions (9) and (26), are proportional to the system volume and have the second order in interaction. Therefore in the set of equations (36) coefficients are of the following orders

$$M^{p}_{in,i'l}(k) \sim \lambda^4, \qquad N^{p}_{in,i'l}(k) \sim \lambda^2, \qquad \nu \sim \lambda^4. \tag{40}$$

and do not depend on volume in the thermodynamic limit. The solution of the set (36) can be searched for in the form $\delta \mathrm{f}^{p}_{in,i'l}(k,t) = C^{p}_{in,i'l}(k)e^{zt}$, $\delta\varepsilon_k(t) = C_k e^{zt}$, it gives the following dispersion equation

$$z = \nu + \sum_{ii'} \int d^3p \frac{M^{p}_{in,i'l}(k) N^{p}_{in,i'l}(k)}{z - z_{i,p+k/2} - z_{i',-p+k/2}}. \tag{41}$$

Its analysis can be based on the account of estimations (40) and on the detailed analysis for the case of small wave vectors; this will be done in another place.

Here we restrict ourselves to the analysis of (36) in the elementary perturbation theory in weak interaction. In order to simplify the formulas, we analyze a set of equations that are similar to the set (36)

$$\dot{x}_i = z_i x_i + a_i y, \qquad \dot{y} = \sum_i b_i x_i + cy \tag{42}$$

where coefficients have the following order in λ: $a_i \sim \lambda^4$, $b_i \sim \lambda^2$, $c \sim \lambda^4$. We search for a solution in the form of series in powers of λ^2. Quantities z_i in Eq. (42) are analogues to quantities $z_{i,p+k/2} + z_{i',-p+k/2}$ in the set (36), the last ones can also be expanded in λ according to the formulas (15). However, we shall not expand z_i in a series in λ while constructing perturbation theory for the set (42). It will allow avoiding partially time secular terms arising in the perturbation theory. Simple calculations give

$$x_i = x_{i0}e^{tz_i} + y_0 \frac{a_i}{z_i}(e^{tz_i} - 1) + O(\lambda^6), \quad y = y_0 + \sum_i \frac{x_{i0}b_i}{z_i}(e^{tz_i} - 1) + cy_0 t + O(\lambda^6) \tag{43}$$

where $x_{i0} \equiv x_i|_{t=0}$, $y_0 \equiv y|_{t=0}$. Secular term presence in the second expression is physically connected with the fact that really corrections to the frequencies z_i occur. Some interesting conclusions can be made from (43). If at the initial time moment the subsystem of field correlations is equilibrium and $x_{i0} = 0$, at the next time moments non-equilibrium correlations take place

$$x_{i0} = 0 \quad \Rightarrow \quad x_i = y_0 \frac{a_i}{z_i}(e^{tz_i} - 1) + O(\lambda^6), \quad y = y_0 + \sum_i + cy_0 t + O(\lambda^6). \tag{44}$$

It means that the medium of two-level emitters radiates waves of correlations. Alternatively, if at the initial time moment the subsystem of emitters is equilibrium and $y_0 = 0$, it is excited under the impact of correlation waves

$$y_0 = 0 \quad \Rightarrow \quad x_i = x_{i0} e^{t z_i} + O(\lambda^6), \quad y = \sum_i \frac{x_{i0} b_i}{z_i} (e^{t z_i} - 1) + O(\lambda^6) . \tag{45}$$

Obtained results show the non-trivial dynamics of the system of emitters and electromagnetic field. Near the equilibrium emitters do not interact with an average field, but interact with binary correlations of the field.

Conclusions

Modes of average transversal electromagnetic field and its correlations in a medium consisting of two-level emitters described by the Dicke type model have been investigated. Four types of correlation modes in the equilibrium medium of emitters have been found. Among them two types of correlation modes correspond to a damping oscillation process and two other correlation modes correspond to decaying waves. In this connection the concept of electromagnetic correlation waves has been proposed. In some sense these waves are close to secondary waves discussed in the condensed matter theory. Particularly, we mean the second sound in a superfluid liquid that can be considered as sound waves in the phonon subsystem (see, for example, [7]).

In the case of a nonequilibrium medium, its coupled oscillations with the oscillations of the field correlations have been studied. The phenomenon of field correlations arising (correlation waves radiation) in the process of emitter medium evolution has been predicted. On the other hand, waves of correlations can break equilibrium state of the emitter subsystem. The obtained dispersion equation (41) is similar to the one investigated in the mode-mode coupling theory. An approach developed in the last theory will be applied to our analysis of this equation in a subsequent paper.

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Received 13.07.2013.