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Mathematical bases of heat-transfer in groundwater

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Processes of transferring thermal energy are common in nature and include an entire complex of transferring heat in the environment, which is due to differences in temperature between separate elements of a system. They are connected with different physical phenomena which exist in geotechnical systems of any level and require accurate study. The article is devoted for possibility possibilities of application of the known differential conformities to law are examined for the decision of practical and theoretical questions of hydrogeology of the urbanized territories. Methodology of estimation of thermal influence is brought around to an underground hydrosphere with an aim possibility of application of certain methodologies of mathematical design for an analysis and prognosis of processes of geohydrology of territories with considerable thermal contamination.
Key words: thermal energy, geotechnical system, heat transfer, convection.

Математичні основи теплопереносу у підземних водах

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Процеси передачі теплової енергії поширені в природі і охоплюють весь комплекс явищ переносу теплоти в просторі, що обумовлено різницею температур окремих елементів системи. Вони пов'язані з різноманітними фізичними явищами, якііснують у геотехнічних системах будь-якогорівня, і потребують докладного дослідження. У статтірозглядаютьсяможливостізастосуваннявідомихдиференціальнихзакономірностей для вирішенняпрактичних та теоретичних питань гідрогеології урбанізованих територій. Наведено методику оцінки термічниго впливу на підземну гідросферу з ціллю можливості застосування певних методик математичного моделювання для аналізу і прогнозу гідрогеологічних процесів територій зі значним тепловим забрудненням.
Ключові слова: теплова енергія, геотехнічна система, теплопередача, конвекція.

Introduction. The study of transferring heat throughthe flow of liquid in porous environments began in the 1920s and 30s.The study covered the influence of filtration speed, size of parts and liquid phase upon the heat conductivity coefficient .Through this study it was experimentally defined that in some cases, due to the size of the components of a porous environment, in fixed conditions efficient heat conduction can depend upon the speed of the water flow, and in the same way, dependencies between thermal parameters of rocks and their moisture appear (Barenblatt, 1972).

Presentation of the general material.The equation of heat transfer within filtration in a porous environment in mathematical relation looks the same as the equation of mass transfer. For a one-dimensional case within a fixed mode of filtration in an isolated water-bearing layer, the roof and the base of which are waterproof and do not conduct heat, the equation of heat transfer will be as follows (Oradovskaya, 1982):

$$\lambda \frac{\partial^2 T}{\partial x^2} - V C_l \frac{\partial T}{\partial x} = C_r \frac{\partial T}{\partial t}, \qquad (1)$$

where T – temperature; V – filtration speed; C_l & C_r – volumetric heat capacities of the liquid and the rock; λ – coefficient of heat conductivity; t – time.

Usually a precondition of instant equalization of the temperature between the rock base and liquid is used. Volumetric heat capacities of the layerC_r, and the liquid$C_l$and base$C_b$and the corresponding specific volumetric heat capacities C'_r, C'_l, C'_b are connected in the equation

$$n_0\rho_l C_l' + (1 - n_0)\,\rho_b C_b' = \rho_r C_r', \quad (2)$$

$$C_r = \rho_r C_r' ; C_l = \rho_l C_l' ; C_b = \rho_b C_b', \quad (3)$$

where n_0 – porosity of the layer; ρ_r, ρ_l, ρ_b – density of the layer, liquid and the framework.

Coefficient of heat conductivity $a = \lambda/C_r$ makes the equation (1)

$$a\frac{\partial^2 T}{\partial x^2} - V\,\overline{C}_l\frac{\partial T}{\partial x} = \frac{\partial T}{\partial t}, \quad (4)$$

where $\overline{C}_l = C_l / C_r$.

The first term of the left part of the equation (4) characterizes conductive heat transfer (molecular movement of the heat); the second one – convectional heat transfer, which depends upon the velocity of the filtration; the right part of the equation (4) reflects the change in the amount of the heat in the layer over time. If the water-bearing layer is not isolated in the thermal aspect and the return of the heat to the roof and the base of the layer is possible, the right part of the equation (1) should be completed with a supplementary term $2\alpha\,(T-T_0)$

$$\lambda\frac{\partial^2 T}{\partial x^2} - V\,C_p\frac{\partial T}{\partial x} = C_п\frac{\partial T}{\partial t} + 2\alpha\,(T-T_0), \quad (5)$$

where α – coefficient of heat transfer; T_0 – initial temperature of the water-bearing layer and rocks, which it includes.

If the heat transfer in the rocks of the roof and the base has only a conductive character, the second term in the right part of the equation (5) will be

$$2\,\frac{\lambda}{m}\,\frac{\partial T}{\partial z}\Big|_{z = m/2},$$

where m – capacity of the water-bearing layer; axis *x* lies along the middle of the layer; all z – vertical.

Solving the fundamental one-dimensional task of convectional heat-transfer in the water-bearing layer with conductive return of the heat to the rock layer of unlimited capacity has the following structure of equation systems (6) and (7), conductive heat-transfer is not included

$$V\,C_l\frac{\partial T}{\partial x} = C_r\frac{\partial T}{\partial t} + 2\,\frac{\lambda}{m}\,\frac{\partial T}{\partial z}\Big|_{z = m/2}, \quad (6)$$

$$\lambda\frac{\partial^2 T}{\partial x^2} = C_r\frac{\partial T}{\partial t}, \quad (7)$$

Within initial and limiting conditions

$$T(x, 0) = T_0 ; T(0, t) = T_{out} ; \frac{\partial T}{\partial z}\Big|_{z \to \infty} = 0, \quad (8)$$

Received as

$$\overline{T} = \frac{T - T_0}{T_{out} - T_0} = \operatorname{erfc}\frac{x\sqrt{\lambda / C_r}}{2m\sqrt{V\overline{C}_l(V\overline{C}_l t - x)}}, \quad (9)$$

The equation (9) is used in the theory of heat and mass transfer, and also for solving different practical tasks of prognosis for distribution of liquid and heat in groundwater (Hydrodynamics and heat exchange of mono- and biphasic streams, 1987). One-dimensional equation of heat-transfer in an isolated water-bearing layer within conditions of changed temperature of waterat the point of its outlet to the layer, if the temperature at the outlet T_{out} periodically changes according to sine wave will be

$$T_{out} = T_0 + \Delta T \sin \frac{2\pi\pi}{\tau_j}, \quad (10)$$

where ΔT - amplitude;τ – frequency of fluctuations;T_0–initial temperature.

The equation`s solution (10) will be

$$\overline{T} = \frac{T - T_0}{\Delta T} = \exp(-a'x) \sin\left(\frac{2\pi\pi}{\tau} - b'x\right), \quad (11)$$

where

$$a' = [(K^2 + 0{,}25)\ \tilde{V}^{4}\ ^{\frac{1}{2}} + 0{,}5\ \tilde{V}^{2}\ ]^{\frac{1}{2}} - \tilde{V}$$

$$b' = [(K^2 + 0{,}25)\ \tilde{V}^{4}\ ^{\frac{1}{2}} + 0{,}5\ \tilde{V}^{2}\ ]^{\frac{1}{2}}$$

$$K = \pi C_r / \tau\lambda;\ \tilde{V} = VC_r/2\lambda$$

This solution may be used for the prognosis of the temperature of groundwater when there is vertical filtration of surface water, the temperature of which periodically changes due to daily and seasonal fluctuations of the air temperature.

Observations of ground water temperature distribution within a separating aquitard layer for defining the speed of vertical water flowing over it is covered by the equation of fixed heat transfer through vertical filtrating flow with constant speed of filtration V

$$-\lambda \frac{\partial^2 T}{\partial x^2} + V C_r \frac{\partial T}{\partial x} = 0, \quad (12)$$

When conditions in the layer roof are T(z = m) =$T_m$and in the base of the layer T(z = 0) = T_0 the solution obtained as

$$\overline{T} = \frac{T - T_0}{T_m - T_0} = \frac{\exp(VC_p z/\lambda) - 1}{\exp(VC_p m/\lambda) - 1}, \quad (13)$$

hasa temperature distribution depending upon V, zand thermal parameters of the layer. The measuring of temperature in several points z_i of the layer temperature T_i, with (13) can define the speed of vertical filtration V.

Two-dimensional heat transfer when there is filtration with constant speed was studied in experiments (Oradovskaya, 1982). Water, heated up to the temperature T_0, was filtrated through soil cores, which were put intoa cylindrical vessel with radius r_0.The differential equation of fixed heat transfer for the abovementioned case will be

$$\lambda_r \left[\frac{\partial^2 T}{\partial^2 r^2} + \frac{1}{r}\frac{\partial T}{\partial r}\right] + \lambda_z \frac{\partial^2 T}{\partial x^2} - V C_r \frac{\partial T}{\partial x} = 0, \quad (14)$$

where λ_r i λ_z – efficient heat conductions in a radial and axis directions.

The environmental temperaturebeyond the coreequals T_∞andstays unchanged. The limiting conditionsare formulated as follows:

$$T = T_0\ ;\ z = 0\ ;\ 0 \le r \le r_0\ ,$$

$$T = T_\infty\ ;\ z \to 0\ ;\ 0 \le r \le r_0\ ,$$

$$\frac{\partial T}{\partial r} = 0\ ;\ r = 0\ ;\ 0 < z \le \infty, \tag{15}$$

$$\lambda_r \frac{\partial T}{\partial r} = -\ \alpha\ (T - T_\infty)\ ;\ 0 < z \le \infty;$$

where λ – coefficient of heat transfer with the environment through the wallsof the cylinder which contains the sample of rock.

Solution of the equation (14) within conditions (15), will be

$$\overline{T} = \frac{T - T_\infty}{T_0 - T_\infty} = 2\sum_{n-1}^{\infty} \frac{J_1(\overline{\alpha}_n)J_0(\overline{\alpha}_n \overline{r})}{\overline{\alpha}_n\left[J_1^2(\overline{\alpha}_n) + J_0^2(\overline{\alpha}_n)\right]} \exp\ [(\delta\beta^2 - \beta\ \sqrt{\delta^2\beta^2 + \overline{\alpha}_n^2})\overline{z}\,]\ , \tag{16}$$

where J_1, J_0–Bessel functions of the first kind of the first and zero order; $\overline{\alpha}_n$ - root of the equation (17).

$$\overline{\alpha}_n\ J_1(\overline{\alpha}_n) = P\ J_1(\overline{\alpha}_n)\ , \tag{17}$$

where $P = \alpha\ r_0/\ \lambda_r$; $\overline{r} = r/r_0$, $\overline{z} = z/l$ (where l -part of the length of the core).

When studying soils with porosity 0.34 - 0.66, with speed of filtration V = 0.05 - 0.19 cm/min, valuesλ_r= 2500 - 5000 W/m*K were defined by correlations $(\lambda_z\ /\lambda_r)$~ 1.1. Most studies mentioned a simultaneous increase in$\lambda_r$with increase in the speed of filtration V.

The change in the temperature of the water, which flows from a river or water body into the water-bearing layer, develops according to several following schemes of uniform non-standard heat transfer in the head water-bearing layer between two water bodies with given constant levels. Due to the change in the layer capacity m(x) and filtration coefficient k(x), the speed of filtration in the layer is observed as a value which varies with the direction of the current. Apart from that,in a roof with low porosity, the influence of surface water infiltration should be considered. The temperature changes in the layer are described by the equation (6) without the term $\lambda\frac{\partial^2 T}{\partial x^2}$, i.e. it includes only convectional heat transfer through filtration flow and extraction of the heat into the rocks which lie under and block the flow.Values of m(x)and k(x) are given as linear or continuous function. The temperature of the roof and the base rocks, and also the initial temperature of the layer are considered constant T_0or have fixed initial distribution T(x).The temperature of the water which flows to the horizon is constant or changes over time according to the law of polygonal chain or trigonometrically functions. All the above mentioned schemes obtained analytical solutions.

A major part of the research into heat transfer was conducted in response to the problem of pumping water used for energetic purposes or pumping of cold groundwater after it had cooled industrial aggregates into the layer. In these cases, the temperature of the water directed to the layer differs from the initial temperature of the layer and distribution of thermal front can affect the water temperature in the next months of pumping. Therefore a prognosis of heat transfer in the water-bearing horizon is required.

The main research into heat transfer within directing heated (or cooled) water to the layer was conducted in accordance with a dual scheme, i.e. when operating one water-injection well or one water withdrawal well. The operation of water-injection wells and water withdrawal wells in a water bearing layer with natural flow of groundwater complicates the scheme of flow. Depending upon the debit of the wells and their dislocation and directions of natural flow, only all or some part of water which was pumped in the water-injection well can turn towards the water withdrawal well. Analytical solutions of heat transfer are possible for a limited number of the simplest dual schemes, for example, when the layer is uniform, the debitsof the wells are uniform, the natural flow of groundwater is not included, etc.

For more complicated schemes (system of water withdrawal and water-injection wells, their different debits) hydrodynamic and thermal calculation is conducted consecutively. On the first stage, using modeling of filtration according to the analog or mathematical model, a hydrodynamic net of filtration is obtained. It includes the marked line of the flow which moves to water withdrawal wells from separate water-injection wellsand from the natural flow. Then the heat transfer by separate tubes is considered – the lines of flow, which are limited by the chosen lines of the current(Bulyandra, 2001). The equation (6) can be used for any tube of the flow, which goes between the wells. We consider the systems of two equations:

- forthe general water-bearing layer, convectional heat-transfer and return of the heat in waterproof rocks of the roof and the base is considered

$$h\, C_r \frac{\partial T_p^{\phi}(\Delta S, t)}{\partial t} + \Delta\varphi\, C_r \frac{\partial T_r^{\phi}(\Delta S, t)}{\partial S} = 2\, \lambda_b \frac{\partial T_r^{\phi}(\Delta S, z, t)}{\partial z}, \tag{19}$$

- for the rocks of the roof and the base, the conductive heat transfer is included

$$\lambda_b \frac{\partial^2 T_b}{\partial z^2} = C_b \frac{\partial T_b}{\partial t}, \tag{20}$$

where h – capacity of the layer; ΔS – area of the flow tube from the point of pumping to the point of calculation within the initial and limiting conditions

$$T(\Delta S, t=0) = T_0;\ T(\Delta S = 0, t) = T_r;$$

$$T_b(\Delta S, z, t) = T_0;\ z \geq h/2;\ T_b(\Delta S, h/2, t) = T_r(\Delta S, h/2, t), \tag{21}$$

$$\lim_{z\to\infty} T_b(\Delta S, z, t) = T_0$$

solutions of systems (19) – (20) for any line of the current from the current line φ to φ + Δφ obtained as

$$\frac{T_b^{\varphi}(t) - T_0}{T_r - T_0} = \operatorname{erfc}\left\{\left[\frac{C_1^2}{\lambda_b C_b}\left(\frac{d\varphi}{dS}\right)^2\left(t - \frac{C_r}{C_1} h \frac{dS}{d\varphi}\right)\right]^{\frac{1}{2}}\right\}, \tag{22}$$

where $T^{\varphi}_b(t)$ – temperature of the water which flows to the waterwithdrawal well by the considered current line.

The temperature of the water which is pumped is defined including the calculation of mixture of the water which comes to the well through different tubes of the flow.

Not many field studies of heat transfer in water bearing layers have been conducted. We know the results of research on heated water pumping into free flow water bearing layers and a consecutive water pump, made for evaluation of efficiency of accumulation of the heat in water-bearing layers. The experiment was conducted in alluvial deposits, the capacity of the water-bearing layer, which is represented with three sandy-gravelly seams was 10 m(Oradovskaya, 1982). There were drilled 2 central and 12 monitor wellsin the area. The inclination of the natural flow was 0.0001 - 0.0006. To the central well, over 223 hours, 494 m^3 of hot water of 51 °C was pumped. During four months was observed the distribution of water temperature in the layer. During 28 days 16,370 m^3 was extracted. The constructed hydrothermal profiles showed a non-uniform distribution of heated water in the layer after pumping, which is due to the heterogeneity of the layer. When pumped out, the more penetrable layers of rocks were faster to cool. The water temperature was 51°C. According to the temperature observation and the amount of injected water and pumped out water, a thermal balance was determined and the heat expenditure was calculated. It is established that in the aeration zone, 2/3 of the heat disperses as a result of heat transfer to heat exchange with the atmosphere. The heat expenditure in the water-bearing layer is mostly due to dispersion. The amount of heat in 16,370 m^3 of the water from a well was about 40% of the incoming amount. After extracting 494 m^3 of the water, only 7% of incoming heat obtained was obtained.

With intense operation of a deep water-bearing layer, the level of groundwater significantly decreases, so for artificial replenishment of the resources, industrial wastewater, which was used for cooling different technological processes, heated to 43° C , was pumped. According to (Oradovskaya, 1982), the heated water from water-injection wells extended not farther than 300m when there was loss of pumping, which changed from 0.4 - 2 m^3/min during five years to 1.2 m^3/min in the following years.

The same data was obtained for observations on the increase in the temperature of water bearing layers when heated water was pumped from installations of air conditioning and technological cooling inside the layers. The worked over warm water of 20 °C was being supplied into gravelly-pebble deposits of a floodplainterrace. Measurement of the temperature according to the system of monitor wells has shown that the heated water extended downwards to a distance of more than 220 m.

Conclusions. The research on the movement of substance and heat in groundwater up to the present has been mostly orientated towardshydro-technical and irrigative construction, water supply, intensifying of coal and oil production, etc. Also, it should be mentioned that quantative methods of solving the equations of heat transfer, and also analog and mathematical modeling have not yet been widely used, though they are necessary for many practical purposes, not only for protecting groundwater from pollution, prognosis of the quality of groundwater, etc, but also for different aspects of optimizing, stability and operation of geotechnical processes, which is especially relevant nowadays. Proving the modeling and efficient usage of analytical methods of calculation requires the corresponding migration parameters of water-bearing and aquitardof layers. Therefore subsequent research should include the accumulation of experimental data covering the processes and parameters of physical-chemical interaction, especially according to the data of field testing. Also the development and testing the physical-chemical models of heat transfer corresponding to the real water-bearing layers and different types of heterogeneity are required.

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