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# CALCULATION OF INDUCED CURRENTS DURING THE INDUCTIONAL HEATING BY A FLAT RECTANGULAR MULTITURN SOLENOID

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***Abstract***. *The analysis of electromagnetic processes and calculation of the induced currents in the system for the induction heating, presented by the flat rectangular multiturn solenoid placed above the plane of the thin-walled sheet-metal has been carried out. Graphic dependences for spatial distribution of the induced currents and coefficient of transformation of current in the area under inductor have been built. The nature of the excitation system has been determined to have no effect on the output characteristics of the induction heating.*

***Key words:*** *induction heating, inductor system, induced current, electromagnetic processes, eddy currents, heating of metal.*

# РАСЧЕТ ИНДУЦИРОВАННЫХ ТОКОВ ПРИ ИНДУКЦИОННОМ НАГРЕВЕ ПЛОСКИМ ПРЯМОУГОЛЬНЫМ МНОГОВИТКОВЫМ СОЛЕНОИДОМ

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***Аннотация***. *Проведен анализ электромагнитных процессов и расчет индуцированных токов в системе для индукционного нагрева, представленной плоским прямоугольным многовитковым соленоидом, помещённым над плоскостью тонкостенного листового металла. Построены графические зависимости для пространственного распределения индуцированных токов и коэффициента трансформации тока в области под токопроводами индуктора. Определено, что характер возбуждения системы не влияет на выходные характеристики индукционного нагрева.*

***Ключевые слова:*** *индукционный нагрев, индукторная система, индуцированный ток, электромагнитные процессы, вихревые токи, нагрев металла.*

# РОЗРАХУНОК ІНДУКОВАНИХ СТРУМІВ ПРИ ІНДУКЦІЙНОМУ НАГРІВІ ПЛОСКИМ ПРЯМОКУТНИМ БАГАТОВИТКОВИМ СОЛЕНОЇДОМ

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***Анотація***. *Проведено аналіз електромагнітних процесів і розрахунок індукованих струмів у системі для індукційного нагріву, представленою плоским прямокутним багатовитковим соленоїдом, розміщеним над плоскістю тонкостінного листового металу. Побудовано графічні залежності для просторового розподілу індукований струмів та коефіцієнту трансформації струму в області під струмопроводами індуктора. Виявлено, що характер збудження системи не впливає на вихідні характеристики індукційного нагріву.*

***Ключові слова:*** *індукційний нагрів, індукторна система, індукований струм, електромагнітні процеси, вихрові струми, нагрів металу.*

## Introduction

Induction heating is the process of non-contact heating an electrically conducting object arranged in a powerful variable (usually high-frequency) field through heat generated in the object by eddy Foucault currents. The flow of eddy currents is accompanied by excretion of heat due to the action of Joule's law, which leads to heating of the metal object [1]. It is widely used in industry to perform a lot of manufacturing operations, for example, by quenching the surfaces of metal products, non-contact heating of liquids, levitation melting of metals, as well as maintenance operations, such as straightening of thin-walled metal, removing of paint from metal surfaces, the parting of the bolt connections, etc. [2-5].

## Literature analysis

The idea of using pre-induction heating in the magnetic-pulse processing of metals has been proposed in 1984.[6]. The authors of the proposal have developed and created a system that initiates the flow of current in the coil of the working tool until the force action. Pre-induction heating may significantly improve the efficiency of deformation by magnetic pulse as a whole.

Implementation of the instrument as inductor in the form of a circular cylinder or solenoid does not solve many practical problems in processing of metals by means of electromagnetic field energy. The systems with the windings of rectangular geometry allowing the excitation of linear induced currents in the metal blanks to be treated. For example, the straightness of circuits means flow transverse uniformity of the excited fields and, accordingly, the temperature distribution or power characteristics depend upon the requirements of the production [7].

The technological necessity mentioned is not only factor that determines the interest in the rectangular inductor systems. Under certain conditions, these systems may be more effective than circular or cylindrical ones.

As shown by the authors [7], the tangential component of the magnetic field through the conductive sheet is hardly penetrates into the free half space in a system where the low-frequency field is presented by a package of a plane electromagnetic waves. If we orient ourselves on the total current law, it will mean that all the induced current in such a system should be determined solely by the magnetic field strength on the source-inductor side.

## Study objectives

The analysis of electromagnetic processes and calculation of the induced currents in the system for the induction heating, presented by the flat rectangular multiturn solenoid placed above the plane of the thin-walled sheet-metal.

## Electromagnetic processes of the system

The zones where rectilinear, parallel and coplanar currents are excited are the areas in the vicinity of sections AA on the inductor schemes (Fig.1,a,c). These sections correspond to the calculated model on the Fig.1,b and Fig.1,d.

It should be noted that differences in the directions of currents, according to the inductor schemes (Fig.1,a,c), mean differences in the characteristics and manufacturing processes. For example, induction heating.

In the first case of symmetric excitation of the system, the identically directed currents in conductors (Fig.1,a,b) induce non-zero currents in a center and, as it appears a priori, must provide more homogeneous heating of sheet-metal along his cross-sectional in a working zone. ( $-b \le y \le b$ , Fig.1,b). Asymmetric excitation of the system takes place in the second case. Opposition of directions of currents in parallel conductors (Fig.1,c,d) will set to the zero of the induced currents in a center ( $y = 0$ ), which means large heterogeneity of heating within a working zone ( $-b \le y \le b$ , Fig.1,d) than in the previous case.

We shall analyze the processes excited by unidirectional currents in accordance with [8].

According to the calculation model (Fig.1,b) formulated assumptions, determine the course of solving the problem according to the algorithm described in [7, 8].

1. We have the geometrical and electrical symmetry of the system with respect to a coordinate plane $ZOX$ , due to design and identity field excitation means for $y < 0$ and $y > 0$ .

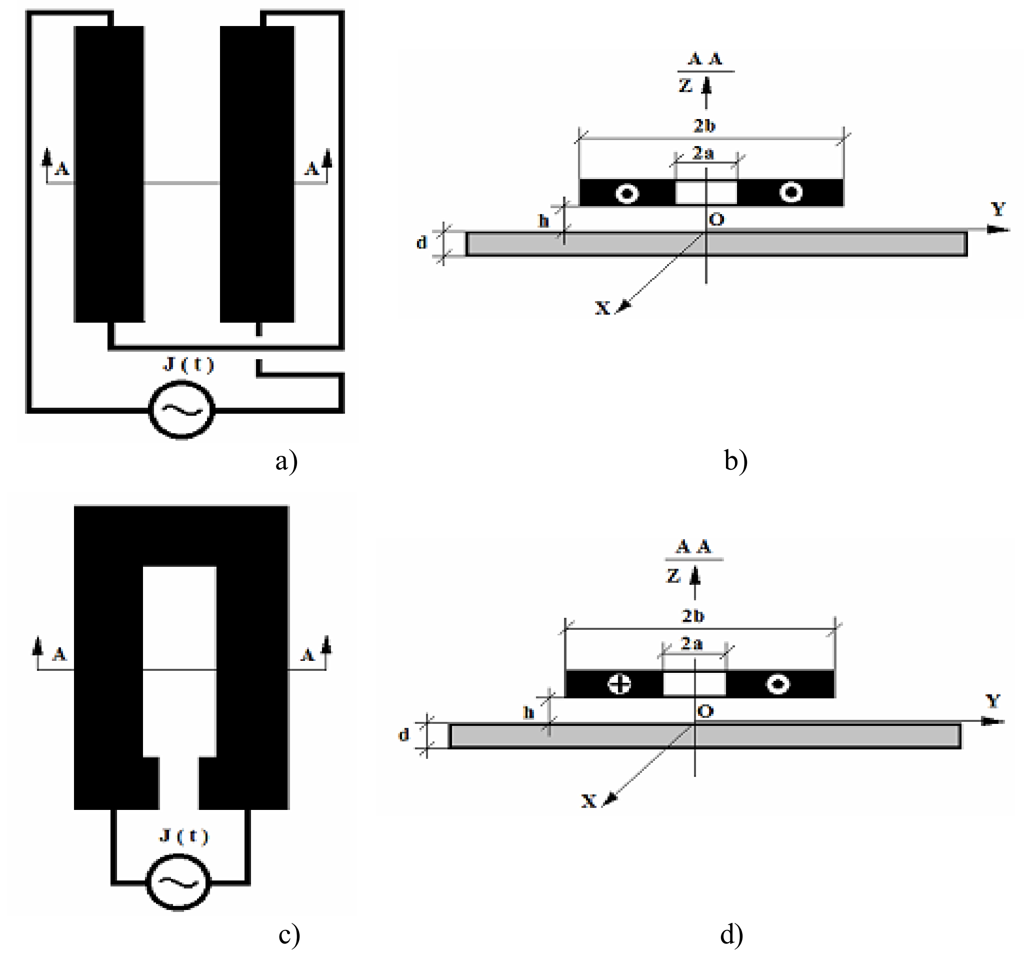



Fig. 1. Sketches of the rectangular inductor systems with different ways of connecting conductors. a) the currents in conductors of working zones are unidirectional; b) corresponding calculation model; c) the currents in conductors of working zones are antidirected; d) corresponding calculation model.

2. The sheet metal has huge cross sizes, thickness $d$ and conductivity $\gamma$ .

3. The current - I(t) with the single coordinate component $I(t) = I_x(t)$, « $y-$distribution» of the induced currents density being equal, flows in the working space of the inductors with the number of coils - W.

4. Along the horizontal axis system has a length which is large enough, so $\frac{\partial}{\partial x}=0.$

5. The condition of the Landau's quasistationarity : $\frac{\omega}{c}\cdot b \ll 1$ , where $\omega -$ is the cyclic frequency, $c-$ is the speed of light, $b-$ is the biggest size of the system.

6. An electromagnetic field with non-zero components of the intensity is induced in the system under consideration: $E_x \neq 0, H_{y,z} \neq 0$ .

Within the framework of the assumptions adopted by the Maxwell equations for the non-zero components of the electromagnetic field, transformed by Laplace.

Solving the Maxwell's equations by the known methods of mathematical analysis and dropping intermediate mathematical transformations (as in [9-13]) eventually we obtain analytical expressions for the induced currents.

Space-time dependence for the currents induced in the metal slab can be written as:

$$\overline{I_x(\varphi)} = -\left(\frac{I_m \cdot w}{(b-a)}\right)\cdot(\pi d)\int_0^\infty f_2^2(\alpha)\cdot e^{-\alpha\frac{h}{d}}\cdot\alpha\times$$

$$\times\sum_{k=0}^{\infty}\delta_k\frac{F_1(\beta_k,\alpha)}{\Phi(\beta_k,\alpha)}\cdot\psi(\varphi)\cdot d\alpha, \quad (1)$$

where $f_2(\alpha)=\frac{4}{\pi\cdot\alpha}\sin\left(\alpha\cdot\frac{b-a}{2d}\right)\cdot\sin\left(\alpha\cdot\frac{b+a}{2d}\right)$

$$F_2(\beta_k,\alpha,z)=\beta_k\cdot\sin\left(\left(\beta_k\left(1-\frac{z}{d}\right)\right)+\right.$$
$$\left.+\left(\frac{\beta_k}{\alpha}\right)\cdot\cos\left(\beta_k\left(1-\frac{z}{d}\right)\right)\right)$$
$$\Phi_k(\alpha)=\cos(\beta_k)\cdot\left[\alpha^2+2\alpha-\beta_k^2\right]-$$
$$-2\cdot\beta_k\cdot\sin(\beta_k)\cdot[1+\alpha]$$
$$\psi(\varphi)=\left(\frac{\omega^2}{p_k^2+\omega^2}\right)\cdot\left[\sin\varphi+\frac{p_k}{\omega}\cdot\left(e^{\frac{p_k}{\omega}\varphi}-\cos\varphi\right)\right],$$

where $\varphi=\omega t-$ is the phase of the signal, $\omega=2\pi f$, $f$ – is the frequency, $I_m$ – is the amplitude of the current in the inductor, $w$ – is the number of coils, $\lambda$ – is the setting of integral transformation, $\beta_k$ – are roots of the equation

$$tg(\beta_k)\cdot\left(\frac{\beta_k}{(\lambda\cdot d)}-\frac{(\lambda\cdot d)}{\beta_k}\right)=2.$$

By integrating the terms (1) on $y\in[a;b]$, we find the value of the current, induced in the sheet metal blank with width $(b-a)$ under inductors conductor.

$$I_x(t,a\le y\le b)=-j_m\cdot(\pi d)\times$$
$$\times\int_0^\infty f_2^2(\alpha)e^{-\alpha\frac{h}{d}}\alpha\sum_{k=0}^\infty\delta_k\frac{F_1(\beta_k,\alpha)}{\Phi(\beta_k,\alpha)}\psi(t)\cdot d\alpha,\quad(2)$$

The coefficient of transformation is defined as the ratio of the amplitudes of the excitated current and the current induced in the metal blank with a width – (b – a) (area: $a\le y\le b$).

$$K(a\le y\le b)=\frac{J_{x\max}}{J_m}=$$
$$=\left(\frac{\pi dw}{(b-a)}\right)\int_0^\infty f_2^2(\alpha)e^{-\alpha\frac{h}{d}}\cdot\alpha\cdot\sum_{k=0}^\infty\delta_k\frac{F_1(\beta_k,\alpha)}{\Phi(\beta_k,\alpha)}\psi(\varphi)\Big|_{\max}d\alpha,\quad(3)$$

where $\psi(\varphi)\big|_{\max}$ – is the maximum value of the function $\psi(\varphi)$;
$\varphi=\omega\cdot t-$ is the phase of the signal; $\omega=2\pi f$, $f$ – is the frequency.

For further analysis of convenience, omitting intermediate identity transformations, we wrote the final term for the averaged value of the induced current.

$$\overline{J_x(\varphi)}=-\left(\frac{I_m\cdot w}{(b-a)}\right)(\pi d)\int_0^\infty f_2^2(\alpha)\cdot e^{-\alpha\frac{h}{d}}\cdot\alpha\times$$
$$\times\sum_{k=0}^\infty\delta_k\frac{F_1(\beta_k,\alpha)}{\Phi(\beta_k,\alpha)}\cdot\psi(\varphi)\cdot d\alpha,\quad(4)$$

Terms (1) – (4) as in symmetrical excitation case, allow to conduct the practical estimation of the parameters of electromagnetic processes in the system for induction heating of thin-walled sheet metal.

**Evaluation of the thermal processes during the induction heating**

Calculations for the following initial benchmark data were made: number of coils - $w=20$, is the transverse sizes of conductor $\{a=0{,}0025m;b=0{,}0225m\}$ – (the transverse size of two conductors coil is ~ 40mm), the thickness of metal sheet is – $d=0{,}001$ m, the gap between inductors plane and metal blank is – $h=0{,}001$ m, specific conductivities of the non-magnetic steel and aluminum are $\gamma_{St}\approx0{,}4\cdot10^7$ 1/Ohm·m, $\gamma_{A\ell}\approx3{,}75\cdot10^7$ 1/Ohm^xx^m, the range of working frequencies is $f\in[1000;50000]$ Hz, amplitude of the current in inductors coil – $I_m=10\div15$ A.

Calculations were made by «Mathematica 5.1» which is the regular application package.

1.An approximate range of the integration variable – $\alpha$, in range where functions $|f_{1,2}(\alpha)|$ are different from the zero $\alpha\in[0;\alpha_{\max}]$ determines from the view of integrands – $f_{1,2}(\alpha)$, which are accordingly the sine or cosine - Fourier images of the radial distribution of initiating current in the inductor, determines Theirs disagreement allows as ~ $5\div10\%$. As shown by numerical estimates for accepted geometry of the inductor – $\alpha\in[0;0{,}8]$.

2. For the value of the summation index k=0,1,2,3... the roots of equation $\beta_k=\beta_k(x)$ are calculated in the value range of $\alpha\in[0;\alpha_{\max}]$

. As shown by numerical estimates for the received geometry of the inductor, the following approximation seems quite satisfactory: $\beta_k \approx k \cdot \pi + \sqrt{2 \cdot x}$ .

3.The dependence $\beta_k = \beta_k(x)$ is obtained and substituted into integrand of the rating formulas.

4. Improper integrals and sums of the series are calculated by the «NIntegrate» and «NSum» regular application package.

5. The summation in the series is performed for a different quantity of higher harmonics. If increasing of the limit value by one is does not leads to a result that differs from the previous by more than 5 ÷ 10 %, their considered quantity is deemed sufficient.

The generalization of calculated results leads to the following conclusions.

**Excitation of the system by unidirectional currents**
**Nonmagnetic steel**

In the range of frequencies $f \in [10000;50000]$ Hz the value of the transformation ratio remains unchanged and equal to $K \approx 18{,}03 \div 18{,}2$ and the time of heating to the $100^o$ by currents value ~10A equals to $\Delta T^o \approx 55 \div 46$ sec.

By lowering the frequencies to 1000Hz the value of the transformation ratio falls to $K \approx 13{,}9$, but time of the heating to $100^o$ with a current of ~10A increases to the $\Delta T^o \approx 3$ min.

**Aluminum**

It is expedient to start the calculation of basic indicators of electromagnetic and thermal processes from the lower frequencies (1000Hz) due to value of aluminums specific conductivity which is higher than steel one.

On the $f = 1000$ Hz the value of the transformation ratio is $K \approx 18{,}9$, but the time of the heating by ~10A current is equal to $\Delta T^o \approx 5{,}5$ min. By increasing the current to ~15A time of the heating falls to $\Delta T^o \approx 2{,}5$ min.

Increasing of the frequency to ~ 50000Hz increases the coefficient of transformation almost to the maximum – $K \approx 19{,}5$ . The time of the heating is $\Delta T^o \approx 1{,}5$ min.

For illustration of the analysis some characteristics are shown below.

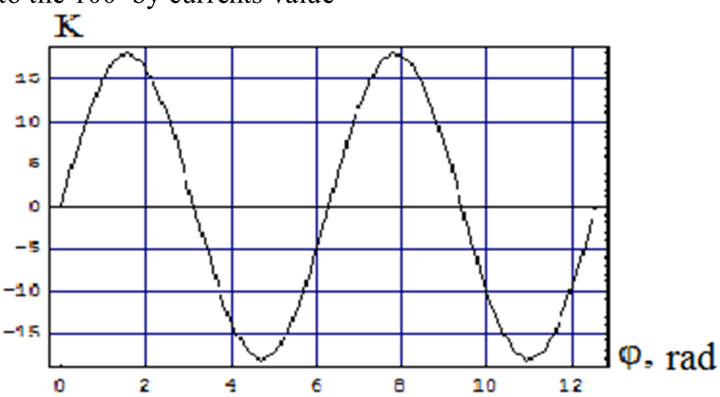
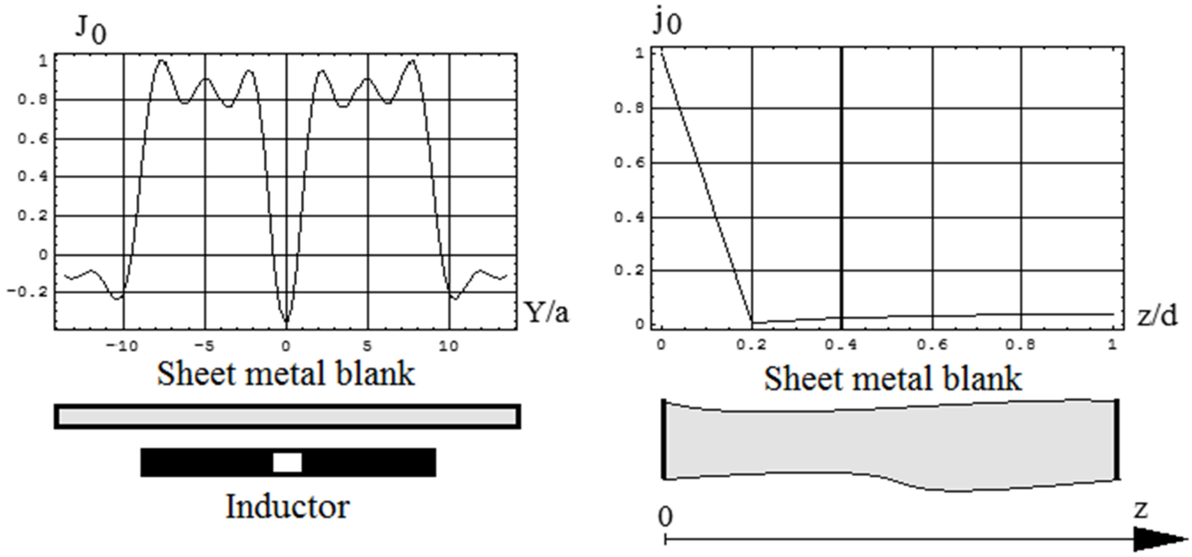



Fig. 2. The coefficient of transformation of current in the area under inductors conductor as a function of phase ($\varphi = 2\pi \cdot f \cdot t$) on the frequency $f = 25000$ Hz

**Excitation of the system by multidirectional currents.**

Calculations of basic indicators of electromagnetic and thermal processes, carried out for both steel and aluminum showed a slight difference from analogues calculated when the system had unidirectional currents. Only the transverse distributions of induced currents are different. (Fig.3). However, in the end, this fact does not affect on temperature indexes, because of their quadratic dependence on floating currents. It means that the nature of the excitation system has no effect on the output characteristics of the induction heating.




a) b)

Fig. 3.Dimensional distribution of the induced currents in the sheet metal on the frequency $f = 25000\,\text{Hz}$ : a) distribution of the relative local current density on the width (normalized to the maximum); b) distribution of the relative local current density on the thickness (normalized to the maximum)

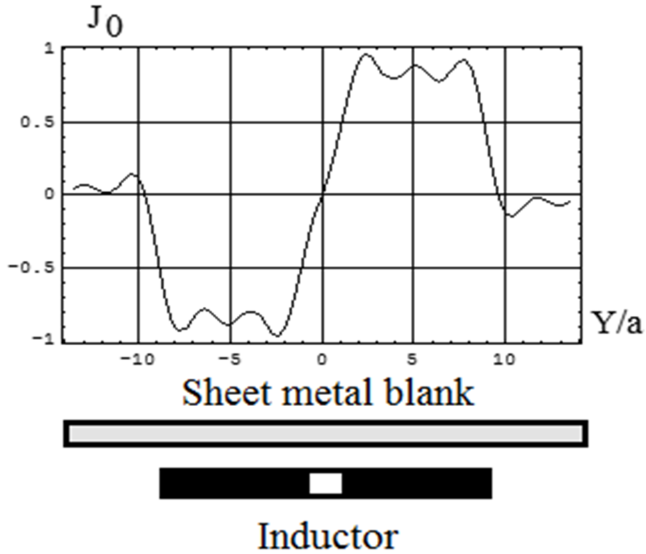



Fig. 4. The distribution of the relative value of the excited current on the width of the working zone on the frequency $f$ = 25000 Hz (normalized to the maximum)

**Conclusions**

1. The analysis of electromagnetic processes and calculation of the induced currents in the system for the induction heating, presented by the flat rectangular multiple-turn solenoid placed above the plane of the thin-walled sheet-metal has been carried out.

2. Graphic dependences for spatial distribution of the induced currents and coefficient of transformation of current in area under inductor are built.

3. It was determined that the nature of the excitation system has no effect on the output characteristics of the induction heating. It was determined that increasing of frequency from 1000 to 50000 Hz is led to increase of coefficient of transformation to – $K \approx 19{,}5$. The time of heating is $\Delta T^{o} \approx 1{,}5$ min, and the nature of the excitation system has no effect on the output characteristics of the induction heating.

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