UDC 533.7

A. I. Sokolovsky, Z. Yu. Chelbaevsky

Oles Honchar Dnipropetrovsk National University, Dnipropetrovsk, Ukraine
e-mail: alexsokolovsky@mail.ru

TO THE PROBLEM OF CALCULATION OF THE EFFECTIVE INITIAL CONDITIONS FOR HYDRODYNAMIC EQUATIONS

Problem of effective initial conditions for hydrodynamic equations is investigated for a rarefied gas described by the Boltzmann kinetic equation. The consideration is based on the Bogolyubov functional hypothesis that leads to a generalization of the Chapman-Enskog method of the hydrodynamic equation construction. Using this hypothesis, a basic integral equation for the effective initial conditions of the theory is obtained. This equation solution is investigated for states close to equilibrium because dynamics of the system, described by the nonlinear Boltzmann equation, cannot be analyzed at the present time. The basic integral equation solution is reduced to the analysis of integral equations of the type similar to ones introduced by the Chapman-Enskog method. Such equations can be solved approximately by an expansion method in the Sonine polynomial series. It is shown that difference between the effective initial conditions and the real ones has the second order in gradients of the hydrodynamic variables. The obtained effective initial conditions generalize results by Grad, obtained by him for a particular case of the Maxwellian molecules.

Keywords: the functional hypothesis, hydrodynamic equations, effective initial conditions, Boltzmann kinetic equation.

Проблема ефективних початкових умов до рівнянь гідродинаміки досліджується для розрідженого газу, що описується кінетичним рівнянням Больцмана. Розгляд ґрунтується на функціональній гіпотезі Боголюбова, яка веде до узагальнення методу Чепмена-Енскоґа побудови рівнянь гідродинаміки. Використовуючи цю гіпотезу, виводиться основне інтегральне рівняння теорії для ефективних початкових умов. Розв'язок одержаного рівняння досліджується для станів, близьких до рівноважного, оскільки динаміка системи, яка описується нелінійним рівнянням Больцмана, не піддається на теперішній час аналізу. Розв'язування основного інтегрального рівняння теорії зводиться до розв'язування інтегральних рівнянь типу тих, до яких веде метод Чепмена-Енскоґа. Ці рівняння можуть бути наближено розв'язані методом розвинення за поліномами Соніна. Показується, що різниця між ефективними початковими умовами і реальними початковими умовами має другий порядок за градієнтами гідродинамічних змінних. Знайдені ефективні початкові умови узагальнюють результати Ґреда, отримані ним для окремого випадку максвелівських молекул.

Ключові слова: функціональна гіпотеза, рівняння гідродинаміки, ефективні початкові умови, кінетичне рівняння Больцмана.

Проблема эффективных начальных условий к уравнениям гидродинамики исследуется для разреженного газа, который описывается кинетическим уравнением Больцмана. Рассмотрение основывается на функциональной гипотезе Боголюбова, которая ведет к обобщению метода Чепмена-Энскога построения уравнений гидродинамики. Используя эту гипотезу, выводится основное интегральное уравнение теории для эффективных начальных условий. Решение полученного уравнения исследуется для состояний, близких к равновесному, поскольку динамика системы, описываемая нелинейным уравнением Больцмана, не поддается в настоящее время анализу. Решение основного уравнения теории сводится к решению интегральных уравнений типа тех, к которым ведет метод Чепмена-Энскога. Эти интегральные уравнения могут быть приближенно решены методом разложения по полиномам Сонина. Показывается, что разность между эффективными начальными условиями и реальными начальными условиями имеет второй порядок по градиентам гидродинамических переменных. Найденные эффективные начальные условия обобщают результаты Грэда, полученные им для частного случая максвелловских молекул.

Ключевые слова: функциональная гипотеза, уравнения гидродинамики, эффективные начальные условия, кинетическое уравнение Больцмана.

1. Introduction

The problem of deriving hydrodynamic equations taking into account dissipative processes from the Boltzmann kinetic equation for one-particle distribution function (DF) $f_p(x,t)$ was discussed by Boltzmann and Lorentz immediately after the equation derivation. Now this problem is solved by the Chapman-Enskog method in which a normal solution of the kinetic equation is investigated, that is a functional $f_p(x,\zeta(t))$ of hydrodynamic variables as functions of coordinates $\zeta_\mu(x,t)$ [1]. The concept of normal solution of the Boltzmann equation was introduced by Hilbert in his pioneering paper [2].

Bogolyubov made a decisive contribution to the understanding of this idea in his method of the reduced description [3]. He formulated the idea of the presence of a sequence of stages with decreasing number of parameters describing the system completely in the evolution of the system. In particular, the hydrodynamic description of the system is possible after time τ_0 that has order of the free path time. In the Bogolyubov theory the Cauchy problem solution for the kinetic equation $f_p(x,t,f_0)$ ($f_p(x,t=0,f_0)\equiv f_{p0}(x)$) and hydrodynamic variables $\zeta_\mu(x,t,f_0)$ as averages taken with DF $f_p(x,t,f_0)$ are introduced. Next, asymptotic values of these functions $f_p^{(+)}(x,t,f_0)$, $\zeta_\mu^{(+)}(x,t,f_0)$ are considered and the above-mentioned *functional* $f_p(x,\zeta)$ *is defined by the formula* $f_p^{(+)}(x,t,f_0)=f_p(x,\zeta^{(+)}(t,f_0))$. *It is assumed that the functional* $f_p(x,\zeta)$ *does not depend on the initial DF* $f_{p0}(x)$ *and DF* $f_p(x,\zeta^{(+)}(t,f_0))$ *is an exact solution of the kinetic equation.* The last ideas express the content of the Bogolyubov *functional hypothesis*. For functions $\zeta_\mu^{(+)}(x,t,f_0)$ one obtains hydrodynamic equations, solution of which has a physical meaning for $t\gg\tau_0$ but allows continuations for times $\tau_0\ge t\ge 0$. This introduces quantities $\zeta_\mu^{(+)}(x,0,f_0)$ that are called the effective initial conditions (**EIC**) (contrary to true ones $\zeta_\mu(x,0,f_0)\ne\zeta_\mu^{(+)}(x,0,f_0)$).

By this way the Bogolyubov functional hypothesis [3] leads to the problem of calculating of the **EIC**. The detailed investigation of this problem for the Boltzmann equation was conducted by Grad [4] as *the problem of the initial layer*. This terminology reflects the thought that the normal solution of the Boltzmann equation does not describe the transition period $\tau_0\ge t\ge 0$ adequately (see discussion of the problem in [5] (p. 122) and in [6] (p. 254)). Bogolyubov discussed the problem in paper [7] with application to the calculation of long-time asymptotics of time correlation functions. Clearly, this problem can be discussed in a general theory of nonequilibriun processes based on the Liouville equation. One can find a review of the Bogolyubov reduced description method with calculation of the EIC for some cases and related applications in book [8].

In the present paper the problem is investigated on the basis of the Bogolyubov functional hypothesis for states close to the equilibrium. As usual in hydrodynamics gradients of hydrodynamic variables are considered as small quantities estimated by the formula $\partial^s/\partial x_{n_1}...\partial x_{n_s}\zeta_\mu(x)\sim g^s$ ($g\equiv l/L$ where l is a free path, L is a characteristic length of distribution of $\zeta_\mu(x)$ in the space). Preliminary results of the investigation were presented at the Ukrainian Mathematical Congress (Kyiv, 2009).

In the Grad's paper [4] the Boltzmann kinetic equation was analyzed following to Hilbert. He considered this equation as a singularly perturbed integro-differential equation with using various time scales (the collision integral was assumed to be proportional to a big parameter). In the Bogolyubov reduced description method, based on the functional hypothesis, using of time scales is not necessary that is an advantage of this method.

The present paper plan is as follows. In the Sec. 2 basic integral equation for the **EIC** is derived for a general nonlinear case. In the Sec. 3 the basic integral equation is transformed for states close to the equilibrium. In the Sec. 4 the basic integral equation close to the equilibrium is solved in the perturbation theory in gradients.

2. Basic integral equation of the theory

The Boltzmann kinetic equation for a rarefied gas

$$\frac{\partial \mathrm{f}_p(x,t)}{\partial t} = -\frac{p_n}{m}\frac{\partial \mathrm{f}_p(x,t)}{\partial x_n} + I_p(\mathrm{f}(x,t)), \tag{1}$$

is considered with standard properties

$$\int d^3p\,\zeta_{\mu p} I_p(\mathrm{f}) = 0; \qquad \zeta_{\mu p}: \quad \zeta_{0p} = \varepsilon_p \equiv p^2/2m, \ \zeta_{np} = p_n, \ \zeta_{4p} = m. \tag{2}$$

$$I_p(w(\xi)) = 0; \qquad w_p(\xi) \equiv \frac{\sigma}{m(2\pi mT)^{3/2}} e^{-\frac{(p-m\upsilon)^2}{2mT}};$$

$$\xi_\mu: \quad \xi_0 = T, \ \xi_n = \upsilon_n, \ \xi_4 = \sigma \equiv nm. \tag{3}$$

The property (2) of the collision integral $I_p(\mathrm{f})$ means that at collisions of system particles the conservation laws of energy, momentum and number of particles hold. The property (3) shows that the Maxwell distribution is the equilibrium solution of the Boltzmann equation.

Hydrodynamic states of the system are investigated on the basis of the Bogolyubov functional hypothesis

$$\mathrm{f}_p(x,t) \xrightarrow[t \gg \tau_0]{} \mathrm{f}_p(x,\zeta(t,\mathrm{f}_0)), \qquad \mathrm{f}_{p0}(x) \equiv \mathrm{f}_p(x,t=0) \tag{4}$$

where hydrodynamic variables are defined by the formulas

$$\int d^3p\,\mathrm{f}_p(x,\zeta)\zeta_{\mu p} = \zeta_\mu(x); \quad \zeta_\mu(x): \ \zeta_0(x) = \varepsilon(x), \ \zeta_n(x) = \pi_n(x), \ \zeta_4(x) = \sigma(x) \tag{5}$$

($\varepsilon(x)$, $\pi_n(x)$, $\sigma(x)$ are densities of energy, momentum and mass of the system; *here and hereafter asymptotic hydrodynamic variables* $\zeta_\mu^{(+)}(x,t,\mathrm{f}_0)$ *are denoted by* $\zeta_\mu(x,t,\mathrm{f}_0)$). Formula (4) describes the DF structure of the system at long times when it becomes a functional of additive motion integrals $\zeta_\mu(x,t,\mathrm{f}_0)$. The functional does not depend on the initial DF $\mathrm{f}_{p0}(x)$. Instead of independent variables $\zeta_\mu(x,t,\mathrm{f}_0)$, new variables $\xi_\mu(x,t,\mathrm{f}_0)$

$$\zeta_\mu(\xi): \qquad \varepsilon(x) = \frac{3}{2}\frac{\sigma(x)T(x)}{m} + \frac{1}{2}\sigma(x)\upsilon(x)^2, \qquad \pi_n(x) = \sigma(x)\upsilon_n(x) \tag{6}$$

are widely used ($T(x)$, $\upsilon_n(x)$ are temperature and mass velocity of the gas). This allows introducing new DF $\tilde{\mathrm{f}}_p(x,\xi)$ defined by relations

$$\tilde{\mathrm{f}}_p(x,\xi) \equiv \mathrm{f}_p(x,\zeta(\xi)), \qquad \int d^3p\tilde{\mathrm{f}}_p(x,\xi)\zeta_{\mu p} = \zeta_\mu(\xi(x)). \tag{7}$$

According to the basic idea of the Bogolyubov reduced description method DF $\mathrm{f}_p(x,\zeta(t,\mathrm{f}_0))$ is an exact solution of the kinetic equation (1) at $t \gg \tau_0$

$$\frac{\partial \mathrm{f}_p(x,\zeta(t,\mathrm{f}_0))}{\partial t} = -\frac{p_n}{m}\frac{\partial \mathrm{f}_p(x,\zeta(t,\mathrm{f}_0))}{\partial x_n} + I_p(\mathrm{f}(x,\zeta(t,\mathrm{f}_0))) \tag{8}$$

($\tilde{\mathrm{f}}_p(x,\xi(t,\mathrm{f}_0))$ satisfies the same equation). According to (2) this formula gives hydrodynamic equations

$$\frac{\partial \zeta_\mu(x,t,\mathrm{f}_0)}{\partial t} = -\frac{\partial \zeta_{\mu n}(x,\zeta(t,\mathrm{f}_0))}{\partial x_n}, \qquad \zeta_{\mu n}(x,\zeta) \equiv \int d^3p\mathrm{f}_p(x,\zeta)\frac{p_n}{m}\zeta_{\mu p};$$

$$\frac{\partial \xi_\mu(x,t,\mathrm{f}_0)}{\partial t} = L_\mu(x,\xi(t,\mathrm{f}_0)) \tag{9}$$

that also are valid for $t \gg \tau_0$ ($\zeta_{\mu n}(x,\zeta)$ are flux densities of the additive motion integrals; an expression for the functional $L_\mu(x,\xi)$ is not given here). By virtue of (8), (9) the functional $\mathrm{f}_p(x,\zeta)$ satisfies the equation

$$-\sum_\mu \int d^3x' \frac{\delta \mathrm{f}_p(x,\zeta)}{\delta \zeta_\mu(x')}\frac{\partial \zeta_{\mu n}(x',\zeta)}{\partial x'_n} = -\frac{p_n}{m}\frac{\partial \mathrm{f}_p(x,\zeta)}{\partial x_n} + I_p(\mathrm{f}(x,\zeta)). \tag{10}$$

Solution of the hydrodynamic equations (9) can be continued for $0 \le t \le \tau_0$. This introduces quantities $\zeta_\mu(x,t=0,\mathrm{f}_0)$ ($\xi_\mu(x,t=0,\mathrm{f}_0)$) which are called the **EIC** for equations (9). Defined by this way solutions $\zeta_\mu(x,t,\mathrm{f}_0)$ ($\xi_\mu(x,t,\mathrm{f}_0)$) have not physical meaning for $0 \le t \le \tau_0$ but they allow investigating the dependence of functions $\zeta_\mu(x,t,\mathrm{f}_0)$ ($\xi_\mu(x,t,\mathrm{f}_0)$) on $\mathrm{f}_{p0}(x)$ for $t \gg \tau_0$. After the continuation Eq. (8) is valid for $t \ge 0$ that follows from (9), (10).

Let us derive an integral equation for the **EIC** $\zeta_\mu(x,0,\mathrm{f}_0)$. The functional hypothesis (4) shows that $\mathrm{f}_p(x,t) - \mathrm{f}_p(x,\zeta(t,\mathrm{f}_0)) \xrightarrow[t \gg \tau_0]{} 0$ and the following relation

$$\mathrm{f}_p(x,\zeta(0,\mathrm{f}_0)) = \mathrm{f}_{p0}(x) + \int_0^{+\infty} dt\{\frac{\partial \mathrm{f}_p(x,t)}{\partial t} - \frac{\partial \mathrm{f}_p(x,\zeta(t,\mathrm{f}_0))}{\partial t}\}$$

is true. Multiplying this equation by $\zeta_{\mu p}$ after integration over p_n with account for kinetic equation in the forms (1) and (8) and the property (2) of the collision integral, one obtains

$$\zeta_\mu(x,0,\mathrm{f}_0) = \int d^3p\mathrm{f}_{p0}(x)\zeta_{\mu p} + \frac{\partial}{\partial x_n}\frac{1}{m}\int_0^{+\infty} dt \int d^3p p_n \zeta_{\mu p}\{\mathrm{f}_p(x,\zeta(t,\mathrm{f}_0)) - \mathrm{f}_p(x,t)\}. \tag{11}$$

This equation is the basic integral equation for the **EIC** $\zeta_\mu(x,0,\mathrm{f}_0)$. In order to solve this equation, one needs the functional $\mathrm{f}_p(x,\zeta)$ known from the construction of hydrodynamics with the help of the Chapman-Enskog method.

3. The effective initial conditions close to the equilibrium

Integral over t in Eq. (11) is defined by dynamics, described by the nonlinear kinetic equation. Therefore, the integral equation (11) analysis is impossible in a general case and we confine ourselves to the particular case of states close to the equilibrium

$$\mathrm{f}_{p0}(x) = w_p(\xi_0(x)), \quad \xi_{\mu 0}(x) = \xi^0_\mu + \delta\xi_{\mu 0}(x), \quad \delta\xi_{\mu 0}(x) \ll \xi^0_\mu . \tag{12}$$

In this situation the basic equations of the theory can be linearized (equilibrium variables $\xi^o_0 \equiv \varepsilon^o$, $\xi^o_n \equiv 0$, $\xi^o_4 \equiv \sigma^o$ do not depend on coordinates). Let us characterize the magnitude of deviations $\delta\xi_{\mu 0}(x)$ by a small parameter λ. Then the initial DF takes the form

$$\mathrm{f}_{p0}(x) = w^o_p + \delta\mathrm{f}_{p0}(x) + O(\lambda^2), \quad w^o_p \equiv w_p(\xi^o), \quad \delta\mathrm{f}_{p0}(x) = \sum_\mu \left.\frac{\partial w_p(\xi)}{\partial \xi_\mu}\right|_{\xi\to\xi^o} \delta\xi_{\mu 0}(x) . \tag{13}$$

Now the kinetic equation (1) leads to an equation for DF $\delta\mathrm{f}_p(x,t)$ defined by the formula

$$\mathrm{f}_p(x,t) = w^o_p + \delta\mathrm{f}_p(x,t) + O(\lambda^2) . \tag{14}$$

This equation can be written in the form

$$\frac{\partial}{\partial t}\delta\mathrm{f}_p(x,t) = \hat{L}\delta\mathrm{f}_p(x,t), \qquad \delta\mathrm{f}_p(x,0) = \delta\mathrm{f}_{p0}(x), \tag{15}$$

where the operator $\hat{L}$ is introduced by the formula

$$\hat{L}h_p(x) = -\frac{p_n}{m}\frac{\partial h_p(x)}{\partial x_n} + \hat{L}_0 h_p(x), \qquad \hat{L}_0 h_p(x) \equiv \int d^3p' M_{pp'}(\xi^o) h_{p'}(x),$$

$$M_{pp'}(\xi) \equiv \left.\frac{\delta I_p(\mathrm{f})}{\delta \mathrm{f}_{p'}}\right|_{\mathrm{f}\to w(\xi)} \tag{16}$$

($h_p(x)$ is an arbitrary function).

In the vicinity of the equilibrium all expressions related to hydrodynamics should be linearized. For hydrodynamic variables $\xi_\mu(x,t,\mathrm{f}_0)$ and DF $\tilde{\mathrm{f}}_p(x,\xi(t,\mathrm{f}_0))$ there are expansions

$$\tilde{\mathrm{f}}_p(x,\xi(t,\mathrm{f}_0)) = w^o_p + \delta\mathrm{f}^{(+)}_p(x,t) + O(\lambda^2), \qquad \xi_\mu(x,t,\mathrm{f}_0) = \xi^o_\mu + \delta\xi_\mu(x,t) + O(\lambda^2) . \tag{17}$$

The first order DF $\delta\mathrm{f}^{(+)}_p(x,t)$ in virtue of (8) and (16) satisfies the equation

$$\frac{\partial}{\partial t}\delta\mathrm{f}^{(+)}_p(x,t) = \hat{L}\delta\mathrm{f}^{(+)}_p(x,t), \tag{18}$$

with the initial condition

$$\delta f_p^{(+)}(x,0)=\sum_\mu \int d^3x' \left.\frac{\delta\tilde{f}_p(x,\xi)}{\delta\xi_\mu(x')}\right|_{\xi\to\xi^o} \delta\xi_\mu(x',0)\,. \tag{19}$$

Entering here values $\delta\xi_\mu(x,0)$ are the **EIC** for hydrodynamic equations for parameters $\delta\xi_\mu(x,t)$ of the linearized theory.

The basic integral equation for the **EIC** (11) after linearization takes the form

$$\delta\zeta_\mu(x,0)=\int d^3p\,\delta f_{p0}(x)\zeta_{\mu p}+\frac{\partial}{\partial x_n}\frac{1}{m}\int_0^{+\infty}dt\int d^3p\,p_n\zeta_{\mu p}\{\delta f_p^{(+)}(x,t)-\delta f_p(x,t)\}\,. \tag{20}$$

Nonlinear relations (6) connecting hydrodynamic variables $\zeta_\mu(x)$ and $\xi_\mu(x)$ taking into account expressions (7) and (17) give

$$\int d^3p\,\delta f_p^{(+)}(x,t)\xi_{\mu p}=\delta\xi_\mu(x,t)\,,\quad \int d^3p\,\delta f_p^{(+)}(x,t)\zeta_{\mu p}=\delta\zeta_\mu(x,t) \tag{21}$$

where the notations

$$\xi_{\mu p}:\qquad \xi_{0p}=\frac{2m}{3\sigma^o}\varepsilon_p-\frac{mT^0}{\sigma^o}\,,\quad \xi_{np}=\frac{1}{\sigma^o}p_n\,,\quad \xi_{4p}=m \tag{22}$$

are introduced. According to (2), (21), and (22) variables $\delta\xi_\mu(x,0)$, $\delta\zeta_\mu(x,0)$ are linearly connected. Therefore, Eq. (20) after substitution $\zeta_{\mu p}\to\xi_{\mu p}$ is true for variables $\delta\xi_\mu(x,0)$. Kinetic equations (15) and (19) describe time evolution of functions $\delta f_p(x,t)$, $\delta f_p^{(+)}(x,t)$ and transform equation (20) in the form

$$\delta\xi_\mu(x,0)=\int d^3p\,\delta f_{p0}(x)\xi_{\mu p}+\lim_{\varepsilon\to+0}\frac{\partial}{\partial x_n}\frac{1}{m}\int d^3p\,p_n\xi_{\mu p}(\hat{L}-\varepsilon)^{-1}\{\delta f_{p0}(x)-\delta f_p^{(+)}(x,0)\}\,. \tag{24}$$

Here before taking the integral over variable t it was regularized according to Abel to avoid complications connected the existence of $\hat{L}^{-1}$. Integral equation (24) is a final form of the basic integral equation for the **EIC** $\delta\xi_\mu(x,0)$ for states close to the equilibrium. It should be solved taking into account the expression (19) for DF $\delta f_p^{(+)}(x,0)$. Eq. (24) is true for an arbitrary initial state of the system described by DF $\delta f_{p0}(x)$ but we restrict ourselves to the case (13).

4. Solution of the basic integral equation for states close to the equilibrium

The integral equation (24) assumes that hydrodynamic DF $\tilde{f}_p(x,\xi)$, entering expression (19) for $\delta f_p^{(+)}(x,0)$, is known. In the perturbation theory in small gradients of hydrodynamic variables $\xi_\mu(x)$ it has the form

$$\tilde{f}_p(x,\xi)=\tilde{f}_p^{(0)}(x,\xi)+\tilde{f}_p^{(1)}(x,\xi)+O(g^2)\,;\qquad \tilde{f}_p^{(0)}(x,\xi)=w_p(\xi(x))\,,$$

$$\tilde{f}_p^{(1)}(x,\xi) = w_p(\xi(x))\left[h_{nl}(p)A_p(\xi(x))\frac{\partial \upsilon_n(x)}{\partial x_l} + p_l B_p(\xi(x))\frac{\partial T(x)}{\partial x_l}\right]_{p\to p-m\upsilon(x)} \quad (25)$$

where the corresponding small parameter, $h_{nl}(p) \equiv p_n p_l - \delta_{nl}p^2/3$ (see, for example, [6]). Functions $A_p(\xi)$, $B_p(\xi)$ are solutions of the integral equations with an additional condition

$$\hat{K}(\xi)h_{nl}(p)A_p(\xi) = -\frac{1}{mT}h_{nl}(p)\,;\; \hat{K}(\xi)p_n B_p(\xi) = \frac{1}{mT}p_n\left(\frac{5}{2} - \frac{\varepsilon_p}{T}\right),\; \langle B_p(\xi)\varepsilon_p\rangle = 0\,. \quad (26)$$

Here integral operator $\hat{K}(\xi)$ and average quantity $\langle h_p\rangle$ with the Maxwell distribution

$$\hat{K}(\xi)h_p = \int d^3p' K_{pp'}(\xi)h_{p'}\,,\;\; w_p(\xi)K_{pp'}(\xi) \equiv -M_{pp'}(\xi)w_{p'}(\xi)\,;\; \langle h_p\rangle = \int d^3p\, w_p(\xi)h_p \quad (27)$$

are introduced (h_p is an arbitrary function).

Solution of the integral equation (24) for $\delta\xi_\mu(x,0)$ is found in the form of a series in gradients assuming that gradients of the initial DF $f_{p0}(x)$ are estimated by the parameter g

$$\delta\xi_\mu(x,0) = \delta\xi_\mu^{(0)}(x,0) + \delta\xi_\mu^{(1)}(x,0) + \delta\xi_\mu^{(2)}(x,0) + O(g^3)\,. \quad (28)$$

For the initial state of the system (13), according to (24) and with account for (3) and (22), in the zero order approximation the formula

$$\delta\xi_\mu^{(0)}(x,0) = \int d^3p\,\delta f_{p0}(x) = \delta\xi_{\mu 0}(x) \quad (29)$$

is true. So, in the main approximation the **EIC** coincide with the real ones.

In view of (16), (25), (27) in the first approximation in gradients Eq. (24) leads to the expression

$$\delta\xi_\mu^{(1)}(x,0) = \lim_{\varepsilon\to+0}\frac{\partial}{\partial x_n}\frac{1}{m}\int d^3p\,p_n\xi_{\mu p}[\hat{L}_0 - \varepsilon]^{-1}\{\delta f_{p0}(x) - \delta f_p^{(+)(0)}(x,0)\}\,. \quad (30)$$

Here it was taken into account that operator $\hat{L}$ in the approximation of the zero order in the gradients can be changed by the operator $\hat{L}_0$ (see definition (16)). According to (19) and (25), entering expression (30) DF $\delta f_p^{(+)(0)}(x,0) = \delta f_{p0}(x)$ and, therefore, $\delta\xi_\mu^{(1)}(x,0) = 0$.

In the second approximation in the gradients, by the analogy with (30), the integral equation (24) gives

$$\delta\xi_\mu^{(2)}(x,0) = -\lim_{\varepsilon\to+0}\frac{\partial}{\partial x_n}\frac{1}{m}\int d^3p\,p_n\xi_{\mu p}[\hat{L}_0 - \varepsilon]^{-1}\delta f_p^{(+)(1)}(x,0)\,. \quad (31)$$

In view of (19) and (25), in this formula DF $\delta f_p^{(+)(1)}(x,0)$ can be written in the form

$$\delta f_p^{(+)(1)}(x,0) = w_p^o\left[h_{nl}(p)A_p(\xi^o)\frac{\partial\delta\upsilon_{n0}(x)}{\partial x_l} + p_l B_p(\xi^o)\frac{\partial\delta T_0(x)}{\partial x_l}\right] \quad (32)$$

where $\delta\upsilon_{n0}(x)$, $\delta T_0(x)$ are real initial deviations of the velocity and temperature from their equilibrium values ξ^o_μ ($w^o_p \equiv w_p(\xi^o)$). As a result the following expression for the contribution of the second order to the **EIC**

$$\delta\xi^{(2)}_\mu(x,0) = A_{\mu mnl}\frac{\partial\delta\upsilon_{m0}(x)}{\partial x_n \partial x_l} + B_{\mu nl}\frac{\partial\delta T_0(x)}{\partial x_n \partial x_l} \qquad (33)$$

is obtained. Here $A_{\mu mnl}$, $B_{\mu nl}$ are some coefficients, approximate calculation of which will be given in another paper. The structure of the expression (33) coincides with the result of investigations by Grad [4].

5. Conclusions

In the present paper the problem of effective initial conditions (the Grad problem of the initial layer [4, 5]) for hydrodynamic equations is investigated on the basis of the Boltzmann kinetic equation, which describes states of a nonequilibrium dilute gas. The problem is formulated here in the terms of the Bogolyubov functional hypothesis. In fact, the concept of the effective initial conditions is a consequence of an asymptotic character of hydrodynamic equations that are valid after some transition process starting from an arbitrary initial state.

Using the functional hypothesis, the basic integral equation for the effective initial conditions of the theory is obtained. Solution of this equation is investigated for states close to the equilibrium because the Cauchy problem solution for the nonlinear Boltzmann equation cannot be analyzed in the necessary details.

Detailed investigation is conducted for the local Maxwell distribution as an initial distribution function of the system. Gradients of hydrodynamic variables, entering this distribution, are assumed to be small quantities. The developed theory generalizes results obtained by Grad [4]. Obtained here effective initial conditions differ from the real ones by quantities of the second order in gradients. However, they can be used to compare predictions of the theory with precision experiments. Among others, it is meant the investigation of a role of the Burnett terms in hydrodynamic equations [1, 5, 6].

References

1. **Chapman, S.** The mathematical theory of non-uniform gases / S. Chapman, T. Cowling. – Cambridge: University Press, 1970. – 422 p.

2. **Hilbert, D.** Begrundung der kinetishen Gastheorie / D. Hilbert // Math. Annalen. – 1912. – Vol. 72. – P. 562 – 577.

3. **Bogoliubov, N. N.** Problems of dynamic theory in statistical physics / N. N. Bogolyubov. – Oak Ridge: Technical Inform. Serv., 1960.

4. **Grad, H.** Asymptotic of the Boltzmann equation / H. Grad // Physics of Fluids. – 1963. – Vol. 6, No. 2. – P. 147-181.

5. **Cercignani, C.** Mathematical methods in kinetic theory / C. Cercignani. – N.Y.: Springer, 1990. – 262 p.

6. **Corban, A.N.** Invariant manifolds for physical and chemical kinetics / A.N. Corban, I.V. Karlin. – Berlin: Springer, 2005. – 495 p.

7. **Bogolyubov, N. N.** On the stochastic processes in the dynamical systems / N. N. Bogolyubov. – Dubna: JINR, E17-10514, 1977. – 130 p.;

8. **Akhiezer, A. I.** Methods of statistical physics (Int. Ser. in Nat. Phil. – Vol. 104) / A. I. Akhiezer, S. V. Peletminsky. – Oxford: Pergamon Press, 1981. – 450 p.

Received 15.04.2014.