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## BACKGROUND COSMIC NEUTRINO AND DARK ENERGY

The 5-th force problem, which accompanies most of the scalar field dark energy (DE) models, does not appear in the scale invariant model in the framework of the so-called Two Measures Field Theory (TMT). In contrast to all other alternative gravity theories, in a very large range of the parameter space TMT satisfies all the classical tests of the Einstein general relativity (GR) without any tuning. The effect of neutrino DE can emerge in the course of evolution of the late time universe filled with the homogeneous scalar field and the cold gas of uniformly distributed non-relativistic neutrinos. A new kind of regime is revealed as an exact asymptotic solution where neutrinos undergo transition to a state with growing neutrino mass. This process is accompanied with the reconstruction of the scalar field potential in such a way that this dynamical regime appears to be energetically more preferable than it would be in the case of the universe with no fermions at all. We show that the deceleration-to-acceleration transition happens about a quite acceptable redshift.

**Keywords:** accelerated universe, dark energy, fifth force problem, growing neutrinos, two measures theory.

### 1. Introduction

Einstein equations in the spatially flat FRW universe with metric  $\tilde{g}_{\mu\nu} = \text{diag}(1, -a^2, -a^2, -a^2)$

$$(\dot{a}/a)^2 = 8\pi G\rho; \quad \ddot{a}/a = -\frac{4\pi G}{3}(\rho + 3P) \quad (1)$$

imply currently observed accelerating expansion[1] if the present day dominated matter component possesses negative pressure and equation of state  $w = P/\rho < -1/3$ . Two approaches have been suggested to understand the nature of the acceleration (for a review see e.g. [2]): (1) Geometrical dark energy (DE) models (modified Einstein's gravity): braneworld,  $f(R)$ , etc. (2) Physical DE models: Cosmological Constant; Dynamical DE usually described by a scalar field (quintessence, k-essence, cosmon, coupled DE, chameleon, etc.).

Special type of the coupled DE model is the Mass-Varying Neutrinos (MaVaNs) one [3], [4], [5]. Ad hoc one postulates that there exists local interaction of neutrinos with DE through the dependence of the neutrino mass  $m_\nu$  on the quintessence field  $\phi$ . In such a way, cosmic relic neutrinos may affect DE. To prevent the emergence of the 5-th force problem one must suppose that the scalar  $\phi$  couples only to neutrinos, but what is the reason of decoupling from other fermions remains unclear. The thermal history of the Universe implies that at the present epoch, the temperature of the cosmic relic neutrinos (and antineutrinos)  $T_{(\nu)} = 1.7 \cdot 10^{-4} eV$  which is less than the neutrino mass  $m_\nu$ . Therefore at present, the cosmic relic neutrinos are nonrelativistic and their energy density  $\rho_\nu = m_\nu(\phi) \cdot n_\nu$  where  $n_\nu$  is the number density of the relic nonrelativistic neutrinos and antineutrinos. One regards both the quintessence potential  $V(\phi)$  and the neutrino mass  $m_\nu(\phi)$  as classical quantities, which already include radiative corrections.

Then neutrinos can be thought as free-falling in a metric  $g_{\alpha\beta}^v = (m_v(\varphi))^2 g_{\alpha\beta}$ . In most of the models one chooses the exponential dependence  $m_v(\varphi) = \tilde{m}_v e^{\beta\varphi}$ . In such MaVaNs quintessence (or growing neutrino quintessence [4], [5]), the attraction between nonrelativistic neutrinos exceeds gravity by a factor  $10^3$ . The growing neutrinos DE models predict a number of new interesting effects (cosmology of the late Universe; impact for structure formation, etc.) In the present paper I review the basic principles of the Two Measures Field Theory (TMT) [6]-[12] and a model of the neutrino DE based on these principles without adding any special assumptions intended to realize the desirable effects, like the absence of the 5-th force problem, the neutrino-to-DE coupling, etc. Distinctive features of the model is that although TMT is an alternative gravity theory, there exists the Einstein frame where the Einstein equations and all other field equations have the canonical GR form. All the novelty consists in the appearance of both a very nontrivial potential of the DE scalar field  $\varphi$  and a coupling of all massive fermions to  $\varphi$ .

It is convenient to define the term 'fermions in regular conditions', which means that the local fermion energy density  $\rho_f$  is many orders of magnitude larger than the vacuum energy density  $\rho_{vac}$ . Notice that all classical tests of GR are fulfilled just with fermions in regular conditions. In contrast to all other known alternative theories, as fermions are in the regular conditions, predictions of our TMT model is undistinguishable from that of GR, and this is realized without fine tuning or special constraints on masses and coupling constants in the underlying Lagrangian. However TMT predicts drastically different physics as  $\rho_f(x)$  is slightly larger than  $\rho_{vac}$  that has a very interesting output in the context of cosmology.

## 2. Main ideas of TMT

TMT is a generally coordinate invariant theory with the action which may be written in the following general form

$$S = \int L_1 \Phi d^4x + \int L_2 \sqrt{-g} d^4x \quad (2)$$

which involves two Lagrangians  $L_1$  and  $L_2$  and two measures of integration:  $(-g)^{1/2}$  and the new one  $\Phi$  independent of the metric.  $\Phi$  being a scalar density may be defined as  $\Phi = \varepsilon^{\mu\nu\alpha\beta} \varepsilon_{abcd} \partial_\mu \phi_a \partial_\nu \phi_b \partial_\alpha \phi_c \partial_\beta \phi_d$   $\phi_a(x)$  ( $a = 1, \dots, 4$ ) are scalar fields. The volume form  $\Phi d^4x \equiv 4! d\phi_1 \wedge d\phi_2 \wedge d\phi_3 \wedge d\phi_4$  exists in the space-time differential manifold even before the manifold is equipped with the metric structure. In TMT becomes clear the reason why the measure  $(-g)^{1/2}$  is used in GR while  $\Phi$  is ignored. One can show that in the vacuum, even with a cosmological constant, the ratio of measures is indeed a constant:  $\Phi / (-g)^{1/2} = const$ . The result remains valid if the underlying action involves gauge fields in canonical manner, as well as massless fermions. This feature of TMT answers the question why in the Riemannian geometry it is enough to work with  $(-g)^{1/2}$ . But in the presence of massive fermions and scalar fields, the scalar  $\zeta \equiv \Phi / (-g)^{1/2}$  appears to be generically a function of some of the matter fields and the metric. This is the key difference between TMT and GR.

Two more additional assumptions are added to the TMT basic principles: (a)  $L_1$  and  $L_2$  are independent of the the measure fields  $\phi_a$ . In such a case, the action possesses an infinite dimensional symmetry which should prevent emergence of a measure fields dependence in  $L_1$  and  $L_2$  after quantum effects are taken into account. (b) Applying the

action principle, one should proceed in the first order (Palatini) formalism. All fields, including metric, connection and the fields  $\varphi_a$  are regarded as independent dynamical variables. All the relations between them are results of equations of motion.

Two main differences of TMT from scalar-tensor and other modified gravity theories: (a)  $L_1$  and  $L_2$  may contain not only the scalar curvature term but also all possible matter field terms. (b) TMT modifies in general both the gravitational sector and the matter sector in the same fashion.

TMT is a constrained dynamical system because  $\Phi$  depends only upon the first derivatives of  $\varphi_a$  and this dependence is linear. Solution of the equation resulting from variation of  $\varphi_a(x)$  is the constraint:

$$L_1 = M^4 = const., \quad (3)$$

where  $M$  is a constant of integration with the dimension of mass. The fields  $\varphi_a$  do not have their own dynamical equations: they are auxiliary fields. All of their dynamical effect is displayed only in the following two ways: (a) in the appearance of the scalar field  $\zeta \equiv \Phi/(-g)^{1/2}$  and its gradient in all equations of motion. (b) in generating the constraint 3 which is actually the algebraic equation describing the  $\zeta$  field as a local function of matter fields and metric. The constraint and  $\zeta$  have a key role in TMT.

### 3. The Model

Our TMT model involves 4D gravity, the quintessence scalar field  $\phi$  and fermions with the following underlying action:

$$\begin{aligned} S = & \int d^4 x e^{\alpha\varphi/M_p} (\Phi + b\sqrt{-g}) \left[ -\frac{1}{16\pi G} R(\omega, e) + \frac{1}{2} g^{\mu\nu} \varphi_{,\mu} \varphi_{,\nu} \right] \\ & - \int d^4 x e^{2\alpha\varphi/M_p} (\Phi V_1 + \sqrt{-g} V_2) + \int d^4 x e^{\alpha\varphi/M_p} (\Phi + k\sqrt{-g}) \frac{i}{2} \sum_i \bar{\Psi}_i \left( \gamma^a e_a^\mu \bar{\nabla}_\mu^{(i)} - \bar{\nabla}_\mu^{(i)} \gamma^a e_a^\mu \right) \Psi_i \\ & - \int d^4 x e^{\frac{3}{2}\alpha\varphi/M_p} (\Phi + h\sqrt{-g}) \sum_i \mu_i \bar{\Psi}_i \Psi_i. \end{aligned} \quad (4)$$

Here  $\Psi_i$  - fermion field of species  $i$ ;  $\mu_i$  - the mass parameters;  $\bar{\nabla} = \bar{\partial} + \frac{1}{2} \omega_\mu^{cd} \sigma_{cd}$ ;

$R(\omega, e) = e^{a\mu} e^{b\nu} R_{\mu\nu ab}(\omega)$  - scalar curvature;  $e_a^\mu$  - vierbein;  $\omega_\mu^{ab}$  - spin-connection;  $g^{\mu\nu} = e_a^\mu e_b^\nu \eta^{ab}$ ;  $R_{\mu\nu ab}(\omega) = \partial_{\mu\nu} \omega_{ab} + \omega_{\mu a}^c \omega_{\nu cb} - (\mu \leftrightarrow \nu)$ ;

The theory is invariant under the global scale transformations:

$$\begin{aligned} e_a^\mu & \rightarrow e^{\theta/2} e_a^\mu, \quad \omega_{ab}^\mu \rightarrow \omega_{ab}^\mu, \quad \phi_a \rightarrow \lambda_{ab} \phi_b, \quad \det(\lambda_{ab}) = e^{2\theta}, \\ \varphi & \rightarrow \varphi - \frac{M_p}{\alpha} \theta, \quad \Psi_i \rightarrow e^{-\theta/4} \Psi_i, \quad \bar{\Psi}_i \rightarrow e^{-\theta/4} \bar{\Psi}_i \end{aligned} \quad (5)$$

where  $\theta = const$ ,  $\lambda_{ab} = const$ ,  $\det(\lambda_{ab}) = e^{2\theta}$ .

Constants  $b$ ,  $k$ ,  $h$  are non specified dimensionless real parameters and we will only assume that  $b > 0$  and they are different but have the same order of magnitude  $b:k:h$ . The dimensionless parameter  $\alpha$  is of the order of unity and  $M_p$  is the Planck mass. Except for the modification of the general structure of the action according to the basic assumptions of TMT, the action does not involve any exotic terms and fields Without changing the

results, one can generalize the model to non-Abelian symmetry adding also gauge and Higgs fields to reproduce the standard model. All equations of motion in the Palatini formalism contain terms proportional to  $\partial_\mu \zeta$  that generically makes the space-time non-Riemannian and equations of motion - non canonical. However, with the new set of variables (Einstein frame)  $\tilde{g}_{\mu\nu} = (\zeta + b)g_{\mu\nu}$ ,  $\Psi'_i = \frac{(\zeta + k)^{1/2}}{(\zeta + b)^{3/4}} \Psi_i$  the spin-connection becomes that of the Einstein-Cartan space-time with the metric  $\tilde{g}_{\mu\nu}$ ; the gravitational equations take the standard GR form; the fermion equations take the standard form of the Dirac equations in the Einstein-Cartan space-time where now the fermion masses become  $\zeta$  dependent

$$m_i(\zeta) = \mu_i \frac{(\zeta + h)}{(\zeta + k)(\zeta + b)^{1/2}} \quad (6)$$

The energy-momentum tensor in the Einstein frame

$$T_{\mu\nu}^{eff} = \varphi_{,\mu} \varphi_{,\nu} - \frac{1}{2} \tilde{g}_{\mu\nu} \tilde{g}^{\alpha\beta} \varphi_{,\alpha} \varphi_{,\beta} + \tilde{g}_{\mu\nu} V_{eff}(\varphi; \zeta) + T_{\mu\nu}^{(f,can)} + T_{\mu\nu}^{(f,noncan)}, \quad (7)$$

where  $V_{eff}(\varphi; \zeta) = [b(M^4 e^{-2\alpha\phi/M_p} + V_1) - V_2] / (\zeta + b)^2$ . It is very important that the  $\phi$ -dependence appears only in the form  $M^4 e^{-2\alpha\phi/M_p}$ , where  $M^4$  results from the spontaneous breakdown of the global scale symmetry (3) which in the Einstein frame is reduced to the spontaneously broken shift symmetry  $\varphi \rightarrow \varphi + const$ .  $T_{\mu\nu}^{(f,can)}$  is the canonical energy-momentum tensor for fermions in curved space-time; The noncanonical contribution of fermions into the energy-momentum tensor has the form of a variable cosmological constant (CC)  $T_{\mu\nu}^{(f,noncan)} = -\tilde{g}_{\mu\nu} \Lambda_{dyn}^{(f)}$ ,  $\Lambda_{dyn}^{(f)} = Z(\zeta)m(\zeta)\overline{\Psi}\Psi$ , where  $Z(\zeta) = [(\zeta - \zeta_1)(\zeta - \zeta_2)] / 2(\zeta + k)(\zeta + h)$  and  $\zeta_{1,2} = \frac{1}{2} \left[ k - 3h \pm \sqrt{(k - 3h)^2 + 8b(k - h) - 4kh} \right]$ .

The scalar field  $\zeta$  is determined by the constraint

$$\frac{1}{(\zeta + b)^2} \left[ (b - \zeta)(M^4 e^{-2\alpha\phi/M_p} + V_1) - 2V_2 \right] = Z(\zeta)m(\zeta)\overline{\Psi}\Psi \quad (8)$$

as a local function  $\zeta(x) = \zeta(\varphi(x), \overline{\Psi}(x)\Psi(x))$ . Here for simplicity we restrict ourself with one fermion species. The quintessence field  $\phi$  equation reads

$$\frac{1}{\sqrt{-\tilde{g}}} \partial_\mu \left( \sqrt{-\tilde{g}} \tilde{g}^{\mu\nu} \partial_\nu \phi \right) - \frac{\alpha}{M_p(\zeta + b)} \left[ M^4 e^{-2\alpha\phi/M_p} - \frac{(\zeta - b)V_1 + 2V_2}{\zeta + b} \right] = -\frac{\alpha}{M_p} Z(\zeta)m(\zeta)\overline{\Psi}\Psi \quad (9)$$

#### 4. Features of the model in two limiting cases

##### 1. Dark Energy in the absence of massive fermions.

In such a case the constraint yields  $\zeta = \zeta_0(\varphi) = b - 2V_2 / [V_1 + M^4 e^{-2\alpha\phi/M_p}]$ . Then

$$V_{eff}^{(0)} \equiv V_{eff}(\phi; \zeta_0) = \frac{(V_1 + M^4 e^{-2\alpha\phi/M_p})^2}{4[b(V_1 + M^4 e^{-2\alpha\phi/M_p}) - V_2]} \quad (10)$$

In the FRW cosmology, the energy density and pressure read

$$\rho_{d.e.}^{(0)} = \frac{1}{2} \dot{\phi}^2 + V_{\text{eff}}^{(0)}(\phi), \quad P_{d.e.}^{(0)} = \frac{1}{2} \dot{\phi}^2 - V_{\text{eff}}^{(0)}(\phi).$$

## 2. General Relativity and fermions in regular conditions.

First of all one can notice that for the case as matter consists of fermions in regular conditions, the standard GR equations are reproduced if  $\Lambda_{\text{dyn}}^{(f)} = Z(\zeta)m(\zeta)\bar{\Psi}\Psi = 0$  or at least  $|T_{\mu\nu}^{(f, \text{noncan})}| \ll |T_{\mu\nu}^{(f, \text{can})}|$ . For a single fermion it happens if  $Z(\zeta) = \frac{(\zeta - \zeta_1)(\zeta - \zeta_2)}{2(\zeta + k)(\zeta + h)} \approx 0$ , that is if  $\zeta \approx \zeta_1$  or  $\zeta \approx \zeta_2$ . Detailed analysis shows that the l.h.s. of the constraint (8) has the order close to the DE density  $\rho_{d.e.}^{(0)}$ , while for the nonrelativistic fermion  $m(\zeta)\bar{\Psi}\Psi$  is of the order of the local fermion energy density  $\rho_f$ . Therefore the constraint (8) should provide the local balance between extremely low DE density  $\rho_{d.e.}$  and very high local fermion energy density  $\rho_f$ . Thus, it requires implementation of exactly the same condition for which Einstein equations are reproduced:  $\zeta \approx \zeta_1$  or  $\zeta \approx \zeta_2$ . Together with this, as it follows from the quintessence field  $\phi$  equation (9), the effective fermion-to-quintessence Yukawa coupling constant becomes unobservable:  $-\frac{\alpha}{M_p} Z(\zeta)m(\zeta) \sim \alpha \frac{m}{M_p} \frac{\rho_{d.e.}}{\rho_f}$ . This is a way how our TMT model solves the 5-th force problem without fine tuning in the parameter space.

## 5. Effect of the neutrino dark energy in the late Universe

**1. General consideration.** In a model of the spatially flat FRW universe filled with the homogeneous scalar field  $\phi$  and a cold gas of uniformly distributed non-relativistic neutrinos the average  $\langle \bar{\Psi}\Psi \rangle$  (both quantum and cosmological) behaves as the dust number density in expanding Universe  $\langle \bar{\Psi}\Psi \rangle \propto \frac{n}{a^3}$ . Then the constraint (8) takes the form

$$\frac{1}{(\zeta + b)^2} \left[ (b - \zeta)(M^4 e^{-2\alpha\phi/M_p} + V_1) - 2V_2 \right] = \frac{(\zeta - \zeta_1)(\zeta - \zeta_2)}{(\zeta + k)^2} \cdot \frac{n}{a^3}. \quad (11)$$

One possible asymptotic solution of the constraint for the expanding Universe as  $a(t) \rightarrow \infty$  is identical to the discussed above DE in the fermion vacuum (when  $\phi$  increases up to  $\infty$ ). The only alternative solution is implemented if the decaying fermion density  $\sim 1/a^3$  is accompanied by approaching  $\zeta \rightarrow -k$  in such a way that  $(\zeta + k)^{-2} \propto a^3$ . The regime when  $(\zeta + k)^{-2} \propto a^3$  we call the Cosmo-Low-Energy-Physics (CLEP) solution. In the CLEP regime, the constraint reads

$$\frac{(b + k)(M^4 e^{-2\alpha\phi/M_p} + V_1) - 2V_2}{(b - k)^2} = (b - k)^{1/2} (h - k) \frac{\mu n}{(\zeta + k)^2 a^3} + O(\zeta + k), \quad (12)$$

while the averaged canonical neutrino energy-momentum tensor decreases as  $\sim 1/a^{3/2}$

$$\langle T_{\alpha\beta}^{v, \text{canon.}} \rangle = \delta_{\alpha}^0 \delta_{\beta}^0 \frac{h - k}{(b - k)^{1/2}} \frac{\mu n}{(\zeta + k)a^3} + O(a^{-3}), \quad (13)$$

but the averaged noncanonical neutrino energy-momentum tensor approaches a constant

$$\langle T_{\alpha\beta}^{V,noncanon} \rangle = -\tilde{g}_{\alpha\beta} \langle \Lambda_{dyn}^V \rangle = -\tilde{g}_{\alpha\beta} (b-k)^{1/2} (h-k) \frac{\mu n}{(\zeta+k)^2 a^3} + O(1/a^{3/2}) \quad (14)$$

So, in the CLEP regime, the total averaged neutrino energy-momentum tensor behaves as

$$\langle T_{\alpha\beta}^{V,noncanon} \rangle = -\tilde{g}_{\alpha\beta} \langle \Lambda_{dyn}^V \rangle = -\tilde{g}_{\alpha\beta} (b-k)^{1/2} (h-k) \frac{\mu n}{(\zeta+k)^2 a^3} + O(1/a^{3/2}). \quad (15)$$

In the CLEP regime,  $V_{eff}(\phi, \zeta)$  instead of the fermion vacuum expression (10) changes to  $V_{eff}^{(CLEP)}(\phi) \equiv V_{eff}(\phi, \zeta)|_{\zeta \rightarrow k} = [b(M^4 e^{-2\alpha\phi/M_p} + V_1) - V_2] / (b-k)^2 + O(1/a^{3/2})$ . This reorganization of the  $\phi$ -dynamics in the presence of the cold neutrino gas is the direct result of general feature of TMT: due to the constraint, in the space-time region occupied by a fermion, the scalar field  $\zeta$  varies so as to satisfy a certain local balance between the energy densities of the DE and of the fermion. Moreover, due to the constraint, the separation of the neutrino and  $\phi$  contributions into the total energy-momentum tensor loses clarity in the CLEP regime, since using the constraint one can represent "the neutrino contribution" in terms of the field  $\phi$  alone. In the FRW universe we get  $\rho_{tot} = \dot{\phi}^2 / 2 + U_{eff}^{tot}(\phi)$   $P_{tot} = \dot{\phi}^2 / 2 - U_{eff}^{tot}(\phi)$ , where  $U_{eff}^{tot}(\phi) \equiv (V_2 - kV_1 - kM^4 e^{-2\alpha\phi/M_p}) / (b-k)^2$ . The remarkable result consists in the fact that

$$V_{eff}^{(0)}(\phi) - U_{eff}^{(tot)}(\phi) \equiv \frac{\left[ (b+k) \left( V_1 + M^4 e^{-2\alpha\phi/M_p} \right) - 2V_2 \right]^2}{4(b-k)^2 \left[ b \left( V_1 + M^4 e^{-2\alpha\phi/M_p} \right) - V_2 \right]} > 0 \quad (16)$$

This means that the universe with the gas of uniformly distributed non-relativistic neutrinos in the CLEP state is energetically more preferable than the Universe without neutrinos at all.

**2. Exact cosmological solution in the CLEP regime.** For the particular value  $\alpha = \sqrt{3/8}$ , the cosmological equations allow the following analytic solution ( $t_0$  is the present time):

$$\varphi(t) = \frac{M_p}{2\alpha} \phi_0 + \frac{M_p}{2\alpha} \ln(M_p t), \quad a(t) \propto \left( \frac{t}{t_0} \right)^{1/3} e^{\lambda t/t_0} \quad (17)$$

$$\text{where } \lambda = \frac{1}{M_p} \sqrt{\frac{\Lambda}{3}}, \quad \Lambda = \frac{V_2 - kV_1}{(b-k)^2}, \quad e^{-\phi_0} = \frac{2(b-k)^2 M_p^2}{\sqrt{3} |k| M^4} \sqrt{\Lambda}. \quad (18)$$

The mass of the neutrino in such CLEP state increases exponentially in time and its  $\phi$  dependence is double-exponential:

$$m|_{clep} \sim (\zeta+k)^{-1} \sim a^{3/2}(t) \sim t^{1/2} e^{\frac{3}{2}\lambda t} \sim \exp \left[ \frac{3\lambda e^{-\phi_0}}{2M_p} \exp \left( \frac{2\alpha}{M_p} \varphi \right) \right]. \quad (19)$$

In this toy model, the expansion law  $a(t)$  in Eq.(17) implies a possibility to estimate the epoch of deceleration-to-acceleration transition:  $\ddot{a} = 0$  as  $\lambda t_{tr} = 0.244$  or  $\lambda t_{tr} = -0.91$ . The value of  $\ddot{a} = raH^3$  as  $\ddot{a} = 0$  is  $\ddot{a}|_{\ddot{a}=0} = \pm \frac{2\lambda}{\sqrt{3}t_{tr}^2} a$ . The upper sign corresponds to deceleration-to-acceleration transition, i. e. it happens as  $\lambda t_{tr} = 0.244$ . With the modern estimations for  $\Omega_{d.e.}^{(0)} \approx 0.7$  we have  $\Lambda = \rho_{cr} \Omega_{d.e.}^{(0)} \approx 0.7 \cdot \frac{3H_0^2}{8\pi G} \lambda = \sqrt{\Lambda/3}/M_p = 0.83H_0 t_{tr} = 0.244/\lambda = 0.294/H_0$ . The appropriate redshift is

$$z_{tr} = \frac{a(t_0)}{a(t_{tr})} - 1 = \left( \frac{t_0}{t_{tr}} \right)^{1/3} e^{-\lambda(t_{tr}-t_0)} - 1 = 1.18(H_0 t_0)^{1/3} e^{0.83(H_0 t_0)} - 1. \quad (20)$$

Using the relation for the age of the Universe

$$H_0 t_0 \approx \frac{2}{3\sqrt{\Omega_{d.e.}^{(0)}}} \cdot \ln \frac{1 + \sqrt{\Omega_{d.e.}^{(0)}}}{\sqrt{1 - \Omega_{d.e.}^{(0)}}} \approx 0.96 \quad (21)$$

we obtain that the deceleration-to-acceleration transition happens in our model about the redshift  $z_{tr} \approx 1.6$  which is quite reasonable from the point of view of the present observational data.

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