ЕЛЕКТРОТЕХНІЧНІ ТА РАДІОТЕХНІЧНІ ВИМІРЮВАННЯ

УДК 621.396

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A WAVE PROCESSES IN METALDIELECTRIC WAVE GUIDE

A number of questions according to the description and studying of the electrodynamic processes happening in a plain metaldielectric wave guide is considered on the basis of beam representations. The beam treatment of the phenomena in a wave guide allowed to define structure of an electromagnetic field in a wave guide, parameters of directing system, the dispersion equations and parameters of own waves, including the main type of H_{10} wave.
Keywords — electromagnetic field; plain metaldielectric wave guide; Poynting's vector

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Волновые процессы в METALDIELECTRIC волновод

А ряд вопросов в соответствии с описанием и изучением электродинамических процессов, происходящих в простом metaldielectric волновода считается на основании представлений пучка. Лечение пучок явлений в волновод позволил определить структуру электромагнитного поля в волноводе, параметры направляющей системы, дисперсионных уравнений и параметров собственных волн, в том числе основного вида H10 волны.
Ключевые слова - электромагнитное поле; равнина metaldielectric волновод; Вектор Пойнтинга

Introduction

It is possible to present the structure of an electromagnetic field in any metaldielectric wave guide of rectangular section including flat, as result of addition of four plain uniform waves called component, repeatedly reflected from its boundary surfaces (Fig 1). The beam treatment of the phenomena in a wave guide allows to define structure of an electromagnetic field in a wave guide, parameters of directing system, the dispersion equations and parameters of own waves. Such approach to questions of the description of electrodynamic processes of the wave guide structures provides physical presentation and simplicity of understanding of a spatial pattern of a field at all stages of the task solution, leading to finite expressions for components of vectors of a field, parameters of wave guides and own waves which are completely coinciding with solutions of Maxwell's equations. Such terms as phase and group velocity, wave length in a wave guide, transverse coefficients, a phase constant, critical length of a wave, types of waves and others take clear physical meaning in this treatment.[1].

Structure of the electromagnetic field in the wave

The direction of Poynting vector of a plain wave is defined by three angles (θ_x, θ_y, θ_z), formed with axes of coordinates (Fig 2). Directional cosines of these angles, sum of squares of which is equal to unit, are functions of wave length, the transverse sizes of a wave guide, electrodynamic parameters of the environments filling the waveguide structure, indexes of own fields.

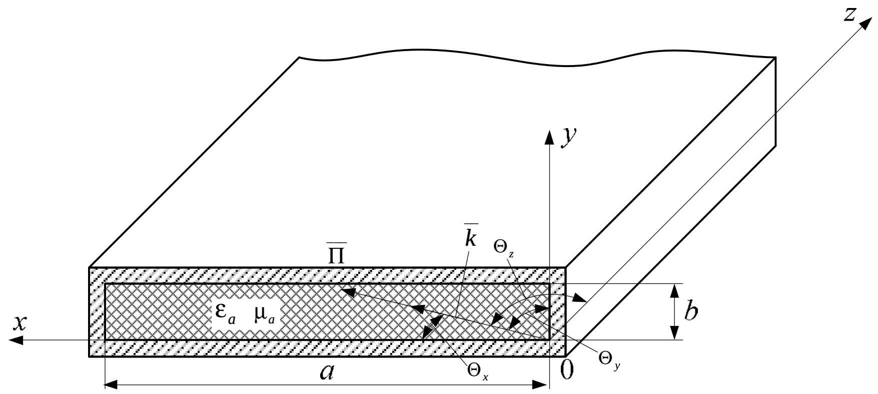

Fig. 1. Schematic drawing of transverse section of wave guide and projections of Poynting vectors of component waves on diametral plane

The component wave generally can have linear, circular or elliptic polarization. The plain wave of any polarization can be presented by the sum of two linearly polarized plain waves, having mutually perpendicular vectors of intensity of electric and magnetic field. Resolution on two orthogonal linearly polarized waves can be made on two any mutually perpendicular unitary vectors lying in cross plane in relation to the direction of propagation and called polarizing basis. It is expedient to choose the plane of polarizing basis perpendicular to the plane S passing through the direction of propagation of a wave and one of axes of coordinates for simplification of mathematical record of projections of vectors of a field on an axis of coordinates. Three variants of S plane positioning are possible.

Above stated couple of two orthogonal linearly polarized component waves is interpreted as a system of own fields of a rectangular wave guide of any cross structure.

Vectors $\overline{E}$ и $\overline{H}$ of plain wave are perpendicular to the propagation direction $\overline{\Pi}$, owing to a vector ratio $\overline{\Pi}=\overline{E}\times\overline{H}$, besides, they are mutually perpendicular. Spread out vector $\overline{E}$ to two orthogonal components $\overline{E}_1$ and $\overline{E}_2$. One of vectors, for example $\overline{E}_1$, will be perpendicular to S plane, then other vector $\overline{E}_2$ will lie in this plane. The wave, vector $\overline{E}_1$ of which is perpendicular to plane S, we will call normally polarized, and other wave with vector $\overline{E}_2$ — parallel polarized.

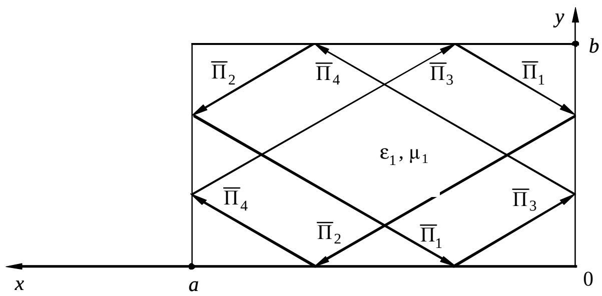

Fig. 2. Structure of plain metaldielectric wave guide

We choose axis $0z$ for the analysis of a studied wave guide (Fig 3).

Vector $\overline{E}_1$, which is perpendicular to plane S, has two components E_{1x} and E_{1y}. The projection of vector $\overline{E}_1$ to axis $0z$ is equal to zero — $E_{1z}=0$. Vector $\overline{H}_1$ is perpendicular to vector $\overline{E}_1$ and lies in plane S. It has three projections H_{1x}, H_{1y}, H_{1z}. У параллельно поляризованной волны Vector $\overline{E}_2$ lies in plane S at parallel polarized wave. It has all three projections to coordinate axes E_{2x}, E_{2y}, E_{2z}. Vector $\overline{H}_2$ of this wave is perpendicular to the plane, therefore it has only two projections H_{2x} and H_{2y}, and $H_{2z}=0$.

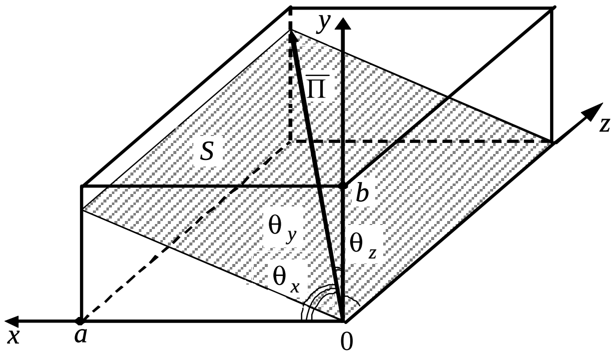

Fig. 3. Rectangular wave guide and plane S, perpendicular to polarizing basis plane

Comparing own fields H_{mn} and E_{mn} in a metal rectangular wave guide with orthogonal linearly polarized flat waves, we see that one of them — normally polarized and has the same projections of vectors, as well as H_{mn}, another — parallel polarized wave corresponds E_{mn}.

The resultant plain wave with vectors $\overline{E}$ and $\overline{H}$ and generally has all six components:

$E_x=E_{1x}+E_{2x}$; $E_y=E_{1y}+E_{2y}$; $E_z=E_{1z}+E_{2z}$; $H_x=H_{1x}+H_{2x}$; $H_y=H_{1y}+H_{2y}$; $H_z=H_{1z}+H_{2z}$.

Superposition of four flat waves extending in the directions $\overline{\Pi}_1$, $\overline{\Pi}_2$, $\overline{\Pi}_3$, $\overline{\Pi}_4$, (Fig 1), forms a resultant non-uniform wave, intensity components of electric or magnetic fields are defined by expression [2]

$$A=A_m\cos\left(k_x x-\frac{\varphi_{x_0}}{2}\right)\cos\left(k_y y-\frac{\varphi_{y_0}}{2}\right)\cos\left(\omega t-k_z z+\frac{\varphi_{x_0}}{2}+\frac{\varphi_{y_0}}{2}\right), \qquad (1)$$

where A_m — amplitude of a component independent on spatial coordinates and time; $k_x=k\cos\theta_x$, $k_y=k\cos\theta_y$, $k_z=k\cos\theta_z$ — projections of a wave vector $\overline{k}$ to axes of coordinates; $\overline{k}=\overline{1}_r k$ — the wave vector which is equal in size to coefficient of wave propagation in this environment $k=\omega\sqrt{\varepsilon_a\mu_a}=\frac{2\pi}{\lambda}$, and coincides in the direction with Poynting's vector $\overline{\Pi}$; $\lambda=\frac{\lambda_0}{\sqrt{\varepsilon}}$ — wave length in the dielectric environment; λ_0 — wave length in free space; φ_{x0}, φ_{y0} — phases of coefficients of reflection of field vectors components from the boundary planes $x=0$ and $y=0$ respectively.

For a metal wave guide φ_{x0} and φ_{y0} can assume the values equal to 0º or 180º. Tangent components of vector $\overline{E}$ and normal components of vector $\overline{H}$ are equal to zero (walls are ideally conductive). Therefore, phases of their coefficients of reflection φ_{x0}, φ_{y0}, are equal 180º. Normal components of vector $\overline{E}$ and tangent components of vector $\overline{H}$ are maximum at boundary surfaces, phases of coefficients of reflection φ_{x0}, φ_{y0} for them, are equal to zero. Thus, on a surface $x=0$ for components E_x, H_y, H_z we have $\varphi_{x0}=0$, and for components — E_y, E_z, H_x — $\varphi_{x0}=180°$.

On a surface $y = 0$ for components $E_y,\ H_x,\ H_z$ — $\varphi_{y0} = 0$, and for components $E_x,\ E_z,\ H_y$ — $\varphi_{y0} = 180°$.

Amplitudes of vectors components $\overline{E}$ and $\overline{H}$ also depend on directing cosines of these vectors. Knowing directing angles of Poynting vector $\overline{\Pi}$: θ_x, θ_y, θ_z and orientation of vectors $\overline{E}_{1,2}$ и $\overline{H}_{1,2}$ normally and parallel polarized waves as regard to chosen plane S, it is possible to define their directing cosines as regard to axes of coordinates: $0x$, $0y$, $0z$. It is expedient to enter new system of coordinates for this purpose: $0x'$, $0y'$, $0z'$, which axes coincide with the direction of vectors $\overline{E}_{1,2}$, $\overline{H}_{1,2}$, $\overline{\Pi}$.

Position of new coordinate system as regard to old one can be completely characterized by three angles entered by L. Euler: nutation angle, procession pure rotation angle. It is possible to define directing cosines of vectors by these angles, $\overline{E}_1$, $\overline{H}_1$ and $\overline{E}_2$, $\overline{H}_2$.

There are three variants of directing cosines according to three variants of polarizing bases positioning.

We specify $\alpha,\ \beta,\ \gamma$ — angles between vector $\overline{E}_{1,2}$ and axes of coordinates $0x$, $0y$, $0z$; $\alpha',\ \beta',\ \gamma'$ — angles between vector $\overline{H}_{1,2}$ and the same axes.

The plane S passes through vector $\overline{\Pi}$ and axis $0z$. Vectors $\overline{E}_1$ and $\overline{H}_1$ of normally polarized wave have the following directing cosines

$$\left.\begin{array}{lll} \overline{E}_1 \to \cos\alpha = \dfrac{\cos\theta_y}{\sin\theta_z}; & \cos\beta = \dfrac{\cos\theta_x}{\sin\theta_z}; & \cos\gamma = 0; \\ \overline{H}_1 \to \cos\alpha' = \dfrac{\cos\theta_x \cos\theta_z}{\sin\theta_z}; & \cos\beta' = \dfrac{\cos\theta_y \cos\theta_z}{\sin\theta_z}; & \cos\gamma' = \sin\theta_z. \end{array}\right\} \qquad (2)$$

Directing cosines of vectors $\overline{E}_2$ and $\overline{H}_2$ of parallel polarized wave are defined by the following expressions:

$$\left.\begin{array}{lll} \overline{E}_2 \to \cos\alpha = \dfrac{\cos\theta_x \cos\theta_z}{\sin\theta_z}; & \cos\beta = \dfrac{\cos\theta_y \cos\theta_z}{\sin\theta_z}; & \cos\gamma = \sin\theta_z; \\ \overline{H}_2 \to \cos\alpha' = \dfrac{\cos\theta_y}{\sin\theta_z}; & \cos\beta' = \dfrac{\cos\theta_x}{\sin\theta_z}; & \cos\gamma' = 0. \end{array}\right\} \qquad (3)$$

The direction of Poynting vector $\overline{\Pi}$ is defined by values of angles θ_x, θ_y, θ_z, with the positive directions of axes of coordinates.

Amplitudes of vectors components $\overline{E}$ and $\overline{H}$ are connected with their amplitude coefficient ratios:

$$\left.\begin{array}{lll} E_{xm} = E_m \cos\alpha; & E_{ym} = E_m \cos\beta; & E_{zm} = E_m \cos\gamma; \\ H_{xm} = H_m \cos\alpha'; & H_{ym} = H_m \cos\beta'; & H_{zm} = H_m \cos\gamma'. \end{array}\right\} \qquad (4)$$

All these amplitudes can be expressed through one of them, for example, as it is made in the analysis of magnetic waves in a metal rectangular wave guide.

All these amplitudes can be expressed through one of them, for example H_{zm}, as it is made in the analysis of magnetic waves H_{mn} in a metal rectangular wave guide.

In this case

$$H_m = \frac{H_{zm}}{\cos\gamma'}; \quad E_m = Z_0 H_m = \frac{Z_0 H_{zm}}{\cos\gamma'}, \qquad (5)$$

where $Z_0 = \sqrt{\dfrac{\mu_a}{\varepsilon_a}}$ — wave resistance of the environment filling a wave guide.

Substituting H_m and E_m from (5) to (4), we receive values of amplitudes of all components, expressed through H_{zm}. Directing cosines thus are defined by expressions (2):

$$\left.\begin{array}{lll} E_{xm} = \dfrac{Z_0 \cos\theta_y}{\sin^2\theta_z} H_{zm}; & E_{ym} = \dfrac{Z_0 \cos\theta_x}{\sin^2\theta_z} H_{zm}; & E_{zm} = 0; \\ H_{xm} = \dfrac{\cos\theta_x \cos\theta_z}{\sin^2\theta_z} \cdot H_{zm}; & H_{ym} = \dfrac{\cos\theta_y \cos\theta_z}{\sin^2\theta_z} H_{zm}; & H_{zm}. \end{array}\right\} \qquad (6)$$

The amplitudes of all components are expressed through E_{zm} during the analysis of electric waves E_{mn}, and directing cosines are defined by expressions (3).

Substituting in (1) values of amplitudes (6) and considering values of phases on surfaces $x = 0$ and $y = 0$, we write down expressions for vectors components of own field H_{mn}:

$$\left.\begin{aligned}
&E_x=\left\{\frac{Z_0\cos\theta_y}{\sin^2\theta_z}\right\}H_{zm}\cos(k_x x)\sin(k_y y)\sin(\omega t-k_z z);\\
&E_y=\left\{\frac{Z_0\cos\theta_x}{\sin^2\theta_z}\right\}H_{zm}\sin(k_x x)\cos(k_y y)\sin(\omega t-k_z z);\\
&E_z=0;\\
&H_x=\left\{\frac{\cos\theta_x\cos\theta_z}{\sin^2\theta_z}\right\}H_{zm}\sin(k_x x)\cos(k_y y)\sin(\omega t-k_z z);\\
&H_y=\left\{\frac{\cos\theta_y\cos\theta_z}{\sin^2\theta_z}\right\}H_{zm}\cos(k_x x)\sin(k_y y)\sin(\omega t-k_z z);\\
&H_z=H_{zm}\cos(k_x x)\cos(k_y y)\cos(\omega t-k_z z).
\end{aligned}\right\} \tag{7}$$

Similarly it is possible to find components of vectors $\overline{E}$ and $\overline{H}$ of own field E_{mn}:

$$\left.\begin{aligned}
&E_x=\left\{\frac{\cos\theta_x\cos\theta_z}{\sin^2\theta_z}\right\}E_{zm}\cos(k_x x)\sin(k_y y)\sin(\omega t-k_z z);\\
&E_y=\left\{\frac{\cos\theta_y\cos\theta_z}{\sin^2\theta_z}\right\}E_{zm}\sin(k_x x)\cos(k_y y)\sin(\omega t-k_z z);\\
&E_z=E_{zm}\sin(k_x x)\sin(k_y y)\cos(\omega t-k_z z);\\
&H_x=\left\{\frac{\cos\theta_y}{Z_0\sin^2\theta_z}\right\}E_{zm}\sin(k_x x)\cos(k_y y)\sin(\omega t-k_z z);\\
&H_y=\left\{\frac{\cos\theta_x}{Z_0\sin^2\theta_z}\right\}E_{zm}\cos(k_x x)\sin(k_y y)\sin(\omega t-k_z z);\\
&H_z=0.
\end{aligned}\right\}$$

According to expression (1) distribution of amplitude of any component along axis $0X$ is described by function

$$A_x=\cos\left(k_x x-\frac{\varphi_{x0}}{2}\right).$$

Then at value $x=0$ we have $A_{x0}=\cos\frac{\varphi_{x0}}{2}$, and at $x=a$ — $A_{xa}=\cos\left(k_x a-\frac{\varphi_{x0}}{2}\right)$. At the same time, at $x=a$ we have $A_{xa}=\cos\frac{\varphi_{xa}}{2}$, where φ_{xa} — phase of reflection coefficient on border $x=a$.

Equality of functions $\cos\left(k_x a-\frac{\varphi_{x0}}{2}\right)=\cos\frac{\varphi_{xa}}{2}$ is reached at the following ratio of arguments: $k_x a-\frac{\varphi_{x0}}{2}=m\pi+\frac{\varphi_{xa}}{2}$. It follows, that $k_x a=m\pi+\frac{\varphi_{x0}}{2}+\frac{\varphi_{xa}}{2}$. Considering that, $\varphi_{xa}=\varphi_{x0}=0$, we receive

$$k_x a=m\pi,\qquad\qquad k_x=\frac{m\pi}{a}.$$

From a condition $\varphi_{x0}=180°$ equality of functions follows $\sin k_x a=\cos 90°=0$, from where it is also possible to receive $k_x a=m\pi$, where $m=0,\ 1,\ 2,\ldots$

From ratios $k_x=k\cos\theta_x$, $k_x=\frac{m\pi}{a}$, $k=\frac{2\pi}{\lambda}$ we define expression for a cosine of the angle θ_x

$$\cos\theta_x=\frac{m\lambda}{2a}.$$

Similarly we define $k_y=\frac{n\pi}{b}$, and directing cosine

$$\cos\theta_y=\frac{n\lambda}{2b},$$

where $n=0,\ 1,\ 2,\ldots$

Directing cosines of these angles are connected among themselves by a known ratio [3]:

$$\cos^2\theta_x+\cos^2\theta_y+\cos^2\theta_z=1,$$

from which the ratio follows

$$\sin\theta_z = \frac{\lambda}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}\ . \tag{8}$$

Thus, the direction of Poynting vector $\overline{\Pi}$ depends on wave length λ, the sizes of a wave guide a and b, and on wave type which is defined by indexes m and n, it is quantity of the half waves which are keeping within the sizes a and b of wave guide walls.

At increase in length of a wave λ value $\sin\theta_z$ increases, and at $\theta_z = 90°$, $\sin\theta_z = 1$. Propagation of an electromagnetic wave along a wave guide stops. Length of a wave with which the angle θ_z reaches 90º, is called as the critical length of a wave $\lambda_{кр}$.

From (8) follows that

$$\lambda_{кр} = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}\,. \tag{9}$$

If the critical length of a wave is known, the angle θ_z can be defined in a different way

$$\sin\theta_z = \frac{\lambda}{\lambda_{кр}}, \qquad \cos\theta_z = \sqrt{1 - \left(\frac{\lambda}{\lambda_{кр}}\right)^2}\ .$$

And vice versa, the critical length of a wave of any mode can be determined on known directing sine of the angle θ_z. Using ratios (8) and (9), we write down expressions for sines and critical lengths of waves for types of modes: H_{10}, H_{20}, H_{01}, H_{30} — $\sin\theta_{z10} = \frac{\lambda}{2a}$, $\lambda_{кр10} = 2a$; $\sin\theta_{z20} = \frac{\lambda}{a}$, $\lambda_{кр20} = a$; $\sin\theta_{z01} = \frac{\lambda}{2b}$, $\lambda_{кр01} = 2b$; $\sin\theta_{z30} = \frac{3\lambda}{2a}$, $\lambda_{кр30} = \frac{2a}{3}$.

Expressions (7) for vectors components $\overline{E}$ and $\overline{H}$ of own field of type H_{mn} can be modified if to multiply numerator and denominator by k^2 and to consider that $k = \omega\sqrt{\varepsilon_а\mu_а}$, $k_x = k\cos\theta_x$, $k_y = k\cos\theta_y$, $k_z = k\cos\theta_z$, and aslo $k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{b}$, $Z_0 = \sqrt{\frac{\mu_а}{\varepsilon_а}}$.

Vectors components of a field of type H_{mn} are written in the form of:

$$\left.\begin{aligned}
E_x &= \left(\frac{\omega\mu_а}{k_c^2}\right)\left(\frac{n\pi}{b}\right)H_{zm}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin(\omega t - k_z z);\\
E_y &= \left(\frac{\omega\mu_а}{k_c^2}\right)\left(\frac{m\pi}{a}\right)H_{zm}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin(\omega t - k_z z);\\
E_z &= 0;\\
H_x &= \left(\frac{k_z}{k_c^2}\right)\left(\frac{m\pi}{a}\right)H_{zm}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin(\omega t - k_z z);\\
H_y &= \left(\frac{k_z}{k_c^2}\right)\left(\frac{n\pi}{b}\right)H_{zm}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin(\omega t - k_z z);\\
H_z &= H_{zm}\cos\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\cos(\omega t - k_z z).
\end{aligned}\right\}$$

It is similarly possible to modify expressions (6) for vectors components of a field of type E_{mn}:

$$\left.\begin{aligned}
&E_x=\left(\frac{k_z}{k_c^2}\right)\left(\frac{m\pi}{a}\right)E_{zm}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin(\omega t-k_z z);\\
&E_y=\left(\frac{k_z}{k_c^2}\right)\left(\frac{n\pi}{b}\right)E_{zm}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin(\omega t-k_z z);\\
&E_z=E_{zm}\sin\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\cos(\omega t-k_z z);\\
&H_x=\left(\frac{\omega\varepsilon_a}{k_c^2}\right)\left(\frac{n\pi}{b}\right)E_{zm}\sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)\sin(\omega t-k_z z);\\
&H_y=\left(\frac{\omega\varepsilon_a}{k_c^2}\right)\left(\frac{m\pi}{a}\right)E_{zm}\cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)\sin(\omega t-k_z z);\\
&H_z=0,
\end{aligned}\right\}$$

where $k_c^2=k_x^2+k_y^2$ — cross wave number.

The received expressions for vectors components $\overline{E}$ and $\overline{H}$ of own fields H_{mn} and E_{mn} on the basis of ray representations completely coincide with the expressions found by the analysis of Maxwell equations. The specified compliance confirms validity of assumptions that own fields are the orthogonal linearly polarized flat waves extending in a wave guide on certain angles θ_z to its axis. Structure of field of a wave of H_{10} type is obtained at substitution in (7) of following parameters: $\cos\theta_y=0$; $\sin\theta_z=\cos\theta_x=\frac{\lambda}{2a}$; $\cos\theta_z=\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}$; $k_x=\frac{\pi}{a}$; $k_z=\frac{2\pi}{\lambda_в}$, obtained at substitution in the corresponding formulas of zero value of index n and m = 1. Thus the constant of wave propagation in a wave guide k_z is equal

$$k_z=k\cos\theta_z=\frac{2\pi}{\lambda}\cos\theta_z=\frac{2\pi}{\lambda_в},$$

where parameter $\lambda_в$ is called the wave length in a wave guide

$$\lambda_в=\frac{\lambda}{\cos\theta_z}=\frac{\lambda}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}}.$$

Expressions for projections of vectors of a field to coordinates axes of a wave of H_{10} type:

$$\left.\begin{aligned}
&E_x=0;\qquad E_z=0;\qquad H_y=0;\\
&E_y=\frac{Z_0}{\sin\theta_z}H_{zm}\sin(k\cos\theta_x x)\sin(\omega t-k\cos\theta_z z)=\\
&=Z_0\frac{2a}{\lambda_в}H_{zm}\sin\frac{\pi x}{a}\sin\left(\omega t-\frac{2\pi}{\lambda_в}z\right);\\
&H_x=\frac{\cos\theta_z}{\sin\theta_z}H_{zm}\sin(k\cos\theta_x x)\sin(\omega t-k\cos\theta_z z)=\\
&\frac{2a}{\lambda_в}H_{zm}\sin\frac{\pi x}{a}\sin\left(\omega t-\frac{2\pi}{\lambda_в}z\right);\\
&H_z=H_{zm}\cos(k\cos\theta_x x)\cos(\omega t-k\cos\theta_z z)=H_{zm}\cos\frac{\pi x}{a}\cos\left(\omega t-\frac{2\pi}{\lambda_в}z\right).
\end{aligned}\right\}$$

H_{10} type wave has simple structure of a field, possibility of a single-wave operating mode, demands the minimum sizes of a wave guide. It is called the main and the large majority of the wave guide devices works at H_{10} type wave.

Parameters of wave guide and own waves extending in

The main parameters of a plain metaldielectric wave guide are: critical length of wave $\lambda_{кр}$, wave length in wave guide $\lambda_в$, propagation constant (or a phase constant) k_z, phase speed of wave $V_ф$, group speed (or power) $V_{гр}$, cross coefficients k_x, k_y. and characteristic resistance of wave guide $Z_в$.

If the direction of propagation of a plain partial wave coincided with the direction of a longitudinal axis of a wave guide, wave length in a wave guide would coincide with wave length in uniform dielectric which fills a wave guide. However propagation of plain waves happens on angle θ_z to a wave guide axis. According to geometrical constructions (Fig 4) it is possible to define interrelation of wave length in the wave guide $\lambda_в$ with a wave length in dielectric λ_1.

In the direction $\overline{\Pi}$ the flat wave with a speed extends

$$V_1 = \frac{1}{\sqrt{\varepsilon_{a1}\mu_{a1}}} = \frac{c}{\sqrt{\varepsilon_1\mu_1}} = \frac{c}{\sqrt{\varepsilon_1}},$$

where c — electromagnetic constant; $\mu_1 = 1$.

The dashed line $1-1'$ shows the wave front in a timepoint $t = 0$. The wave front $1-1'$ is perpendicular to Poynting's vector $\overline{\Pi}$ (line AB). For a short time $\Delta t = T$, where T — the period, the front of a wave will reach position $2-2'$. During this time the plain wave will pass a way in the direction of the propagation, equal to AB. We assume, that the wave phase was equal in a point A to zero; as the wave is plain, and in all points of the front $1-1'$ will be a zero phase. The front of a wave moves parallel to itself, therefore, in line points $2-2'$ the zero phase of a wave also is observed.

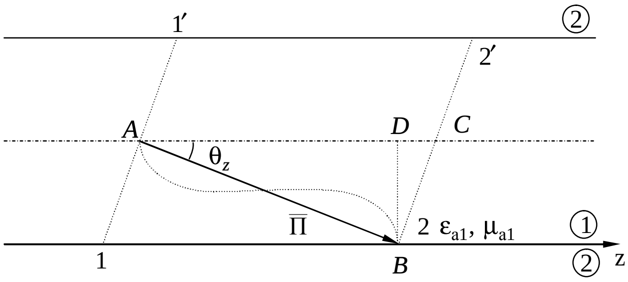

Fig. 4. Wave length determination in plain metaldielectric wave guide

Speed of movement of wave front (phase speed in direction AB) coincides with the speed of propagation of energy and is equal to $V_1 = \frac{AB}{\Delta t}$. Speed of propagation of the front along an axis $0z$ is defined by a ratio $V_ф = \frac{AC}{\Delta t}$. Speed of propagation of energy along a wave guide (group speed) is equal to $V_{гр} = \frac{AD}{\Delta t}$. Speeds V_1, $V_ф$, $V_{гр}$, it is easy to connect among themselves; as $AD = AB\cos\theta_z$.

$$V_{гр} = \frac{AB\cos\theta_z}{\Delta t} = V_1\cos\theta_z; \tag{10}$$

$$V_ф = \frac{AB}{\Delta t\cos\theta_z} = \frac{V_1}{\cos\theta_z}. \tag{11}$$

If on piece AB one length of wave λ_1 keeps, that along a longitudinal axis of a wave guide length of a wave $\lambda_в$ keeps on a piece AC, that is,

$$\lambda_в = \frac{\lambda_1}{\cos\theta_z}. \tag{12}$$

As it seen from expressions (10) — (12), phase and group speeds, wave length in a wave guide depend on value of a directing cosine $\cos\theta_z$, which, in turn, is defined by type of the waveguide structure, electromagnetic parameters of dielectric, and also the sizes of a wave guide a and b, wave length in free space, a class of fields, wave type.

The constant of wave propagation in a wave guide k_z is defined by length $\lambda_в$

$$\lambda_в = \frac{\lambda}{\sqrt{\varepsilon_1}\cos\theta_z}, \tag{13}$$

$$k_z = \frac{2\pi}{\lambda_в} = k_1\cos\theta_z = \frac{2\pi}{\lambda}\sqrt{\varepsilon_1}\cos\theta_z. \tag{14}$$

Cross coefficients k_x and k_y are determined by formulas:

$$k_x = k\cos\theta_x, \qquad k_y = k\cos\theta_y. \tag{15}$$

Characteristic resistance of a wave guide $Z_в$ for H_{10} type wave is equal to the relation of cross components of intensity of electric and magnetic field.

$$Z_в = \frac{E_y}{H_x} = \frac{Z_0\lambda_в}{\lambda} = \frac{Z_0}{\sqrt{1-\left(\frac{\lambda}{2a}\right)^2}} = \frac{Z_0}{\cos\theta_z}.$$

As for H_{10} type wave $\cos\theta_y = 0$, as $n = 0$ the sine of the angle θ_z is equal to a cosine of the angle θ_x. Therefore $\sin\theta_z = \cos\theta_x = \frac{\lambda}{2a}$, $\cos\theta_z = \sqrt{1-\left(\frac{\lambda}{2a}\right)^2}$ and in ratios (10) — (15) for wave length, phase and group speeds in a plain metaldielectric wave guide to substitute value

$$\cos\theta_z = \sqrt{1-\left(\frac{\lambda}{2a}\right)^2}.$$

On the basis of the offered and theoretically studied plain metaldielectric wave guide it is necessary to develop, first of all, a number of elements of paths of measuring instruments of wave guide parameters, two-detector microwave converter for construction on its basis of the measuring instrument of wave length in PLMDW, the measuring instrument of complex coefficients of reflection and transfer of developed elements and devices.

Such elements and devices are: transitions from the coaxial line and a hollow standard wave guide to the plain metaldielectric wave guide, the coordinated loading, a detector head through passage and the microwave converter.

The structure of an electromagnetic field in a plain metaldielectric wave guide is presented as result of addition of four plain uniform waves called partial, repeatedly reflected from its boundary surfaces. On the basis of ray representations inclined falling of a plain (partial) wave with any type of polarization is investigated. Thus liberally polarized plain wave was presented in the form of two orthogonal linearly polarized waves. Decomposition is made on the polarizing basis plane of which is perpendicular to the direction of a wave and a longitudinal axis of coordinates. The structure of an electromagnetic field in a wave guide is defined. Expressions for component strengths of electric and magnetic fields of resultant non-uniform waves of H_{mn} and E_{mn} types which correspond to the orthogonal linearly polarized waves are obtained. The dispersion equations from which directing angles of Poynting vectors of partial waves are found.

The found ratios for directing corners allowed to determine all parameters of waves of H_{mn} and E_{mn} types and the main H_{10} wave: critical waves lengths, phase and group speeds, waves lengths in a wave guide, phase constants, cross coefficients and characteristic resistance of a wave guide.

The applied approach to questions of the description of the electrodynamic processes happening in a wave guide, provides physical presentation and simplicity of understanding of a spatial picture of a field at all stages of the task solution, leading to final expressions for vectors components of a field, parameters of wave guides and own waves which are completely coinciding with solutions of Maxwell equations. Such terms as phase and group speeds, wave length in a wave guide, cross coefficients, a phase constant, critical length of a wave, types of waves and others get clear physical sense in new representation.

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Рецензія/Peer review : 17.5.2014 р. Надрукована/Printed :25.6.2014 р.