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O. D. Onopriienko[1], **Y. N. Lapusta**[2], Dr. Sci. (Phys.-math.),
V. V. Loboda[1] Dr. Sci. (Phys.-math.)

[1]*Oles Honchar Dnipropetrovsk National University*
[2]*Université Clermont Auvergne, SIGMA Clermont*
(ex- IFMA, French Institute of Advanced Mechanics)

ANTIPLANE PROBLEM FOR AN INTERFACE CRACK IN A PIEZOELECTROMAGNETIC BIMATERIAL

Antiplane problem for piezoelectromagnetic bimaterial with a tunnel crack at the material interface is considered. The presentations of mechanical, electrical, and magnetic factors via a sectionally-analytic vector function are constructed. The problems of linear relationship corresponding to electrically and magnetically conducting conditions on the crack faces are formulated and solved in an analytical form. The variations of mechanical, electrical and magnetic factors along the crack and it continuations are presented.

Key words: piezoelectromagnetic material, interface crack, antiplane problem.

Introduction. Piezoelectric and piezoelectromagnetic materials are widely used in modern micro electronics. Generally, piezoelectrics are encountered in a combination with other materials and the adhesion fracture has a significant influence to the construction's strength. In some cases the stress-strain state can be characterized as the anti-plane state. In such cases, the mathematical models of the problems under consideration are greatly simplified and analytical solutions of the associated problems can often be obtained. In [1; 2; 5; 6] an antiplane problem for a piezoelectric material was studied in the cases of moving and curved motionless electro-conductive interface crack and also for an interface crack in magneto-electro-elastic bimaterial.

In the present paper a practically important case of electro-magneto-conductive crack is explored. Such model is eligible, when a crack is covered with electrodes or filled with conductive grounded liquid which has a magnetic field evenly distributing along the crack.

The constitutive relations for the piezo-electro-magnetic material are

$$\sigma_{ij} = c_{ijks}\varepsilon_{ks} - e_{sij}E_s - h_{sij}H_s \;, \quad D_i = e_{iks}\varepsilon_{ks} + \alpha_{is}E_s + d_{is}H_s \;,$$
$$B_i = h_{iks}\varepsilon_{ks} + d_{is}E_s + \gamma_{is}H_s$$

where σ_{ij}, ε_{ij} – components of stress and strain tensors; D_i, B_i – the components of the electric and magnetic inductions; E_i, H_i – intensities of the electric and magnetic fields, c_{ijks} – elastic, e_{iks} – piezoelectric, h_{iks} – piezomagnetic, d_{is} – electromagnetic constants, α_{is}, γ_{is} – electric and magnetic permeability.

The equilibrium equations in the absence of body forces and free charges are:

$$\sigma_{ij,j} = 0, \quad D_{i,i} = 0, \quad B_{i,i} = 0.$$

The expressions for the deformation, electric and magnetic fields have the form:

$$\varepsilon_{ij} = 1/2 \cdot \left(u_{i,j} + u_{j,i}\right), \quad E_i = -\varphi_{,i}, \quad H_i = -\psi_{,i}$$

where u_i – the components of the displacement vector, φ, ψ – the electric and magnetic potentials.

In an antiplane case we have

$$u_1 = u_2 = 0, \quad u_3 = u_3(x_1, x_2), \quad \varphi = \varphi(x_1, x_2), \quad \psi = \psi(x_1, x_2).$$

Then, the constitutive relations take the form

$$\begin{Bmatrix} \sigma_{3i} \\ D_i \\ B_i \end{Bmatrix} = \mathrm{R} \begin{Bmatrix} u_{3,i} \\ -\varphi_{,i} \\ -\psi_{,i} \end{Bmatrix},$$

where $i = 1, 2$; $\boldsymbol{R} = \begin{bmatrix} c_{44} & e_{15} & h_{15} \\ e_{15} & -\alpha_{11} & -d_{11} \\ h_{15} & -d_{11} & -\gamma_{11} \end{bmatrix}$.

Introducing the vectors

$$\boldsymbol{u} = \left[u_3, \varphi, \psi\right]^T, \qquad \boldsymbol{t} = \left[\sigma_{32}, D_2, B_2\right]^T, \tag{1}$$

one can write

$$\boldsymbol{t} = \boldsymbol{R}\boldsymbol{u}_{,2}. \tag{2}$$

As the functions u_3, φ and ψ are harmonic, then, taking into account (2), the following presentations are valid:

$$\boldsymbol{u} = \boldsymbol{\Phi}(z) + \overline{\boldsymbol{\Phi}}(\overline{z}), \qquad \boldsymbol{t} = \boldsymbol{B}\boldsymbol{\Phi}'(z) + \overline{\boldsymbol{B}}\overline{\boldsymbol{\Phi}}'(\overline{z}) \tag{3}$$

where $\boldsymbol{\Phi}(z) = \left[\Phi_1(z), \Phi_2(z), \Phi_3(z)\right]^T$ – an arbitrary analytical vector-function of the complex variable $z = x_1 + ix_2$, $\boldsymbol{B} = i\boldsymbol{R}$.

The solution of the problem for electrically and magnetically conducting crack. Suppose that there is a crack $|x_1| < a$, $x_2 = 0$ with free faces in a bonded interface between two half-spaces (Fig. 1).

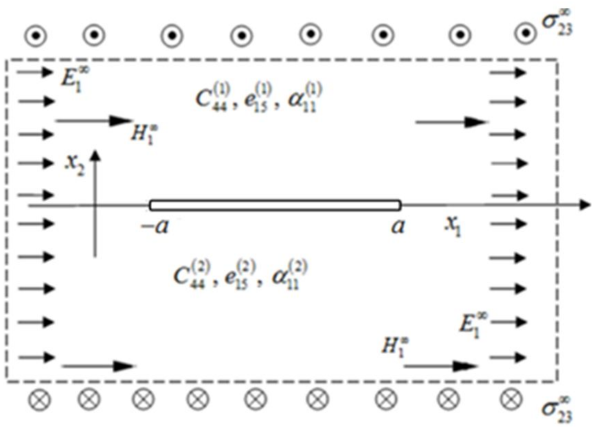

Fig. 1. – A crack between two piezoelectromagnetic materials

The crack is covered with electrodes or filled with conductive fluid, which is grounded and the magnetic field has constant distribution along the crack faces. Then, the boundary conditions on the interface can be written in the form:

$$\sigma_{23}^{(1)} = \sigma_{23}^{(2)} = 0, \quad E_1^{(1)} = E_1^{(2)} = 0, \quad H_1^{(1)} = H_1^{(2)} = 0 \text{ for } |x_1| < a \tag{4}$$

$$\langle \sigma_{32} \rangle = 0, \ \langle D_2 \rangle = 0, \ \langle B_2 \rangle = 0, \ \langle \varepsilon_{31} \rangle = 0, \ \langle E_1 \rangle = 0, \ \langle H_1 \rangle = 0 \text{ for } |x_1| > a \tag{5}$$

It is also assumed, that the vector $\boldsymbol{P}^{\infty} = \left[\sigma_{32}^{\infty}, E_1^{\infty}, H_1^{\infty}\right]$ at infinity is given.

Introducing the vectors

$$\boldsymbol{\upsilon}' = \left[u_3', D_2, B_2\right]^T, \qquad \boldsymbol{P} = \left[\sigma_{32}, \varphi', \psi'\right]^T \tag{6}$$

(the derivatives in (6) are implicit in x_1), then, on the basis of (3), we have

$$\boldsymbol{\upsilon}' = \boldsymbol{M}\boldsymbol{\Phi}'(z) + \bar{\boldsymbol{M}}\bar{\boldsymbol{\Phi}}'(\bar{z}) \tag{7}$$

$$\boldsymbol{P} = \boldsymbol{N}\boldsymbol{\Phi}'(z) + \bar{\boldsymbol{N}}\bar{\boldsymbol{\Phi}}'(\bar{z}) \tag{8}$$

where, the matrixes $\boldsymbol{M}$ and $\boldsymbol{N}$ have the structure

$$\boldsymbol{M} = \begin{bmatrix} 1 & 0 & 0 \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}, \qquad \boldsymbol{N} = \begin{bmatrix} B_{11} & B_{22} & B_{33} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $B_{kl} = iR_{kl}$, where R_{kl} are real.

Using now the representations of the form (7), (8) for the upper and lower half-planes and performing the transformations similar to [5], we obtain

$$\langle \boldsymbol{\upsilon}'(x_1) \rangle = \boldsymbol{\omega}^+(x_1) - \boldsymbol{\omega}^-(x_1) \tag{9}$$

$$\boldsymbol{P}^{(1)}(x_1, 0) = \boldsymbol{S}\boldsymbol{\omega}^+(x_1) - \bar{\boldsymbol{S}}\boldsymbol{\omega}^-(x_1) \tag{10}$$

where $\boldsymbol{\omega}(z)$ – an arbitrary vector-function analytic in each half-plane; $\boldsymbol{S} = \boldsymbol{N}^{(1)}\boldsymbol{E}^{-1}$, $\boldsymbol{E} = \boldsymbol{M}^{(1)} - \bar{\boldsymbol{M}}^{(2)}\left(\bar{\boldsymbol{N}}^{(2)}\right)^{-1}\boldsymbol{N}^{(1)}$. From the last relations it follows that

$$\boldsymbol{S} = \left[\boldsymbol{M}^{(1)}\left(\boldsymbol{N}^{(1)}\right)^{-1} - \bar{\boldsymbol{M}}^{(2)}\left(\bar{\boldsymbol{N}}^{(2)}\right)^{-1}\right]^{-1}. \tag{11}$$

The upper indexes (1) and (2) are related to the upper and lower half-planes, respectively. Transform further the relation (10) according to the method of the work [3]. Consider the matrix $\boldsymbol{L} = \left[L_1, L_2, L_3\right]$ and the composition

$$\boldsymbol{L}\boldsymbol{P}^{(1)}(x_1, 0) = \boldsymbol{L}\boldsymbol{S}\boldsymbol{\omega}^+(x_1) - \boldsymbol{L}\bar{\boldsymbol{S}}\boldsymbol{\omega}^-(x_1) \tag{12}$$

Denoting $\boldsymbol{Y} = \left[Y_1, Y_2, Y_3\right] = \boldsymbol{L}\boldsymbol{S}$, we introduce the function

$$F(z) = \boldsymbol{Y}\boldsymbol{\omega}(z) \tag{13}$$

Let assume that $\boldsymbol{L}\overline{\boldsymbol{S}} = -\gamma \boldsymbol{L}\boldsymbol{S}$ and transpose last equation for convenience. It gives

$$\left(\gamma \boldsymbol{S}^T + \overline{\boldsymbol{S}}^T\right)\boldsymbol{L}^T = 0 \tag{14}$$

This is an eigenvalue problem for finding an eigenvalue γ and an eigenvector $\boldsymbol{L}^T$. The eigenvalues are the roots of the equation

$$\det\left(\gamma \boldsymbol{S}^T + \overline{\boldsymbol{S}}^T\right) = 0 \tag{15}$$

Denote them as $\gamma_1, \gamma_2, \gamma_3$. Eigenvectors $\boldsymbol{L}_j^T = \left[L_{j1}, L_{j2}, L_{j3}\right]^T$ $(j = 1,2,3)$, which correspond to the eigenvalues γ_j, are found from the system (14).

Denoting

$$\boldsymbol{Y}_j = \boldsymbol{L}_j \boldsymbol{S}, \tag{16}$$

we obtain from (12)

$$\boldsymbol{L}_j \boldsymbol{P}^{(1)}(x_1, 0) = \boldsymbol{Y}_j \boldsymbol{\omega}^+(x_1) + \gamma_j \boldsymbol{Y}_j \boldsymbol{\omega}^-(x_1)$$

or, taking into account (13), one gets

$$\boldsymbol{L}_j \boldsymbol{P}^{(1)}(x_1, 0) = F_j^+(x_1) + \gamma_j F_j^-(x_1) \tag{17}$$

where

$$F_j(z) = \boldsymbol{Y}_j \omega(z) \tag{18}$$

Because $F^+(x) = F^-(x) = F(x)$ for $x \notin (-a, a)$, we obtain from (17) the following condition at infinity

$$F_j(z)\Big|_{z\to\infty} = \frac{1}{1+\gamma_j}\left(L_{j1}\sigma_{32}^{\infty} + L_{j2}E_1^{\infty} + L_{j3}H_1^{\infty}\right) \tag{19}$$

Consider the system (10) in an expanded form

$$\left.\begin{aligned}
\sigma_{32}^{(1)}(x_1,0) &= is_{11}\omega_1^+(x_1) + s_{12}\omega_2^+(x_1) + is_{11}\omega_1^-(x_1) - s_{12}\omega_2^-(x_1) + is_{13}\omega_3^+ + is_{13}\omega_3^- \\
-E_1^{(1)}(x_1,0) &= s_{21}\omega_1^+(x_1) + is_{22}\omega_2^+(x_1) - s_{21}\omega_1^-(x_1) + is_{22}\omega_2^-(x_1) + is_{23}\omega_3^+ + is_{23}\omega_3^- \\
-H_1^{(1)}(x_1,0) &= s_{31}\omega_1^+(x_1) + is_{32}\omega_2^+(x_1) - s_{31}\omega_1^-(x_1) + is_{32}\omega_2^-(x_1) + is_{33}\omega_3^+ + is_{33}\omega_3^-
\end{aligned}\right\} \tag{20}$$

Here is taken into account that the matrix $\boldsymbol{S}$ has the following structure

$$\boldsymbol{S} = \begin{bmatrix} is_{11} & s_{12} & s_{13} \\ s_{21} & is_{22} & is_{23} \\ s_{31} & is_{32} & is_{33} \end{bmatrix},$$

where all s_{kl} $(k,l=1,2,3)$ are real. Then for the presentation (10) the system (14) takes the form

$$\begin{bmatrix} (\gamma-1)\,\mathrm{i}s_{11} & (\gamma+1)s_{21} & (\gamma+1)s_{31} \\ (\gamma+1)s_{12} & (\gamma-1)is_{22} & (\gamma-1)is_{32} \\ (\gamma+1)s_{13} & (\gamma-1)is_{23} & (\gamma-1)is_{33} \end{bmatrix} \begin{Bmatrix} L_1 \\ L_2 \\ L_3 \end{Bmatrix} = 0 \qquad (21)$$

and equation (15) is reduced to the relation

$$\det\left(\gamma S^T + \bar{S}^T\right) = \begin{vmatrix} is_{11}(\gamma-1) & s_{12}(\gamma+1) & s_{13}(\gamma+1) \\ (\gamma+1)s_{21} & (\gamma-1)is_{22} & (\gamma-1)is_{23} \\ (\gamma+1)s_{31} & (\gamma-1)is_{32} & (\gamma-1)is_{33} \end{vmatrix} = 0\,.$$

The last equation gives the following roots $\gamma_1 = 1$, $\gamma_2 = \dfrac{\delta+1}{\delta-1}$, $\gamma_3 = \dfrac{\delta-1}{\delta+1}$, where

$$\delta = \sqrt{t_2/t_1}\,,\quad t_1 = s_{21}s_{32}s_{13} + s_{12}s_{23}s_{31} - s_{31}s_{22}s_{13} - s_{12}s_{21}s_{33}\,,$$
$$t_2 = s_{11}s_{22}s_{33} - s_{23}s_{32}s_{11}\,.$$

For the eigenvalue γ_1 we get the following eigenvector $\mathrm{L}_1 = [0, 1, m_{13}]$, and for γ_2 – $\mathrm{L}_2 = [1, im_{22}, im_{23}]$,
where $m_{13} = -s_{21}/s_{31}$, $m_{22} = m(s_{12}s_{23} - s_{13}s_{32})/D_0$, $m_{23} = m(s_{22}s_{13} - s_{23}s_{12})/D_0$, $m = (\gamma_2+1)/(\gamma_2-1)$, $D_0 = s_{22}s_{33} - s_{23}s_{32}$.
It should be noted, that for the eigenvalue γ_3 the eigenvector is $\mathrm{L}_3 = [1, im_{32}, im_{33}]$, where $m_{32} = -m_{22}$, $m_{33} = -m_{23}$.

Then the relation (17) takes the form

$$\begin{cases} E_1^{(1)}(x_1,0) - \dfrac{s_{21}}{s_{31}} H_1^{(1)}(x_1,0) = F_1^+(x_1) + F_1^-(x_1) \\ \sigma_{32}^{(1)}(x_1,0) - im_{22}E_1^{(1)}(x_1,0) - im_{23}H_1^{(1)}(x_1,0) = F_2^+(x_1) + \gamma_2 F_2^-(x_1) \end{cases}$$

Satisfying the boundary conditions (4), one gets

$$F_j^+(x_1) + \gamma_j F_j^-(x_1) = 0, \qquad j = 1,2. \qquad (22)$$

The conditions at infinity have a form

$$F_1(z)\Big|_{z\to\infty} = \frac{1}{2}\left(E_1^\infty - \frac{s_{21}}{s_{31}} H_1^\infty\right) \qquad (23)$$

$$F_2(z)\Big|_{z\to\infty} = \frac{1}{1+\gamma_2}\left(\sigma_{32}^\infty - im_{22}E_1^\infty - im_{23}H_1^\infty\right) \qquad (24)$$

The solution of the problems (22), (24) is obtained by the method [4] in the form

$$F_j\left(z\right)=F_j(z)\big|_{z\to\infty}\cdot\frac{z-2i\varepsilon_j a}{\sqrt{z^2-a^2}}\left(\frac{z+a}{z-a}\right)^{i\varepsilon_j} \tag{25}$$

where $\varepsilon_j=\dfrac{\ln\gamma_j}{2\pi}$ (j=1,2).

The stress, electric and magnetic fields on the interface can be easily found from the system

$$\begin{cases} E_1^{(1)}\left(x_1,0\right)-\dfrac{s_{21}}{s_{31}}H_1^{(1)}\left(x_1,0\right)=\dfrac{E_1^\infty-H_1^\infty s_{21}/s_{31}}{2}\dfrac{x_1}{\sqrt{x_1^2-a^2}}, \\ \sigma_{32}^{(1)}\left(x_1,0\right)-im_{22}E_1^{(1)}\left(x_1,0\right)-im_{23}H_1^{(1)}\left(x_1,0\right)= \\ \qquad =\dfrac{\sigma_{32}^\infty+im_{22}E_1^\infty+im_{23}H_1^\infty}{1+\gamma_2}\dfrac{x_1-2i\varepsilon_2 a}{\sqrt{x_1^2-a^2}}\left(\dfrac{x_1+a}{x_1-a}\right)^{i\varepsilon_2}, \end{cases}$$

and to avoid cumbersomeness they are not written out here.

Define further the displacement jump. It follows from (18)

$$F_j^+\left(x_1\right)-F_j^-\left(x_1\right)=\sum_{k=1}^{3}Y_{jk}\left\{\omega_k^+\left(x_1\right)-\omega_k^-\left(x_1\right)\right\}\left(j=1,2,3\right), \tag{26}$$

where

$$\boldsymbol{Y}=L\mathrm{S} \tag{27}$$

and $\boldsymbol{L}=\begin{bmatrix}0 & 1 & m_{13}\\ 1 & im_{22} & im_{23}\\ 1 & im_{32} & im_{33}\end{bmatrix}$.

The analysis of (27) shows that for the considered class of material $Y_{11}=0$, Y_{jk} are real and Y_{1k}, Y_{j1} are pure imaginary. Therefore, introducing the following designations $n_{1k}=-iY_{1k}$, $n_{j1}=-iY_{j1}, n_{jk}=Y_{jk}$ ($j,k=2,3$) and taking into account (9) the Eq. (26) for $j=1,2$ can be written in the form

$$i\,n_{12}\left\langle D_2\left(x_1,0\right)\right\rangle+i\,n_{13}\left\langle B_2(x_1,0)\right\rangle=F_1^+\left(x_1\right)-F_1^-\left(x_1\right) \tag{28}$$

$$in_{21}\left\langle u_3'\left(x_1,0\right)\right\rangle+n_{22}\left\langle D_2\left(x_1,0\right)\right\rangle+n_{23}\left\langle B_2(x_1,0)\right\rangle=F_2^+\left(x_1\right)-F_2^-\left(x_1\right) \tag{29}$$

Then from (29) we obtain

$$in_{21}\left\langle u_3'\left(x_1,0\right)\right\rangle+n_{22}\left\langle D_2\left(x_1,0\right)\right\rangle+n_{23}\left\langle B_2(x_1,0)\right\rangle=$$
$$=\frac{\gamma_2+1}{\gamma_2}\left(\sigma_{32}^\infty+im_{22}E_1^\infty-im_{23}H_1^\infty\right)\frac{x_1-2i\varepsilon_2 a}{\sqrt{a^2-x_1^2}}\left(\frac{a+x_1}{a-x_1}\right)^{i\varepsilon_2}.$$

Integrating and selecting the imaginary part, we obtain an expression for the displacement jump

$$\left\langle u_3\left(x_1,0\right)\right\rangle=\frac{\gamma_2+1}{\gamma_2 n_{11}}\cdot\operatorname{Im}\left\{\left(\sigma_{32}^{\infty}+im_{22}E_1^{\infty}-im_{23}H_1^{\infty}\right)\cdot\left(\frac{a+x_1}{a-x_1}\right)^{i\varepsilon_2}\sqrt{a^2-x_1^2}\right\}. \quad (30)$$

The jump in the components of the electric and magnetic induction vectors on the base of (28), (29) can be found from the system

$$\begin{cases} n_{12}\left\langle D_2\left(x_1,0\right)\right\rangle+n_{13}\left\langle B_2\left(x_1,0\right)\right\rangle=\dfrac{2x_1}{\sqrt{a^2-x_1^2}}\sigma_{32}^{\infty}, \\ n_{22}\left\langle D_2\left(x_1,0\right)\right\rangle+n_{23}\left\langle B_2\left(x_1,0\right)\right\rangle=\dfrac{\gamma_2+1}{\gamma_2\sqrt{a^2-x_1^2}}\left[\mathrm{A}\cos\omega_2+x_1 B\sin\omega_2\right], \end{cases}$$

where

$$\omega_2=\varepsilon_2\ln\left(\frac{a+x_1}{a-x_1}\right), \qquad A=\sigma_{32}^{\infty}+2\varepsilon_2 a\left(m_{22}E_1^{\infty}+m_{23}H_1^{\infty}\right),$$

$$B=\sigma_{32}^{\infty}2\varepsilon_2 a-\left(m_{22}E_1^{\infty}-m_{23}H_1^{\infty}\right).$$

Finally we obtain

$$\left\langle D_2\left(x_1,0\right)\right\rangle=\frac{1}{n_{12}}\left(\frac{2x_1\sigma_{32}^{\infty}}{\sqrt{a^2-x_1^2}}-n_{13}\left\langle B_2\left(x_1,0\right)\right\rangle\right),$$

$$\left\langle B_2\left(x_1,0\right)\right\rangle=\frac{n_{12}}{n_{23}n_{12}-n_{13}n_{22}}\left(\frac{\gamma_2+1}{\gamma_2\sqrt{a^2-x_1^2}}\left[A\cos\omega_2+B\sin\omega_2\right]-\frac{n_{22}}{n_{12}}\frac{2x_1\sigma_{32}^{\infty}}{\sqrt{a^2-x_1^2}}\right).$$

Numerical realization. The numerical realization was carried out for $\sigma_{32}^{\infty}=1\cdot10^5$ Pa, $H_1^{\infty}=3\cdot10^5$ A/m, $l=0,02\,m$. The results of the calculations for $E_1^{\infty}=3\cdot10^5\,V/m$ (curve 1), $E_1^{\infty}=4\cdot10^5\,V/m$ (curve 2), $E_1^{\infty}=6\cdot10^5\,V/m$ (curve 3), $E_1^{\infty}=8\cdot10^5\,V/m$ (curve 4) are shown in Fig. 2 – 5. The materials with the following characteristics were chosen

$$c_{44}^{(1)}=43,7\cdot10^9\,Pa,\quad e_{15}^{(1)}=17\frac{C}{m^2},\quad \alpha_{11}^{(1)}=15,1\cdot10^{-9}\frac{C}{V\cdot m},$$

$$d_{11}^{(1)}=0,\quad h_{15}^{(1)}=165\frac{N}{a\,m},\quad \gamma_{11}^{(1)}=180,5\cdot10^{-6}\frac{N\cdot s^2}{C^2},$$

$$c_{44}^{(2)}=42,47\cdot10^9\,Pa,\quad e_{15}^{(2)}=-0,48\frac{C}{m^2},\quad \alpha_{11}^{(2)}=0,0757\cdot10^{-9}\frac{C}{V\cdot m},$$

$$d_{11}^{(2)}=0,\quad h_{15}^{(2)}=385\frac{N}{a\,m},\quad \gamma_{11}^{(2)}=414,5\cdot10^{-6}\frac{N\cdot s^2}{C^2}.$$

In particular, the displacement jump $\left\langle u_3\left(x_1,0\right)\right\rangle$ is shown in Fig. 2. Variations of the electric field intensity value E_1 are shown in Fig. 3. Variations of the magnetic field intensity value H_1 are shown in Fig. 4.

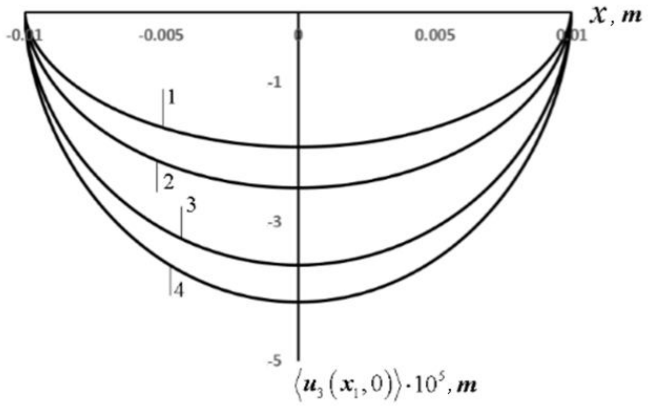

Fig. 2. – The displacement jump $\left\langle u_3\left(x_1,0\right)\right\rangle$ for different values of E_1^{∞}

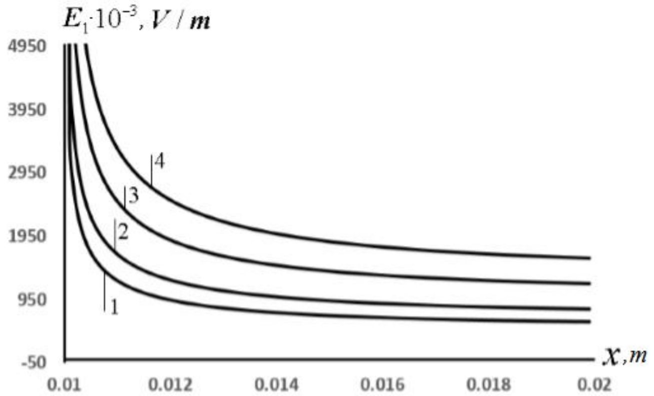

Fig. 3. – The electric field intensity value E_1 for different values of E_1^{∞}

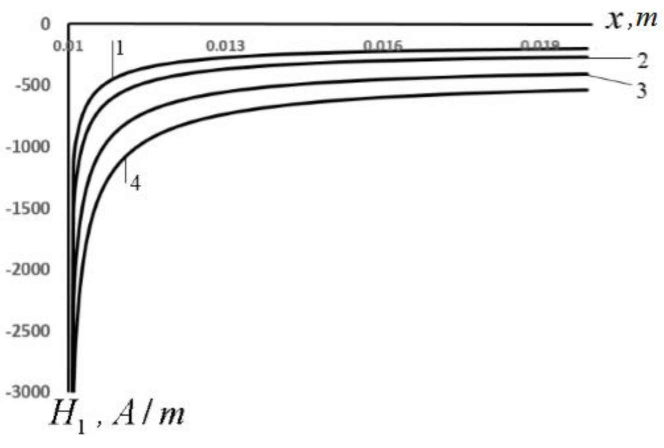

Fig. 4. – The magnetic field intensity H_1 for different values of E_1^{∞}

Variations of the stress value $\sigma_{32}^{(1)}$ on the continuation of the crack are shown in Fig. 5.

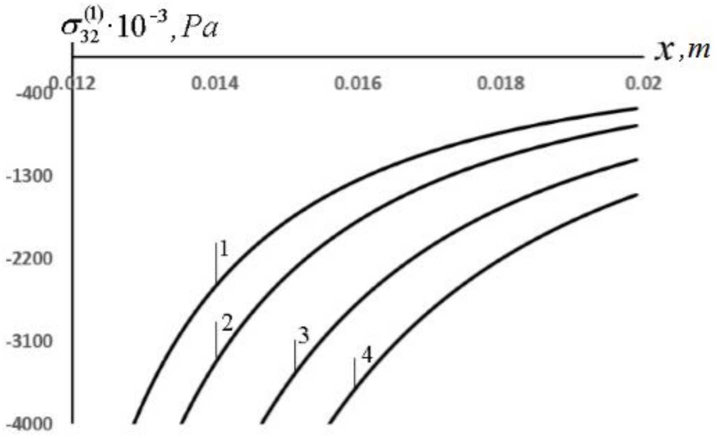

Fig. 5. – The stress value $\sigma_{32}^{(1)}$ for different values of E_1^{∞}

It is seen from the results that the magnitude of the electric field intensity applied at infinity significantly affects the voltage, the magnetic field intensity and also the electric field intensity (Fig. 3 – 5). The change in the sign of the displacement jump which is not very clearly manifested in Fig. 2 is connected with the phenomenon of oscillations.

Conclusion. Antiplane problem for piezoelectromagnetic bimaterial with a tunnel crack at the material interface has been considered. The presentations (9), (10) were formulated for mechanical, electrical, and magnetic factors via a sectionally-analytic vector function. The variations of mechanical, electrical and magnetic factors along the crack and it continuations are presented. The displacement jump, the electric and magnetic fields intensity and the stress are calculated along the corresponding parts of the material interface and presented in a graphical form.

References

1. **Hao R. J.** Interaction of a screw dislocation with a semi-infinite interfacial crack in a magneto-electro-elastic bimaterial / R. J. Hao, J. X. Liu // Mechanics Research Communications. – 2006. – Vol. 33. – P. 415–424.
2. **Li R.** The Mode III Interface Crack in Piezo-Electro-Magneto-Elastic Dissimilar Bimaterials / R. Li, G. A. Kardomateas // Transactions of the ASME. – 2006. – Vol. 73. – P. 220–227.
3. **Loboda V. V.** Antiplane problem for an interface crack in a piezoelectromagnetic biomaterial / V. V. Loboda, A. G. Kryvoruchko, Y. N. Lapusta // Bull. Dnipropetrovsk University. Ser.: Mechanics. – 2015. − Vol. 23. – № 5. − P. 106–117.
4. **Muskhelishvili N. I.** Some Basic Problems of Mathematical Theory of Elasticity / N. I. Muskhelishvili. – Noordhoff International Publishing, Leyden, 1977. – 707 p.
5. **Wang X.** A conducting arc crack between a circular piezoelectric inclusion and an unbounded matrix / X. Wang, Z. Zhong // International Journal of Solids and Structures. – 2002. – № 39. – P. 5895–5911.

6. **Wang X.** A moving conducting crack at the interface of two dissimilar piezoelectric materials / X. Wang, Z. Zhong, F. L. Wu // International Journal of Solids and Structures. – 2003. – № 40. – P. 2381–2399.

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АНТИПЛОСКА ЗАДАЧА ДЛЯ МІЖФАЗНОЇ ТРІЩИНИ В П'ЄЗОЕЛЕКТРОМАГНІТНОМУ БІМАТЕРІАЛІ

О. Д. Онопрієнко[1], **Ю. М. Лапуста**[2], д-р фіз.-мат. наук,
В. В. Лобода[1], д-р фіз.-мат. наук.

[1]*Дніпропетровський національний університет імені Олеся Гончара, Дніпро, Україна*
[2]*Університет SIGMA Клермон, Франція*

Розглянуто антиплоску задачу для пьєзоелектромагнітного біматеріалу з тунельною тріщиною в області розділу матеріалів. Побудовано представлення механічних, електричних і магнітних факторів через одну кусоково-аналітичну вектор-функцію. Сформульовані і ров'язані задачі лінійного спряження в аналітичній формі, що відповідають електрично- та магнітнопровідним умовам на берегах тріщини. Представлені варіації механічних, електричних та магнітних факторів уздовж тріщини та її продовження.

Ключові слова: п'єзоелектромагнітний матеріал, міжфазна тріщина, антиплоска задача.

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АНТИПЛОСКАЯ ЗАДАЧА ДЛЯ МЕЖФАЗНОЙ ТРЕЩИНЫ В ПЪЕЗОЭЛЕКТРОМАГНИТНОМ БИМАТЕРИАЛЕ

О. Д. Оноприенко[1], **Ю. Н. Лапуста**[2] , д-р физ.-мат. наук,
В. В. Лобода[1], д-р физ.-мат. наук

[1]*Днепропетровский национальный университет имени Олеся Гончара, Днепр, Украина,*
[2]*Университет SIGMA Клермон, Франция*

Рассмотрена антиплоская задача для пьезоэлектромагнитного биматериала с туннельной трещиной в области раздела материалов. Построены представления механических, электрических и магнитных факторов через одну кусочно-аналитическую вектор-функцию. Сформулированы и решены задачи линейного сопряжения в аналитической форме, которые соответствуют электро и магнитопроводящим условиям на берегах трещины. Представлены вариации механических, электрических и магнитных факторов вдоль трещины и ее продолжения.

Ключевые слова: пьезоэлектромагнитный материал, межфазная трещина, антиплоская задача.