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THE MODEL OF DARK GALACTIC HALO BASED ON EQUILIBRIUM DISTRIBUTION FUNCTION

The model of a galactic halo as a statistical ensemble of small collisionless particles moving in their own gravitational field is constructed. The particles take part only in the gravitational interaction and cannot be detected by modern methods of registration. However, their amount is so great that they make a significant contribution to the mass of the galaxy and its gravitational field, forming a halo of dark matter (DM). The stationary solution of the kinetic equation for such a system leads to the equilibrium Maxwell-Boltzmann distribution function. Using this distribution function, we construct a model of the galactic halo of DM in the form of spherically symmetric equilibrium ensemble of moving particles. Incidentally, for the effective equation of equilibrium arising in it, according to the "hydrodynamic analogy," the velocity dispersion plays the role of "pressure." The proposed model corresponds to the phenomenological model with a linear equation of state. At the same time, the plateau of the rotation curves is interpreted as an observed manifestation of the DM.

Keywords: rotation curves, dark matter, kinetic equation, Maxwell-Boltzmann distribution, the equilibrium equation**.**

Будується модель галактичного гало як статистичної сукупності малих частинок без зіткнень, що рухаються у власному гравітаційному полі. Ці частинки, окрім як гравітаційним чином, ні з чим не взаємодіють і не піддаються сучасним методам реєстрації. Проте їх настільки багато, що вони забезпечують істотний внесок у повну масу галактики та її гравітаційне поле, утворюючи гало темної матерії (ТМ). Стаціонарний розв'язок кінетичного рівняння для такої системи призводить до рівноважної функції розподілу Максвелла-Больцмана. З використанням цієї функції розподілу будується модель галактичного гало ТМ у вигляді сферично-симетричної рівноважної сукупності частинок. Тут виникає ефективне рівняння рівноваги, в якому, згідно з "гідродинамічною аналогією", дисперсія швидкостей виконує роль "тиску". Запропонована модель відповідає феноменологічній моделі з лінійним рівнянням стану. При цьому плато у кривих обертання інтерпретується як спостережуваний прояв ТМ.

Ключові слова: криві обертання, темна матерія, кінетичне рівняння, розподіл Максвелла-Больцмана, рівняння рівноваги**.**

Строится модель галактического гало в виде статистической совокупности малых бесстолкновительных частиц, которые движутся в собственном гравитационном поле. Эти частицы, кроме как гравитационным образом, ни с чем не взаимодействуют и не поддаются современным методам регистрации. Однако их так много, что они вносят существенный вклад в массу галактики и её гравитационное поле, образуя гало темной материи (ТМ). Стационарное решение кинетического уравнения для такой системы приводит к равновесной функция распределения Максвелла-Больцмана. На основе этой функции распределения строится модель галактического гало ТМ в виде сферически-симметричной равновесной совокупности движущихся частиц. Здесь возникает эффективное уравнение равновесия, в котором, согласно "гидродинамической аналогии", дисперсия скоростей играет роль "давления". Предложенная модель соответствует феноменологической модели с линейным уравнением состояния. При этом плато у кривых вращения интерпретируется как наблюдаемое проявление ТМ.

Ключевые слова: кривые вращения, темная материя, кинетическое уравнение, распределение Максвелла-Больцмана, уравнение равновесия.

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1. Introduction

The rotational speed V of an object on a stable Keplerian orbit with the radius r around the galaxy can be found from the formula

$$V = \sqrt{GM/r} \tag{1}$$

where G is the gravitational constant. Thus, if the radius r lies outside the visible part of the galaxy, one might expect that the rotational speed $V \sim 1/\sqrt{r}$. This disagrees with the data of observational astronomy where we have $V = const$. For our solar system, this velocity is $V \approx 220\, km/s$. The above inconsistency is explained by the presence of a hidden mass – dark matter (DM) distributed in the form of a dark galactic halo (see, e.g. [1,2]). It cannot be otherwise detected, except through its gravitational impact on stars and others objects. Let us assume that DM is distributed in the galaxy in a spherically symmetric manner. Then the mass of DM inside the sphere of radius r is $M = M(r)$. Here r is a distance from the galactic center. Thus, ignoring the visible matter, we have

$$V = \sqrt{\frac{GM(r)}{r}} = const. \tag{2}$$

Hence

$$M(r) = \frac{V^2}{G} r. \tag{3}$$

On the other hand, the mass of the DM sphere is

$$M(r) = 4\pi \int_{r_0}^{r} r^2 \rho_{DM}(r) dr \tag{4}$$

where $\rho_{DM}(r)$ is the density of the DM, r_0 is the lower limit of the plateau in the rotation curve. From these two relations the following formula for the density of the DM arises

$$\rho_{DM}(r) = \frac{V^2}{4\pi G r^2} \qquad (r > r_0). \tag{5}$$

Let us find the gravitational field φ of DM. In the Newtonian gravity it is described by the Poisson equation

$$\Delta\varphi = 4\pi G \rho_{DM}. \tag{7}$$

In the spherically symmetric case, for the density of DM (5) we have $\varphi = V^2 \ln r - \frac{C_1}{r} + C_2$.

The second term of this formula corresponds to the central source. We are looking for the gravitational potential generated by a cloud of DM with $r > r_0$. Therefore, we can set $C_1 = 0$, and choose the constant C_2 so that $\varphi(r_0) = 0$. Thus, the assumption of a spherically symmetric distribution of DM and the fact of the plateau on the rotation curves lead to the potential

$$\varphi = V^2 \ln\frac{r}{r_0} \qquad (r > r_0). \tag{8}$$

Let us now consider the equilibrium conditions of DM. To do this we must make an assumption about the nature of DM. The simplest assumption is reduced to the introduction of a DM cloud in the form of an ideal gas of nonrelativistic particles with density ρ and pressure P. Then the equilibrium condition for the cloud has the form $\nabla P = -\rho_{DM} \nabla \varphi$. Hence, the equilibrium condition of a spherical cloud of DM follows

$$\frac{dP}{dr} = -\rho_{DM} \frac{d\varphi}{dr}. \tag{9}$$

Using the expression for the density (5), we find the pressure of DM

$$P = \frac{V^4}{8\pi G r^2} \qquad (r > r_0), \tag{10}$$

whence using (5), we obtain a linear equation of state for the DM in the form

$$P = \frac{1}{2} V^2 \rho_{DM}. \tag{11}$$

The upper boundary of DM cloud can be estimated with the condition $\rho_{DM} \geq \rho_{GDM}$ where ρ_{GDM} is the density of the intergalactic DM. Then, we obtain the radius of the DM cloud $r_G = \frac{V}{2\sqrt{\pi G \rho_{GDM}}}$. It is obvious that the described phenomenological model of DM is not complete. The meaning of DM pressure is not clear as well.

2. The model of dark galactic halo

We now consider the DM as the collision-free gas of nonrelativistic particles. By assumption, they are neutral, spinless, and massive particles of very small sizes and they can interact only by gravitation. Elementary black holes (BH's) with masses of order of the Planck mass can be considered as a candidate to be such particles. They can possibly be the remnants of BH's evaporation. Stable elementary BHs may play the role of maximally heavy elementary particles and, possibly, DM particles (primordial BHs, maximons, friedmons etc.) [3-7]. Elementary BH's are characterized by an extremely small scattering cross-section of the order of $10^{-66}\ cm^2$ [5].

We apply the kinetic approach [1, 8, 9] to the ensemble of such particles. Here, the basic value is the distribution function $\psi(\vec{r}, \vec{v}, t)$ where $\vec{v}$ is the velocity and $\vec{r}$ is the radius-vector of particles. The particle number density in the space with coordinates $\{\vec{r}, \vec{v}\}$ is as follows: $f(\vec{r}, \vec{v}) = N\psi(\vec{r}, \vec{v}, t)$ where N is the total number of particles. The DM mass density is $\rho_{DM}(\vec{r}, t) = m\int f(\vec{r}, \vec{v}, t) d\vec{v}$ where m is the particle mass. The distribution function satisfies the collisionless kinetic equation

$$\frac{\partial \psi}{\partial t} + \left(\vec{v} \cdot \frac{\partial \psi}{\partial \vec{r}}\right) + \left(\vec{F} \cdot \frac{\partial \psi}{\partial \vec{v}}\right) = 0. \tag{12}$$

Here $\vec{F} = -\nabla\varphi$ is the gravitational force, φ is the gravitational potential that satisfies the Poisson equation (7). This set of equations is the complete set of equations describing a self-gravitating collisionless system of particles. If the distribution function $\psi(\vec{r}, \vec{v}, t)$ is found, the components of "stress" tensor can be obtained with using the formula

$$T_{\alpha\beta} = m\int f\, v_\alpha v_\beta d\vec{v}. \tag{13}$$

In the case of the equilibrium configurations, the distribution function and the mass density are independent of time $\psi = \psi_0(\vec{r}, \vec{v})$, $\rho = \rho_0(\vec{r})$ and the equation for the equilibrium distribution function has the form

$$\left(\vec{v} \cdot \frac{\partial \psi_0}{\partial \vec{r}}\right) - \left(\frac{\partial \varphi_0}{\partial \vec{r}} \cdot \frac{\partial \psi_0}{\partial \vec{v}}\right) = 0 \tag{14}$$

where φ_0 is the self-consistent potential satisfying the Poisson equation $\Delta\varphi_0 = 4\pi G\rho_0$. In this case, we deal with anisotropic pressure given by the formula

$$P_r = m\int f_0 v_r^2 d\vec{v}, \qquad P_t = T_{\theta\theta} = T_{\alpha\alpha} = \frac{1}{2} m\int f_0 v_\perp^2 d\vec{v}. \tag{15}$$

Strictly speaking, this is not the pressure, but the velocity dispersion!

The equilibrium distribution function is a function of energy E and the other possible single-valued integrals of motion. In the isotropic case, the distribution function can depend only on the energy $\psi = \psi_0(E)$, in this case $P_r = P_t = P_0$. Then the mass density and pressure are given by following formulae

$$\rho_0 = 4\pi\sqrt{2}m\int_{\varphi_0}^{\infty} f_0(E)(E-\varphi_0)^{1/2}\, dE, \qquad P_0 = \frac{8\pi\sqrt{2}}{3} m\int_{\varphi_0}^{\infty} f_0(E)(E-\varphi_0)^{3/2}\, dE. \tag{16}$$

Differentiating the last equality in (16) and comparing the result with the previous formula in (16), we get the known condition of hydrodynamic equilibrium

$$\frac{dP_0}{dr} = -\rho_0 \frac{d\varphi_0}{dr} \tag{17}$$

which was used above when we considered the equilibrium conditions of a DM cloud. It is a hydrodynamic analogy [9].

Let us consider a partial solution of the kinetic equation − Maxwell-Boltzmann distribution [10]

$$\psi(\vec{r},\vec{v}) = Ae^{-E/\theta} = \frac{1}{J}\left(\frac{m}{2\pi\theta}\right)^{3/2} exp\left(-\frac{m}{\theta}\left(\frac{\vec{v}^2}{2} + \varphi_0(\vec{r})\right)\right) \tag{18}$$

where

$$J = \int exp\left(-\frac{m}{\theta}\varphi_0(\vec{r})\right) d\vec{r}, \tag{19}$$

$\theta = kT$ is the module of the canonical distribution. The function $\psi(\vec{r},\vec{v})$ is the probability density of a certain state of a particle. The mean particle number density in the space with coordinates $\{r,v\}$ is described by the formula $f(\vec{r},\vec{v}) = N\psi(\vec{r},\vec{v})$ where N is the total number of particles in the system. Mass density of the DM in the cloud is given by

$$\rho_{DM}(\vec{r}) = m\int f(\vec{r},\vec{v})d\vec{v} = \frac{mN}{J}\left(\frac{m}{2\pi\theta}\right)^{3/2} \int exp\left(-\frac{m}{\theta}\left(\frac{\vec{v}^2}{2} + \varphi_0(\vec{r})\right)\right) d\vec{v} \tag{20}$$

where m is the mass of DM particle. Then the total mass of the cloud equals to $M = mN$. Hence, we get

$$\rho_{DM}(\vec{r}) = \frac{M}{J} e^{-m\varphi_0/\theta}. \tag{21}$$

The Poisson equation for the self-consistent spherically symmetric field φ_0 takes the form

$$\Delta\varphi = \frac{d^2\varphi_0}{dr^2} + \frac{2}{r}\frac{d\varphi_0}{dr} = 4\pi G \frac{M}{J} e^{-m\varphi_0/\theta}. \tag{22}$$

This nonlinear equation has the following particular solution

$$\varphi_0 = \varphi_{DM} = \frac{2\theta}{m}\ln\left(\frac{r}{r_0}\right). \tag{23}$$

Herewith the module of the canonical distribution should be equal

$$\theta = 2\pi m G r_0^2 \frac{M}{J}. \tag{24}$$

Potential (23) is similar to the potential (8) of the phenomenological model. Using the hydrodynamic analogy and comparing these potentials, we obtain $\theta = mV^2/2$. This expression for the modulus θ and (24) lead to the relation for the total mass of DM cloud

$$M_{DM} = \frac{JV^2}{4\pi G r_0^2}. \tag{25}$$

Hence, using (21) and (23) for the density of dark matter, we obtain expression (5) that corresponds to the phenomenological picture.

3. Conclusions

Thus, we can consider as a model of the galactic halo the collision-free system of very small and very heavy neutral spinless DM particles interacting only by gravitation (perhaps, primordial BH's). The ensemble of DM particles satisfies the Maxwell-Boltzmann distribution. The galaxy has the atmosphere of DM, i.e. the DM galactic halo that is actually transparent. The DM particle number density in the atmosphere is

$$n_{DM}(r) = \frac{M}{mJ} e^{-m\varphi_0/\theta} = \frac{V^2}{4\pi m G r^2}. \tag{26}$$

In the statistical approach, the velocity dispersion (15) plays the role of pressure P (10). This pressure provides stability in the phenomenological picture of the halo. This correspondence is provided by the hydrodynamic analogy following from equations (16) and (17).

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