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A. N. Turinov

Oles Honchar Dnipropetrovsk National University, Dnipropetrovsk, Ukraine
e-mail: jen1275@mail.ru

TOLMAN'S VOIDS IN THE FRIEDMAN UNIVERSE

Discovery of regions in the Universe with density of matter which is significantly lower than that of their surroundings (so-called voids) enhances theoretical studies of these objects and their effect on the evolution of the Universe. As it becomes clear, similar violation of the homogeneity and isotropy properties of the Universe takes place also at significantly large (compared with distances between galaxies) scales (the tens and hundreds of megaparsec). Voids in the Friedman Universe are simulated by a spherically symmetric regions described by Tolman dust space-times. In this paper, we consider the modeling of a void by matching two different metrics and studying their temporal evolution. As matching conditions the Lichnerowicz–Darmois conditions are used. It is found that the space curvature of a void should be of the same sign as the external space one. The voids cannot exist in the flat Friedman Universe. The voids of two different classes in the hyperbolic Friedman Universe are built. Matter in such voids is always "older" than in their environment. The results of studying the void model parameters are presented.

Keywords: cosmological models, metric, Tolman Universe, voids, evolution.

Відкриття у Всесвіті областей з густиною матерії, значно нижчою за густину навколишнього простору (так званих порожнин), активізувало теоретичне дослідження цих об'єктів та їх вплив на еволюцію Всесвіту. Як стало зрозуміло аналогічне порушення властивостей однорідності й ізотропії Всесвіту відбувається й на досить великих (порівняно з відстанями між галактиками) масштабах (десятки та сотні мегапарсек). У Всесвіті Фрідмана порожнини моделюються як області, що описуються простором-часом Толмена. В даній роботі пропонується побудова моделі порожнини шляхом зшивки двох різних метрик і розгляду їх еволюції в часі. В ролі умов зшивки використовуються умови Ліхнеровича–Дармуа. Просторова кривина порожнин має бути такою самою, як і просторова кривина зовнішнього простору. У Всесвіті Фрідмана нульової просторової кривини порожнин не існує. Побудовано моделі порожнин двох різних класів у Всесвіті Фрідмана від'ємної просторової кривини. Речовина в цих порожнинах завжди "старіша", ніж в оточуючому просторі. Наведено результати дослідження параметрів моделей розглядуваних порожнин.

Ключові слова: космологічні моделі, метрика, толменівський Всесвіт, порожнини, еволюція.

Открытие во Вселенной областей с плотностью материи значительно ниже плотности окружающего пространства (так называемых пустот) активизировало теоретическое исследование этих объектов и их влияние на эволюцию Вселенной. Стало ясно, что аналогичное нарушение свойств однородности и изотропии Вселенной происходит и на достаточно больших (в сравнении с расстояниями между галактиками) масштабах (десятки и сотни мегапарсек). Во Вселенной Фридмана пустоты моделируются как области, описываемые пространством-временем Толмена. В данной работе предлагается построение модели пустоты путем сшивки двух разных метрик и рассмотрения их эволюции во времени. В качестве условий сшивки используются условия Лихнеровича–Дармуа. Пространственная кривизна пустот должна быть такой же, как и пространственная кривизна внешнего пространства. Во Вселенной Фридмана нулевой пространственной кривизны пустот не существует. Построены модели пустот двух различных классов во Вселенной Фридмана отрицательной пространственной кривизны. Вещество в этих пустотах всегда "старее", чем в окружающем пространстве. Приведены результаты исследования параметров моделей рассматриваемых пустот.

Ключевые слова: космологические модели, метрика, толменовская Вселенная, пустоты, эволюция.

1. Introduction

In recent years there were many astronomical observations of large spherical regions in the Universe with density of the luminous matter, which is far less, than that of their surrounding, so-called voids [1-2]. This data enhanced theoretical studies of the evolution and effect of such regions in models of the expanding Universe. There are four main directions of investigations: small perturbation of homogeneous cosmologies, use of the Einstein-Straus vacuole, use of Tolman space-time, representation of the void boundary as the thin wall approximation [3-4]. As a result investigators come to contradictory conclusions concerning the formation and evolution of voids [5-7].

In this paper we obtain some results regarding voids in cosmological models. We use the voids, constructed by matching of Tolman solution for nonhomogeneous dust (description of void space-time) and Friedman solution for homogeneous dust (description of space-time in the surrounding Universe) as a special case of the Tolman one. To gain a continuity of the first and second fundamental forms of the matched metrics, we impose the Lichnerowicz–Darmois matching conditions. The voids described by the Minkowski space-time, cannot exist in the Friedman Universe [8]. But it is possible to choose the Tolman Universe with exotic parameters, and such Universe can contain the voids described by empty space-time. The Friedman Universe also cannot have the voids, which are described by another Friedman space-time.

2. The Tolman solution

To construct the model of voids we use the mass function method proposed in the paper [9] and considered in [10].

The Tolman solution for the motion of spherically symmetric nonuniform dust for all types of the space curvature in the curvilinear coordinate system has the form

$$ds^2 = dt^2 - \frac{r'^2(R,t)}{f^2(R)}dR^2 - r^2(R,t)(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

where

$$r(R,t) = \frac{m(R)}{1-f^2(R)}\begin{Bmatrix} \sin^2(\alpha/2) \\ -\sinh^2(\alpha/2)\end{Bmatrix} for \begin{Bmatrix} f^2(R)<1 \\ f^2(R)>1\end{Bmatrix}; \tag{2}$$

$$t - t_0(R) = \frac{m(R)}{2\left|1-f^2(R)\right|^{3/2}}\begin{Bmatrix} \alpha - \sin\alpha \\ \sinh\alpha - \alpha\end{Bmatrix} for \begin{Bmatrix} f^2(R)<1 \\ f^2(R)>1\end{Bmatrix}; \tag{3}$$

$$r(R,t) = \left[\pm\frac{3}{2}m(R)^{1/2}(t-t_0(R))\right]^{2/3} for\ f^2(R)=1. \tag{4}$$

The energy density is

$$\varepsilon(R,t) = \frac{1}{8\pi\gamma}\frac{m'(R)}{r^2(R,t)r'(R,t)}. \tag{5}$$

The speed of light $c=1$. The constant $a_0 = 4\gamma M/3\pi c^2$. It has the sense of the maximum radius of the world for a positive space curvature. The mass function $m(R)$ is an active gravitational mass inside the sphere R (the rest-mass plus rest gravitational binding energy), $f(R)$ is an arbitrary integration function. It is the total energy of a particle inside the layer R. The function $t_0(R)$ is initial moment of the collapse of layer R, a so-called temporal shift.

The Friedman solution describing the uniform dust distribution is a special case of Eqs. (1)-(5) with $t_0(R) = 0$ and the mass function

$$m(R) = a_0 \begin{Bmatrix} \sin^3 R \\ \sinh^3 R \\ R^3 \end{Bmatrix} \text{for} \begin{Bmatrix} f^2(R) = \cos^2 R \\ f^2(R) = \cosh^2 R, \\ f^2(R) = 1 \end{Bmatrix} \quad (6)$$

for positive, negative, and zero space curvature, accordingly.

3. The matching conditions

As matching conditions of our metrics we choose the Lichnerowicz–Darmois conditions [11]. They represent the equality of the first and the second quadratic forms of intrinsic and external metrics on the matching hypersurface. As the boundary of the void we takes the hypersurface $R = R_{\text{b}} = \text{const}$. From matching conditions it follows that on this hypersurface the following conditions should be fulfilled

$$r_{\text{T}}(R_{\text{b}}, t_{\text{T}}) = r_{\text{F}}(R_{\text{b}}, t_{\text{F}}),\ f_{\text{T}}(R_{\text{b}}) = f_{\text{F}}(R_{\text{b}}),\ m_{\text{T}}(R_{\text{b}}) = m_{\text{F}}(R_{\text{b}}) \quad (7)$$

where the characters T and F denote values concerning to Tolman and Friedman space-time, accordingly. After applying the matching conditions (7) the Tolman solution remains enough arbitrary. However, these conditions require the space curvature of the voids to be of the same sign as the space curvature of external space. Without loss of generality it is possible to suppose the function $f(R)$ be the same in both metrics (1)-(6). Starting from (7), we obtain that on the matching surface $\alpha_{\text{T}}(R_{\text{b}}, t_{\text{T}}) = \alpha_{\text{F}}(t_{\text{F}})$, therefore

$$t_{\text{T}} - t_0(R_{\text{b}}) = t_{\text{F}}. \quad (8)$$

We can conclude that if $t_0(R) \neq 0$ matter in the voids is "older" than in environment. For the first time this fact was noted by Bonnor [8, 12]. Let us emphasize that if the times for Tolman and Friedman spaces on the matching surface are equal, i. e. $t_0(R) = \text{const}$, the voids do not exist.

4. Voids in the flat Friedman Universe

The mean density of matter in the voids is

$$\bar{\varepsilon}_{\text{T}} \equiv \bar{\varepsilon}(t_T) = \frac{M_T}{V_T}, \quad (9)$$

where $M = \int_0^{R_b} \varepsilon \sqrt{-g}\, dR d\theta d\varphi$ and $V = \int_0^{R_b} \sqrt{-g}\, dR d\theta d\varphi$ are the mass and the size of the void. Taking into account Eqs. (1)-(5), we can rewrite the expressions for mass and size of the void in the case of Tolman space as

$$M = \int_0^{R_b} \frac{m_T'(R)}{f_T(R)} dR, \quad V = \int_0^{R_b} \frac{r_T^2(R,t) r_T'(R,t)}{f_T(R)} dR. \tag{10}$$

Substituting expression (10) in (9), we obtain

$$\bar{\varepsilon}_{\mathrm{T}} = 3m_T(R_b) \big/ r_T^3(R_b, t_T). \tag{11}$$

The expression for mean density (11) is fulfilling for any Tolman solution, including the Friedman one.

Due to the homogeneity of the Friedman space we get $\varepsilon_F(t) = \dfrac{m_F'(R)}{r_F^2 r_F'} = \dfrac{3m_F(R)}{r_F^3(R,t_F)}$.

As $\varepsilon_{\mathrm{F}}(t)$ does not depend on R, we always can replace R by R_{b}

$$\bar{\varepsilon}_{\mathrm{F}} \equiv \varepsilon_F(t) = 3m_F(R_b) \big/ r_F^3(R_b, t_F). \tag{12}$$

Comparing Eqs. (12) and (11), we can conclude that in the Universe with zero space curvature it is impossible to build voids by our method. In this case the homogeneous energy density of external space is equal to the void mean energy density.

5. The model of voids in the hyperbolic Friedman world

To describe the external space we consider the Friedman solution

$$r_{\mathrm{F}}(R, t_{\mathrm{F}}) = a_0 \sinh(R) \sinh^2(\alpha_F/2), \; t_{\mathrm{F}} = \frac{a_0}{2}(\sinh\alpha - \alpha). \tag{13}$$

To take the Tolman solution for void description we should choose the mass function. The last one can be arbitrary enough because it is constrained only by the matching conditions (7)

$$m_{\mathrm{T}}(R) = a_0 \left\{ \frac{\sinh^{n+1} R}{\sinh^{n-2} R_b} \frac{\Psi(R)}{\Psi(R_b)} + (F(R) - F(R_b))^L \right\} \tag{14}$$

where $\Psi(R)$, $\Psi(R_{\mathrm{b}})$ and $F(R)$ are arbitrary functions without singularities at $0 \le R \le R_{\mathrm{b}}$; n and L are arbitrary numbers.

Let us consider a simple case $m_{\mathrm{T}}(R) = a_0 \dfrac{\sinh^{n+1} R}{\sinh^{n-2} R_b}$, $\dfrac{\Psi(R)}{\Psi(R_b)} = 1$ and $F(R) = F(R_b) = 0$. Then

$$r_{\mathrm{T}}(R, t_{\mathrm{T}}) = a_0 \frac{\sinh^{n+1} R}{\sinh^{n-2} R_b} \sinh^2(\alpha_T/2), \tag{15}$$

$$t_{\mathrm{T}} - t_0(R) = \frac{a_0}{2} \left(\frac{\sinh R}{\sinh R_b} \right)^{n-2} (\sinh\alpha_T - \alpha_T). \tag{16}$$

From these equation it follows that there are two different classes of the Tolman solutions: for $n > 2$ and $n < 2$. Case $n = 2$ corresponds to the Friedman solution (13).

Sizes of voids. According to astronomical data, the observable voids in the Universe have sizes ranging from 25 Mpc up to 100 Mpc ($7.5\cdot 10^{25}$ cm – $3.74\cdot 10^{26}$ cm). The greatest observable void in Boötes constellation has the size of 124 Mpc. Within the framework of the considered model the sizes of voids are described by the dimensionless value R_b. The above-mentioned data should be treated as the sizes of the voids measured by an observer in the Friedman Universe. But such a size depends on the value R_b as follows

$$r_T(R_b, t_T) = r_F(R_b, t_F) = a_0 \sinh R_b \sinh^2(\alpha_F/2), \tag{17}$$

where α_F is taken from Eq. (14), $a_0 = 1.0\cdot 10^{28}\,\text{cm} = 3.5\cdot 10^{3}$ Mpc. From Eq. (17) it follows that the void size depends on the value R_b, varies with time and does not depend on the parameter n.

In Table 1 the temporal evolutions of void sizes depending on the value R_b are presented. The dimensionless value t_f is connected with time in the Friedman Universe by means of the relation

$$t_F = t_f \times 3\cdot 10^{17}\ \text{s}. \tag{18}$$

Table 1

Temporal evolution of void sizes in the Friedman Universe with negative space curvature

t_f	Size of voids (cm)		
	$R_b = 0.005$	$R_b = 0.01$	$R_b = 0.02$
0.1	$1.5\cdot 10^{25}$	$3\cdot 10^{25}$	$6\cdot 10^{25}$
0.5	$4.8\cdot 10^{25}$	$1\cdot 10^{26}$	$2\cdot 10^{26}$
1.0	$8.1\cdot 10^{25}$	$1.6\cdot 10^{26}$	$3.2\cdot 10^{26}$
1.5	$1.1\cdot 10^{26}$	$2.2\cdot 10^{26}$	$4.5\cdot 10^{26}$

The selected row represents the values of sizes at present. We can make the conclusion that the models of the voids satisfying the observational data should be characterized by the value R_b varying within the range of $0.004 < R_b \le 0.02$ (the void in Boötes constellation corresponds to $R_b = 0.023$).

Let us consider models of both classes in details.

The models of the voids for the case $n < 2$. For the model of the voids with $n = 1$ the Eqs. (14) and (15)-(16) take the form

$$m_T(R) = a_0 \sinh^2 R \sinh R_b, \tag{19}$$

$$r_T(R, t_T) = a_0 \sinh R_b \sinh^2(\alpha_T/2),\ t_T - t_0(R) = \frac{a_0}{2}\frac{\sinh R_b}{\sinh R}(\sinh \alpha_T - \alpha_T). \tag{20}$$

In the class under consideration this is the unique satisfactory solution (for integer values of n), as all the solutions with $n < 1$ have a singularity: $r \to \infty$ with $R \to 0$.

Then the mass of voids is $M_T(R_b) = 2a_0 \sinh R_b(\cosh R_b - 1)$. The configuration size can be presented in the following form

$$V_{\rm T}(R_{\rm b},t_{\rm T})=a_0^3(A_1(R_{\rm b},t_{\rm T})+B_1(R_{\rm b},t_{\rm T})).\qquad(21)$$

The function $A_1(R_{\rm b},t_{\rm T})=\dfrac{\sinh^3 R_b \sinh^6(\tilde{\alpha}_T/2)}{3\cosh R_b}$ is taken on the void boundary, therefore in order to calculate the function $\tilde{\alpha}_{\rm T}$ on the boundary, it is necessary to use the equation

$$t_{\rm T}-t_0(R_{\rm b})=a_0(\sinh\tilde{\alpha}_T-\tilde{\alpha}_T)/2=t_F.\qquad(22)$$

Thus, we can make changes $\tilde{\alpha}_{\rm T}\to\alpha_{\rm F}$ and $A_1(R_{\rm b},t_{\rm T})\to A_1(R_{\rm b},t_{\rm F})$. To calculate the function $\alpha_{\rm T}$ in expression $B_1(R_{\rm b},t_{\rm T})=\int\limits_0^{R_b}\dfrac{\sinh^3 R_b}{3}\dfrac{\sinh R\sinh^6(\alpha_T/2)}{\cosh^2 R}dR$ it is necessary to use the first equation (20).

The Friedman solution gives the following values for the mass and size of the configuration

$$M_{\rm F}(R_{\rm b})=\frac{3a_0}{2}\left(\frac{\sinh 2R_b}{2}-R_b\right),\ V_{\rm F}(R_{\rm b},t_{\rm F})=a_0^3(C_1(R_{\rm b},t_{\rm F})+D_1(R_{\rm b},t_{\rm F}))\qquad(23)$$

where to calculate $\alpha_{\rm F}(t_{\rm F})$ in expressions $C_1(R_{\rm b},t_{\rm F})=\dfrac{\sinh^3 R_b\sinh^6(\alpha_F/2)}{3\cosh R_b}$ and $D_1(R_{\rm b},t_{\rm F})=\int\limits_0^{R_b}\dfrac{\sinh^4 R\sinh^6(\alpha_F/2)}{3\cosh^2 R}dR$ the following condition is used

$$t_{\rm F}=a_0(\sinh\alpha_F-\alpha_F)/2.\qquad(24)$$

Therefore, $A_1(R_{\rm b},t_{\rm F})\equiv C_1(R_{\rm b},t_{\rm F})$.

Using the mass of voids and Eq. (21) the mean density of matter in the voids, described by Tolman solution, is calculated. In the same manner, using Eqs. (23), the density of matter in the region of the same radius, but described by Friedman solution, is calculated.

The constructed model of the hollow can be considered as the model of void provided that $\dfrac{\bar{\varepsilon}_{\rm T}}{\bar{\varepsilon}_{\rm F}}=\dfrac{M_T(R_b)}{M_F(R_b)}\dfrac{V_F(R_b,t_F)}{V_T(R_b,t_T)}<<1$. For the model under consideration we have

$$\frac{\bar{\varepsilon}_{\rm T}}{\bar{\varepsilon}_{\rm F}}=\frac{8\sinh R_b(\cosh R_b-1)}{3\sinh 2R_b-2R_b}\left(\frac{C_1(R_{\rm b},t_{\rm F})+D_1(R_{\rm b},t_{\rm F})}{C_1(R_{\rm b},t_{\rm F})+B_1(R_{\rm b},t_{\rm F})}\right).\qquad(25)$$

It should be noted that if $t_0(R)=\text{const}$ the voids do not exist. The voids can exist only if $t_0(R)\neq\text{const}$. Let us consider some models, each of which is characterized by the different value of the configuration radius $R_{\rm b}$ (0.005 and 0.01) and temporal shift $t_0(R)=R$. In Table 2 the numerical calculations of time evolutions of parameters in the considered configurations are presented. The current instant corresponds to $t_F = 3\cdot 10^{17}$ s. In all surveyed models the voids arise relatively recently, not earlier than in the moment of $0.75\cdot 3\cdot 10^{17}$ s. As a matter of record we can assert that the voids are conceived not so long ago and persist in the future.

Table 2

The parameters of voids (T) and Friedman space-time (F) for $n = 1$

R_b	t_f	V_T/V_F	M_T/M_F	$\bar{\varepsilon}_T/\bar{\varepsilon}_F$
0.005	0.10	1.013	1.0	0.987
	0.50	3.33		0.3
	1.0	41.7		0.024
	1.50	303.03		0.0033
0.01	0.10	1.011	1.0	0.989
	0.50	2.36		0.423
	1.0	22.22		0.045
	1.50	156.25		0.0064

We see that the total masses of the configurations are equal for Tolman or Friedman solution. However, the size of configuration described by the Tolman solution is much greater than the size of configuration described by the Friedman one (with the same R_b). As the result, we obtain the models of the voids satisfying the necessary requirements.

The models of the voids for the case $n > 2$. Let us consider the model of the voids with the mass function (14) and $n = 3$. In this case Eqs. (14)-(16) are reduced to

$$m_T(R) = a_0 \frac{\sinh^4 R}{\sinh R_b}, \tag{26}$$

$$r_T(R,t_T) = a_0 \frac{\sinh^2 R}{\sinh R_b}\sinh^2(\alpha_T/2),\; t_T - t_0(R) = \frac{a_0}{2}\frac{\sinh R}{\sinh R_b}(\sinh\alpha_T - \alpha_T) \tag{27}$$

Then the void mass is $M_T(R_b) = \frac{8a_0}{3}\frac{\sinh^3(R_b/2)}{\cosh(R_b/2)}(\cosh R_b + 2)$. The configuration size can be written in the form $V_T(R_b,t_T) = a_0^3(A_1(R_b,t_T) + B_2(R_b,t_T))$ where the function $A_1(R_b,t_T)$ is taken on the voids boundary, and to calculate α_T in the function $B_2(R_b,t_T) = \int_0^{R_b}\frac{\sinh^7 R\sinh^6(\alpha_T/2)}{3\sinh^3 R_b\cosh^2 R}dR$, it is necessary to use Eq. (20). In order to calculate the total mass and size of the configuration described by the Friedman solution we use Eqs. (23) and (24).

The investigation of models of the given class was carried out for $5\cdot 10^{-3} < R_b \le 2\cdot 10^{-2}$. The numerical calculations give the grounds to confirm that such models describe the voids which were existing only in the early Universe. Now they are filled with matter. These features is inherent to all models of the class $n > 2$.

6. Conclusions

Using the matching conditions of two different metrics, the models of the voids are constructed. Such voids are described by Tolman space-time with the mass function (19), the external space-time is the Friedman one. From matching conditions it follows that the space curvature in the void should be of the same sign as in the Friedman space-time, i. e. positive, negative or zero. However, space curvature of the voids can essentially differ from the curvature of the Friedman space. It is shown that in the flat Friedman world the voids, constructed by matching of Tolman and Friedman solutions, cannot exist.

The models of the voids in the hyperbolic Friedman Universe are built. It is shown that matter in these voids is always "older" than in external space. If $t_0(R) = \text{const}$, the voids do not exist. The voids described by the Tolman solution with the mass function (19) exist no more than one quarter of all the Universe lifetime and persists in the future. Such void sizes correspond to the observational data.

It is interesting to emphasize that the voids appear not because the mass of the configuration with the Tolman solution is less than the corresponding mass in the Friedman world, but because of the great difference of sizes of these configurations.

The voids described by the Tolman solution with the mass function (26) do not exist at present.

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