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# ON PHOTON KINETICS IN EQUILIBRIUM PLASMA MEDIUM

The photon kinetics in equilibrium plasma is investigated on the basis of the Bogolyubov reduced description method. A starting point of the analysis is a kinetic equation which takes into account the Compton and the bremsstrahlung photon processes. The photon polarization, electron spin phenomena and plasma ion effects are neglected. It is assumed that all processes in the system can be considered in quasi-relativistic approximation. In this approximation change of a photon (an electron) energy in the Compton process is small that is described by small parameter  $\lambda$ . The states of the system are considered as spatially weakly non-uniform that is described by small parameter g. The situation is investigated in which contribution of the bremsstrahlung processes is small compared to the Compton processes (for simplicity this is described by the small parameter g too). Reduced description of photons in the plasma by photon energy distribution is investigated. The consideration is based on the Bogolyubov functional hypothesis that leads to a generalization of the Chapman-Enskog method. The kinetic equation for the photon energy distribution that describes the photon diffusion similar to the known neutron diffusion equation is obtained.

**Keywords:** photon diffusion, completely ionized plasma, Compton scattering, bremsstrahlung process, reduced description method, functional hypothesis.

На основі метода скороченого опису Боголюбова досліджується кінетика фотонів у рівноважному плазмовому середовищі. Стартовою точкою аналізу є кінетичне рівняння, яке враховує комптонівські та тормозні фотонні процеси. Поляризацією фотона, спіном електрона та впливом іонів нехтуємо. Вважається, що всі процесі в системі можуть розглядатися у квазірелятивістскому наближенні. В цьому наближенні зміна енергії фотона (електрона) в комптонівському процесі мала, що описується малим параметром  $\lambda$ . Стани системи вважаються просторово слабко неоднорідними, що описується малим параметром g. Вивчається ситуація, в якій внесок тормозних процесів є малий порівняно з внеском комптонівських процесів (для простоти це описується також малим параметром g). Досліджується скорочений опис фотонів у плазмі за допомогою функції розподілу фотона за енергією. Розгляд ґрунтується на ідеї функціональної гіпотези Боголюбова, що веде до узагальнення метода Чепмена-Енскога. Отримується кінетичне рівняння для функції розподілу фотона за енергією, яке описує дифузію фотонів і подібне до відомого рівняння дифузії нейтронів.

**Ключові слова:** дифузія фотонів, повністю іонізована плазма, комптонівське розсіяння, тормозний процес, метод скороченого опису, функціональна гіпотеза..

На основе метода сокращенного описания Боголюбова исследуется кинетика фотонов в равновесной плазменной среде. Стартовой точкой анализа является кинетическое уравнение, которое учитывает комптоновские и тормозные процессы. Поляризацией фотонов, спином электронов и влиянием ионов пренебрегаем. Считается, что все процессы в системе можно рассматривать в квазирелятивистском приближении. В этом приближении изменение энергии фотона (электрона) в комптоновском процессе мало, что описывается малым параметром  $\lambda$ . Состояния системы считаются пространственно слабо неоднородными, что описывается малым параметром g. Изучается ситуация, в которой вклад тормозных процессов является малым по сравнению с вкладом комптоновских процессов (для простоты это описывается также малым параметром g). Исследуется сокращенное описание фотонов в плазме с помощью функции распределения фотона по энергиям. Рассмотрение основывается на идее функциональной гипотезы Боголюбова, что ведет к обобщению метода Чепмена-Энскога. Получается кинетическое уравнение для функции распределения фотона по энергиям, которое описывает диффузию фотонов и подобно известному уравнению диффузии нейтронов.

**Ключевые слова:** диффузия фотонов, полностью ионизированная плазма, комптоновское рассеяние, тормозной процесс, метод сокращенного описания, функциональная гипотеза.

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#### 1. Introduction

The study of kinetics of electromagnetic field in a medium is an actual problem. It has several areas of focus. A number of fundamental problems of the quantum theory of electromagnetic field is under active discussion now (see, for example, review [1]). An important direction of the studies is the radiative transfer theory, in which the problem of choice of parameters describing the radiation and of consistent parameters of the medium is still under consideration [2, 3]. Many problems of the electromagnetic field kinetics are considered in quantum optics revealing the complexity of the quantum electromagnetic field even at low energies [4]. In this direction the paper by Kompaneets [5], in which kinetics of photons in a plasma medium was investigated with taking into account bremsstrahlung and Compton processes, deserves attention.

It is important to conduct mentioned studies in terms of the modern theory of non-equilibrium processes and, in particular, on the basis of the Bogolyubov method of reduced description of nonequilibrium states (see a review of this method in [6]). Kinetic equation for photons in such medium on the basis of the reduced description method was obtained by Akhiezer and Peletminsky [7].

In this paper the reduced description of the photon gas kinetics in equilibrium plasma is investigated taking into account the Compton scattering and the bremsstrahlung in the quasi-relativistic approximation. It is assumed that the system evolution is determined by the Compton scattering with small corrections related to the bremsstrahlung. In spatially non-uniform states the proposed theory describes diffusion of photons in plasma. The mentioned photon processes are widely discussed in the literature (see, for example, [8]).

The plan of the paper is as it follows: in the Sec. 2 the basic equations of the reduced description of nonequilibrium states of the system and small parameters of the theory are introduced; in Sec. 3 the perturbation theory is constructed and the kinetic equation for photon energy distribution is obtained.

# 2. Basic equations of the theory

We assume that the evolution of the photon system in rarefied completely ionized equilibrium plasma is determined by photon bremsstrahlung processes and Compton scattering of photons. Ion dynamics is neglected. The kinetic equation for the distribution of photons  $f_p(x,t)$  in the medium has the form [5, 7]

$$\frac{\partial \mathbf{f}_{p}(x,t)}{\partial t} = -c \frac{p_{l}}{p} \frac{\partial \mathbf{f}_{p}(x,t)}{\partial x_{l}} + I_{p}^{B}(\mathbf{f}(x,t)) + I_{p}^{C}(\mathbf{f}(x,t))$$
(1)

with the Compton  $I_p^c(f)$  and the bremsstrahlung  $I_p^B(f)$  collision integrals

$$I_p^C(\mathbf{f}) = \int d^3 p_1 d^3 p_1' d^3 p' W(p, p_1; p', p_1') \left\{ \mathbf{f}_{p'}(1 + \mathbf{f}_p) w_{p_1'} - \mathbf{f}_p(1 + \mathbf{f}_{p'}) w_{p_1} \right\} \times$$

$$\times \delta(p + p_1 - p' - p_1') \delta(\varepsilon_p + E_{p_1} - \varepsilon_{p'} - E_{p_1'}), \qquad (2)$$

$$I_p^B(\mathbf{f}) = -\left(\mathbf{f}_p - n_{\varepsilon_n}\right) / \tau_{\varepsilon_n}^B. \tag{3}$$

Here  $E_p \equiv c[(mc)^2 + p^2]^{1/2}$ , and  $\varepsilon_p \equiv cp$  are electron and photon energies;  $n_{\varepsilon} = [e^{\varepsilon/T} - 1]^{-1}$  is the Planck distribution;  $\tau_{\varepsilon_p}^B$  is a characteristic time). Polarization phenomena are ne-

glected, it is assumed that the Wigner distribution function of photons  $f_p(x,t)$  does not depend on polarization indexes and is normalized by the condition

$$\frac{2}{h^3} \int d^3 p f_p(x,t) = n_{ph}(x,t)$$
 (4)

where  $n_{ph}(x,t)$  is the total number density of the photons. Similarly, we assume that the equilibrium distribution function of electrons does not depend on the spin indexes and is given by a Maxwell distribution

$$w_p = \frac{nh^3}{2(2\pi mT)^{3/2}} e^{-E_p/T}$$
 (5)

(*n* is the total number density of the electrons). In this case,  $W(p, p_1; p', p'_1)$  can be considered as the probability of the scattering process of a photon by an electron per unit of time, summed over the final polarizations and averaged over their initial values.

The problem is investigated in quasi-relativistic approximation. In this situation one may formally consider the inverse light velocity 1/c as a small parameter and build the corresponding perturbation theory. With this end in view let us consider photon energy change at a collision  $\varepsilon' = F(\varepsilon, p_1, n, n')$  ( $p = \varepsilon n/c$ ,  $p' = \varepsilon' n'/c$ ; n, n' are unit vectors). Energy and momentum conservation laws in (2) give an equation

$$\varepsilon + E_{p_1} = \varepsilon' + E_{(\varepsilon'n' - \varepsilon n)/c - p_1} \tag{6}$$

from which the expression follows

$$\varepsilon' = \varepsilon + \frac{\varepsilon}{mc} p_1(n'-n) + \frac{\varepsilon}{m^2 c^2} (p_1 n') p_1(n'-n) - \frac{\varepsilon^2}{mc^2} (1-nn') + O(1/c^3). \tag{7}$$

So, in the Compton process photon energy change is small in the quasi-relativistic approximation. The electron momentum enters the Maxwell distribution and can be estimated as  $p_1 \leq (mT)^{1/2}$ . Therefore, dimensionless small quantity of the presented theory is  $\lambda = (T/mc^2)^{1/2}$ . The forth term in (7) can be estimated as a second order contribution at photon energies  $\varepsilon \leq mc^2\lambda^2 = T$  under consideration. In this case  $\varepsilon' - \varepsilon \leq \lambda T$ ,  $|E_{p_1'} - E_{p_1}| \leq \lambda T$  ( $E_{p_1}, E_{p_1'} \leq T$ ) and the Compton collision integral (2) can be approximated on the basis of expansion

$$\delta(\varepsilon_{p'} - \varepsilon_p + E_{p'_1} - E_{p_1}) = \sum_{0 \le s \le n} \frac{1}{s!} \delta^{(s)}(\varepsilon_{p'} - \varepsilon_p) (E_{p'_1} - E_{p_1})^s + O(\lambda^{n+1}).$$
 (8)

The corresponding series expansion of the Compton collision integral can be written in the form

$$I_p^C(\mathbf{f}) = \sum_{s=0}^{\infty} I_{sp}(\mathbf{f}), \qquad I_{sp}(\mathbf{f}) \sim \lambda^s,$$

$$I_{sp}(f) = \int d^3 p' \{ W_s(p, p') f_{p'}(1 + f_p) + (-1)^{s+1} W_s(p', p) f_p(1 + f_{p'}) \} \delta^{(s)}(\varepsilon_p - \varepsilon_{p'});$$
 (9)

$$W_{s}(p,p') = \frac{1}{s!} \int d^{3}p_{1}d^{3}p'_{1}W(p,p_{1};p',p'_{1})w_{p'_{1}}\delta(p+p_{1}-p'-p'_{1})(E_{p_{1}}-E_{p'_{1}})^{s}.$$
 (10)

Here it was taken into account that due to the detailed balance relation  $W(p, p_1; p', p'_1) = W(p', p'_1; p, p_1)$  holds.

Further it is convenient to express momenta of photons through their energy and introduce notations

$$f_{p}(x,t)|_{p=n\varepsilon/c} = \tilde{f}_{\varepsilon}(n,x,t), \quad I_{p}^{C}(f)|_{p=n\varepsilon/c} = I_{\varepsilon}^{C}(n,\tilde{f}), \quad I_{sp}(f)|_{p=n\varepsilon/c} = I_{s\varepsilon}(n,\tilde{f}),$$

$$I_{p}^{B}(f)|_{p=n\varepsilon/c} = I_{\varepsilon}^{B}(n,\tilde{f}). \tag{11}$$

The most simple expression is obtained from (9) and (10) for the functions

$$I_{0\varepsilon}(n,\tilde{\mathbf{f}}) = \frac{\varepsilon^2}{c^3} \int d\Omega_n \Phi_0(\varepsilon,\varepsilon,nn') \{\tilde{\mathbf{f}}_{\varepsilon}(n') - \tilde{\mathbf{f}}_{\varepsilon}(n)\}, \quad I_{\varepsilon}^B(n,\tilde{\mathbf{f}}) = -[\tilde{\mathbf{f}}_{\varepsilon}(n) - n_{\varepsilon}]/\tau_{\varepsilon}^B \quad (12)$$

where

$$\Phi_{s}\left(\varepsilon,\varepsilon',nn'\right) \equiv W_{s}\left(p',p\right)\Big|_{p=\varepsilon n/c,\ p'=\varepsilon'n'/c} \tag{13}$$

The function  $I_{0\varepsilon}(n,\tilde{\mathbf{f}})$  has the property

$$\int d\Omega_n I_{0\varepsilon}(n,\tilde{\mathbf{f}}) = 0. \tag{14}$$

Let us consider the relation

$$\frac{\partial}{\partial t} \int d\Omega_n \tilde{\mathbf{f}}_{\varepsilon}(n, x, t) = -c \int d\Omega_n n_l \frac{\partial \tilde{\mathbf{f}}_{\varepsilon}(n, x, t)}{\partial x_l} + \int d\Omega_n [I_{\varepsilon}^C(n, \tilde{\mathbf{f}}(x, t)) + I_{\varepsilon}^B(n, \tilde{\mathbf{f}}(x, t))]$$
(15)

following from (1), (2), and (11). In this paper the situation is considered in which the Compton process is slower than the bremsstrahlung one. From this point of view,  $I_{\varepsilon}^{B}(n,\tilde{\mathbf{f}}(x,t))$  contribution to (15) is small. In the present paper weakly non-uniform states of the photon system are investigated. In order to take into account the last two ideas, the additional small parameter g is introduced by estimates

$$\partial^s \tilde{\mathbf{f}}_{\varepsilon}(n,x,t)/\partial x_L...\partial x_l \sim g^s, \quad I_{\varepsilon}^B(n,\tilde{\mathbf{f}}) \sim g^1.$$
 (16)

The first estimate introduces the parameter g in a standard manner as the value of the order of the ratio of the mean free path of a photon to the characteristic distance at which the function varies significantly. The second estimate is made for simplicity. In paper [5] it was argued that this corresponds to photons with  $\omega \gg \omega_0$  where  $\omega_0$  is a frequency defined by relation  $\tau_{\hbar\omega_0}^B = \tau_C$ . Here  $\tau_C \equiv (mc^2/T)(l/c)$  is the Compton process characteristic time (l is photon free path defined by the Compton scattering). According to [5]  $\tau_{\hbar\omega}^B$  is an increasing function of the frequency  $\omega$ . Therefore, at times  $t \gg \tau_0 \equiv \tau_{\hbar\omega_0}^B$  all photons with  $\omega \leq \omega_0$  will be in an equilibrium state and for  $\omega \gg \omega_0$  the relation

 $\tau_{h\omega}^{B} \gg \tau_{C}$  is true. So, in this situation photon evolution is determined by the Compton processes with small corrections from the bremsstrahlung processes.

Introduced small parameters of the theory show that the quantity  $\int d\Omega_n \tilde{\mathbf{f}}_{\epsilon}(n,x,t)$  changes slowly with time in weakly nonuniform states, when parameters  $\lambda$ , g are small. This result suggests that the reduced description of the system is possible by the photon energy distribution  $\varphi_{\epsilon}(x,t)$ . The corresponding theory can be based on the Bogolyubov idea of the functional hypothesis in the form

$$\tilde{\mathbf{f}}_{\varepsilon}(n,x,t) \xrightarrow{t \gg \tau_0} \tilde{\mathbf{f}}_{\varepsilon}(n,x,\varphi(t)), \qquad \varphi_{\varepsilon}(x) \equiv \frac{1}{4\pi} \int d\Omega_n \tilde{\mathbf{f}}_{\varepsilon}(n,x,\varphi)$$
 (17)

where  $\tau_0$  is a characteristic time defined above by the formula  $\tau_0 = \tau_C$  (functional  $\tilde{\mathbf{f}}_{\varepsilon}(n,x,\phi)$  does not depend on  $\tilde{\mathbf{f}}_{\varepsilon}(n,x,0)$ ). In this situation the distribution function  $\tilde{\mathbf{f}}_{\varepsilon}(n,x,\phi)$  is a functional of the photon energy distribution  $\phi_{\varepsilon}(x)$ . The functional hypothesis (17) and the equations (1), (2) lead to the following kinetic equation for  $\phi_{\varepsilon}(x,t)$ 

$$\frac{\partial \varphi_{\varepsilon}(x,t)}{\partial t} = L_{\varepsilon}(x,\varphi(t)),$$

$$L_{\varepsilon}(x,\varphi) = \frac{1}{4\pi} \int d\Omega_{n} \left\{ -cn_{l} \frac{\partial \tilde{f}_{\varepsilon}(n,x,\varphi)}{\partial x_{l}} + I_{\varepsilon}^{C}(n,\tilde{f}(x,\varphi)) + I_{\varepsilon}^{B}(n,\tilde{f}(x,\varphi)) \right\}. \tag{18}$$

The functional  $\tilde{\mathbf{f}}_{\varepsilon}(n,x,\varphi)$  according to (1), (17), and (18) satisfies the equation

$$\int d^3x' d\varepsilon' \frac{\delta \tilde{\mathbf{f}}_{\varepsilon}(n, x, \varphi)}{\delta \varphi_{\varepsilon'}(x')} L_{\varepsilon'}(x', \varphi) = -cn_l \frac{\partial \tilde{\mathbf{f}}_{\varepsilon}(n, x, \varphi)}{\partial x_l} + I_{\varepsilon}^C(n, \tilde{\mathbf{f}}(x, \varphi)) + I_{\varepsilon}^B(n, \tilde{\mathbf{f}}(x, \varphi))$$
(19)

which should be solved with taking into account the second formula in (17) as an additional condition. This equation is nonlinear integro-differential one and its solution can be found only approximately.

# 3. Perturbation theory

The basic equations of the theory should be solved in a double perturbation theory in parameters g and  $\lambda$ . An important role is played by arguments of rotational invariance and based on them relations

$$I_{cc}(n,\varphi) = I_{cc}(\varphi)$$
,

$$I_{s\varepsilon}(\varphi) \equiv \int_{0}^{\infty} d\varepsilon' \varepsilon'^{2} \left[ F_{s}(\varepsilon, \varepsilon') (1 + \varphi_{\varepsilon}) \varphi_{\varepsilon'} + (-1)^{s+1} F_{s}(\varepsilon', \varepsilon) \varphi_{\varepsilon} (1 + \varphi_{\varepsilon'}) \right] \delta^{(s)}(\varepsilon - \varepsilon'),$$

$$F_s(\varepsilon, \varepsilon') = \frac{1}{c^3} \int d\Omega_{n'} \Phi_s(\varepsilon, \varepsilon', nn') = \frac{1}{c^3} \int_{-\infty}^{\infty} du \Phi_s(\varepsilon, \varepsilon', u)$$
 (20)

derived from the definitions (9), (10), and (13).

The solution of Eq. (19) and calculation of the right-hand side of Eq. (18) are based on expansions of the distribution function  $\tilde{f}_s(n,x,\varphi)$  in parameters g and  $\lambda$ 

$$f_{\varepsilon}(n,x,\varphi) = f_{\varepsilon}^{(0)} + f_{\varepsilon}^{(1)} + O(g^{2}), \qquad f_{\varepsilon}^{(m)} = f_{\varepsilon}^{(m,0)} + f_{\varepsilon}^{(m,1)} + O(g^{m}\lambda^{2});$$

$$L_{\varepsilon}(x,\varphi) = L_{\varepsilon}^{(0)} + L_{\varepsilon}^{(1)} + O(g^{2}), \qquad L_{\varepsilon}^{(m)} = L_{\varepsilon}^{(m,0)} + L_{\varepsilon}^{(m,1)} + O(g^{m}\lambda^{2}). \tag{21}$$

In the zero approximation in the gradients the distribution function  $f_{\epsilon}(n, x, \varphi)$  and the right-hand side of Eq. (18) have the structure

$$f_s^{(0)}(n, x, \varphi) = h_s(n, \varphi(x)), \qquad L_s^{(0)}(x, \varphi) = M_s(\varphi(x))$$
 (22)

where  $h_{\epsilon}(n,\varphi)$ ,  $M_{\epsilon}(\varphi)$  are some functions. In the considered case the integral equation (19) and the second relation (17) give the following equation for  $h_{\epsilon}(n,\varphi)$  with the additional condition

$$\int d\varepsilon' \frac{\partial h_{\varepsilon}(n,\varphi)}{\partial \varphi_{\varepsilon'}} M_{\varepsilon'}(\varphi) = I_{\varepsilon}(n,h(\varphi)), \qquad \varphi_{\varepsilon} = \frac{1}{4\pi} \int d\Omega_n h_{\varepsilon}(n,\varphi)$$
(23)

where according to (18)

$$M_{\varepsilon}(\varphi) = \frac{1}{4\pi} \int d\Omega_n I_{\varepsilon}(n, h(\varphi)). \tag{24}$$

The solution of Eq. (23) is sought in the form of a power series  $\lambda$ . Simple consideration shows that

$$f_{\varepsilon}^{(0)}(n, x, \varphi) = \varphi_{\varepsilon}(x) \tag{25}$$

because solution of the equation  $0 = I_{0\varepsilon}(n,h)$  is an arbitrary function of  $\varepsilon$ .

At the same time, according to (20), (22), and (24), the right side of Eq. (18) in the zero order approximation in gradients is given by the formula

$$L_{\varepsilon}^{(0)}(x,\varphi) = \sum_{1 \le s < \infty} I_{s\varepsilon}(\varphi(x))$$
 (26)

In other words, in a spatially uniform case the right side of the time equation for  $\phi_{\varepsilon_p}(x,t)$  is given by substituting this function into the collision integral (2).

Thus, in the zero approximation in the gradients (in other words, in the spatially uniform case) the distribution function of the photon after a transition process with duration  $\tau_0$  (17) ceases to depend on the direction of the photon momentum. This justifies the initial assumptions of [5], where the author is limited to consideration of an isotropic distribution of photons in a spatially uniform case without the discussion of this view generality.

We limit calculations in the first order in the gradients to the main contributions of the perturbation theory in  $\lambda$ , that is by  $f_{\epsilon}^{(1,0)}(n,\phi)$ . To do this, we first note that, according to (18) with (12) and (14)

$$L_{\varepsilon}^{(1,0)} = \frac{1}{4\pi} \int d\Omega_n \left\{ -cn_l \frac{\partial \varphi_{\varepsilon}}{\partial x_l} + I_{0\varepsilon}(\tilde{\mathbf{f}}^{(1,0)}) - \frac{\varphi_{\varepsilon} - n_{\varepsilon}}{\tau_{\varepsilon}^B} \right\} = -\frac{\varphi_{\varepsilon} - n_{\varepsilon}}{\tau_{\varepsilon}^B}.$$
 (27)

Taking into account (26) and (27), the integral equation (19) and the additional condition (17) give the equation for  $f_{\epsilon}^{(1,0)}(n,\varphi)$ 

$$0 = -cn_l \frac{\partial \varphi_{\varepsilon}}{\partial x_l} + I_{0\varepsilon}(n, \tilde{\mathbf{f}}^{(1,0)}), \qquad \int d\Omega_n \mathbf{f}_{\varepsilon}^{(1,0)}(n, \varphi) = 0$$
 (28)

Arguments of rotational invariance show that their solution can be sought in the form

$$f_{\varepsilon}^{(1,0)}(n,x,\varphi) = a_{\varepsilon} n_{l} \frac{\partial \varphi_{\varepsilon}(x)}{\partial x_{l}}$$
(29)

where  $a_{\varepsilon}$  is a function. Really, according to (12) the formula

$$\frac{\varepsilon^2}{c^3} \int d\Omega_{n'} \Phi_0(\varepsilon, \varepsilon, nn') (n'_l - n_l) = -n_l / \tau_{\varepsilon}$$
(30)

with a function  $\tau_{\epsilon}$  is true (according to (10) and (13)  $\tau_{\epsilon} > 0$ ). So, Eq. (30) gives  $a_{\epsilon} = -c\tau_{\epsilon}$  and therefore

$$\mathbf{f}_{\varepsilon}^{(1,0)}(n,x,\mathbf{\phi}) = -c\mathbf{\tau}_{\varepsilon}n_{l}\frac{\partial\mathbf{\phi}_{\varepsilon}(x)}{\partial x_{l}}.$$
(32)

According (14) and (17) the corresponding contribution to the function  $L_{\varepsilon}(x,\varphi)$  from (18)

$$L_{\varepsilon}^{(2,0)} = \frac{1}{4\pi} \int d\Omega_n \left\{ -cn_l \frac{\partial \tilde{\mathbf{f}}_{\varepsilon}^{(1,0)}}{\partial x_l} + I_{0\varepsilon}(\tilde{\mathbf{f}}^{(2,0)}) - \frac{\tilde{\mathbf{f}}_{\varepsilon}^{(2,0)}}{\tau_{\varepsilon}^B} \right\} = \frac{\tau_{\varepsilon}}{3} \Delta \varphi_{\varepsilon}(x)$$
(33)

Giving the final results, we restrict ourselves by calculation of  $L_{\varepsilon}^{(0)}(x,\varphi)$  on the basis of (26) up to the second order in  $\lambda$ . Taking this and contribution (27), (33) into account, gives the following kinetic equation for the photon energy distribution function  $\varphi_{\varepsilon}(x,t)$ 

$$\frac{\partial \varphi_{\varepsilon}}{\partial t} = \frac{1}{\varepsilon^{2}} \frac{\partial}{\partial \varepsilon} \left\{ \varepsilon^{4} \left[ \left( g_{1}^{00}(\varepsilon) + g_{2}^{10}(\varepsilon) - g_{2}^{01}(\varepsilon) \right) \varphi_{\varepsilon}(1 + \varphi_{\varepsilon}) + g_{2}^{00}(\varepsilon) \frac{\partial \varphi_{\varepsilon}}{\partial \varepsilon} \right] \right\} +$$

$$-(\varphi_{\varepsilon} - n_{\varepsilon})/\tau_{\varepsilon}^{B} + D(\varepsilon)\Delta\varphi_{\varepsilon}(x) + O(g^{0}\lambda^{3}, g^{1}\lambda^{1}, g^{2}\lambda^{1})$$
(34)

where denoted

$$g_s^{mn}(\varepsilon) = \partial^{m+n} F_s(\varepsilon, \varepsilon') / \partial \varepsilon^m \partial \varepsilon'^n |_{\varepsilon' = \varepsilon}, \qquad D(\varepsilon) = \tau_{\varepsilon} / 3$$
 (35)

Equation (34) can be called the photon diffusion equation. It is an analogue of the neutron diffusion equation (see, for example, [6]). The contribution of zero approximation in the gradients in Eq. (34) does not coincide with the expression obtained in [5]. This is a consequence of an assumption made in this paper that it should vanish by substitution the Planck distribution  $\varphi_{\varepsilon} \to n_{\varepsilon}$ ,  $n_{\varepsilon} \equiv (e^{\varepsilon/T} - 1)^{-1}$ . However, substitution  $f_p \to n_{\varepsilon_p}$  reduces to zero only the full collision integral (2). This property is connected with full  $\delta$ -function entering (2) and is absent in finite sum of the series (8). For investigating the processes close to the equilibrium it is more convenient to use an equation of the type (34), but obtained from the linearized kinetic equation (1).

#### 5. Conclusions

The photon kinetics in equilibrium plasma medium is investigated on the basis of the Bogolyubov reduced description method. The analysis is based on kinetic equation for the photon Wigner distribution function  $f_n(x,t)$  which takes into account the Compton and the bremsstrahlung photon processes. The photon polarization, electron spin phenomena and plasma ion effects are neglected. It is assumed that all processes in the system can be considered in quasi-relativistic approximation. In this approximation change of a photon (an electron) energy in the Compton process is small quantity of the order  $\lambda \equiv (T/mc^2)^{1/2}$ . The states of the system are considered as weakly non-uniform in the space and gradients of the distribution  $f_n(x,t)$  are estimated by small parameter g. A situation is investigated in which contribution of the bremsstrahlung processes is small compared with the Compton processes. It takes place at photon frequencies  $\omega \gg \omega_0$ where the characteristic frequency  $\omega_0$  was estimated by Kompaneets [5]. For simplicity of the consideration as the corresponding small parameter the parameter g is chosen. Reduced description of photons in the plasma by photon energy distribution  $\varphi_s(x,t)$  is investigated. The consideration is based on the Bogolyubov idea of the functional hypothesis that leads to a generalization of the Chapman-Enskog method. The characteristic time  $\tau_0$  in the functional hypothesis is chosen in the present paper following to nontrivial arguments that give  $\tau_0 = \tau_C$ . The kinetic equation for the photon energy distribution is obtained that describes the photon diffusion similar to the known neutron diffusion equation (see, for example, [6]). The calculation is conducted in perturbation theory in small parameters g and  $\lambda$ .

# References

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