УДК 621.391

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# THE NATURE OF QUANTUM STATE UNCERTAINTY IN TELECOMMUNICATION SYSTEM

***Annotation –*** *The paper studies methodological issues of uncertainty state occurred in a digital telecommunication trunk while critical increasing the symbol transmission rate. A math model proposed for quantum state description of a telecommunication system in a form of conditional probability matrix. This model has a relevant geometric interpretation as a linear vector set that simulates symmetric quantum entanglement of discrete states in transmitting-receiving system. Given approach intends to advance researches in the realm of data transmission over digital communication channels.*
*Keywords: quantum state uncertainty, telecommunication system, data transmission.*

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## СУЩНОСТЬ СОСТОЯНИЯ КВАНТОВОЙ НЕОПРЕДЕЛЕННОСТИ В ТЕЛЕКОММУНИКАЦИОННОЙ СИСТЕМЕ

*В статье рассматриваются методологические аспекты возникновения состояния неопределенности в цифровой телекоммуникационной системе при критическом увеличении скорости передачи. Предложена математическая модель для описания квантовых состояний телекоммуникационной системы в форме матрицы условных вероятностей. Эта модель имеет соответствующую геометрическую интерпретацию в виде линейной комбинации векторов, которая симулирует симметричную квантовую запутанность дискретных состояний в приемо-передающей системе. Данный подход продолжает исследования в области передачи данных по цифровым каналам связи.*
*Ключевые слова: квантовая неопределенность состояния, телекоммуникационная система, передача данных.*

## 1. Introduction

Consider a conventional digital telecommunication system $S$, which comprises three principal entities: sender of numerical symbols $SNS$, digital channel for symbol transmission $DC$ and receiver of symbols $RS$, $S := \{SNS, DC, RS\}$, fig.1. The typical example of this type of system is an LTE downlink wireless trunk formed by evolved base station (eNB) acting as $SNS$, OFDM digital radio channel as $DC$ and terminal user equipment UE playing the role of $RS$ entity.

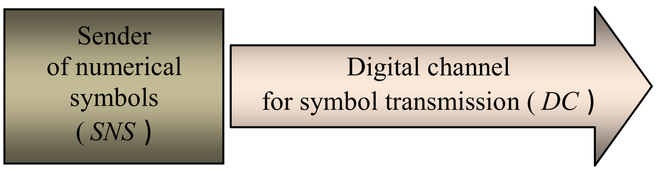


**Fig.1 – The digital telecommunication system**

The widely used legacy communication facilities exploit the primitive one-bit symbols type of coding for digital data transmission [1−2]; however, advanced technologies like coherent optical communication (COC) and fourth generation mobile communication (4G) provide extended two-bit and more modulation techniques [3]. For instance, the LTE standard includes three optional types of one subcarrier symbol coding: the 4-states quadrature phase shift keying (QPSK) with petty two-bit symbols, the16-states quadrature amplitude manipulation (16QAM) with four-bit symbols and 64-states quadrature amplitude manipulation (64QAM) with six-bit symbols. Thus, taking the given symbol transmission rate ($STR$) measured in baud units (the number of symbols or transport informational blocks per one second), the three options of channel bit rate are available in LTE trunk: $STR_{QPSK}$ ×bps= $2\cdot\dfrac{\text{bps}}{\text{baud}}$ × $STR$ ×baud= $2\cdot STR$ ×bps; $STR_{16QAM}$ ×bps=4× $STR$ ×bps; $STR_{64QAM}$ ×bps=6× $STR$ ×bps. In coherent optical communication, the one-symbol information capacity (SC) varies from 2 up to 8 bits per symbol and more; because of that, the overall digital channel productivity in symbol performance channel depends on symbol transmission rate STR and symbol capacity.

In turn, the two entities (STR and SC) are mutually predetermined by the limited physical channel properties, such as frequency bandwidth $\Delta F$, maximal power of transmitted signal $SP$, signal attenuation $SA$ and interference level. To some extent, the last three entities can be uniformly rendered by so called signal-to-noise-ratio (SNR) parameter. Commonly, the frequency bandwidth $\Delta F$ implies a dedicated or fixed channel resource available for data transmission utilization, e.g. 100 MHz optical trunk in DWDM optic fiber, or 5MHz ether trunk in LTE radio access network. Due to specific spectrum allocation $f(s)$ of modulated signal within the given bandwidth $\Delta F$, the transmission channel is considered either analogue or discrete one. Of course, this classification is rather contingent and marginal. The analogue channel supposes continuous and fairly smooth parceling the signal spectrum in the band $\Delta F$ ; the pulse amplitude manipulation of GSM frequency division multiplexing (FDM) or code division multiple access (CDMA) driven 3G radio trunk are a good patterns of this case. Instead, the modern methods of signal modulation like orthogonal frequency division multiplexing (OFDM) in frequency domain, or the alternative Nyquist raised cosine filtering in time domain, expose a near to discrete spectrum allocation by modulated electromagnetic carrier wave [4].

The character of signal spectrum allocation within any given bandwidth $\Delta F$ is critical towards estimation of signal-to-noise ratio SNR due to the widely applicable of noise/interference model with quasi Gaussian distribution of spectral power density $\sigma^2(f)$ as the function of frequency $f$ , fg.2. The signal with spread continuous spectrum is painfully sensitive to the Gaussian noise, while pure discrete spectrum signal tolerates Gaussian noise incredibly better. Typically, two main factors limit the telecommunication system performance on the $SNS$ party, i.e. output peak/average power and frequency bandwidth. On the receiving party of a telecommunication system, the key performance restrictor is input signal-to-noise ratio SNR; the later crucially depends on the channel physical length.

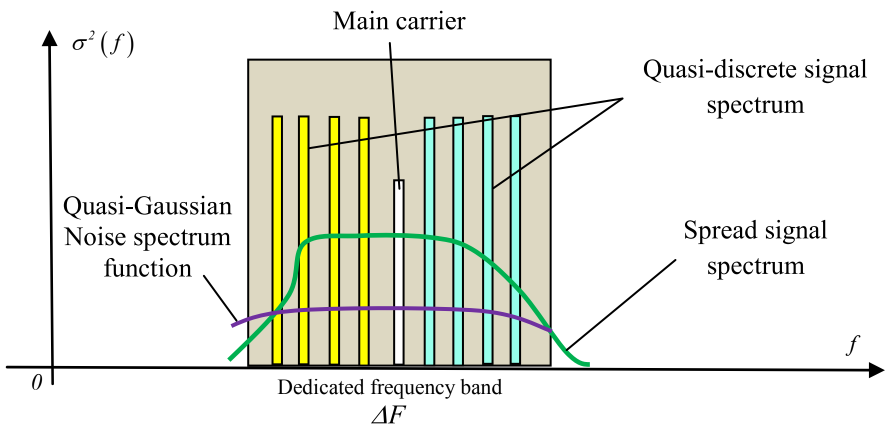


**Fig.2 – The quasi Gaussian noise spectral density**

The potential of a telecommunication system with spread signal spectrum reflects the Shannon-Hartley formula

$$C = \Delta F \cdot log_2\left(1 + SNR\right) \times bps\ . \tag{1}$$

where $C$ is the maximal possible channel capacity calculated in bit per second units (bps); the signal-to-noise ratio $SNR$ is determined as $SNR = \dfrac{S}{N}$; $S$ is the average in-band signal power, $N$ is the average in-band noise power [5]. Often, the relative channel capacity is used in form of spectral efficiency $\gamma = \dfrac{C}{\Delta F} = log_2\left(1 + SNR\right) \times bps / Hz$ . Figure 3 illustrates function $\gamma$ behavior with respect to formula (1):

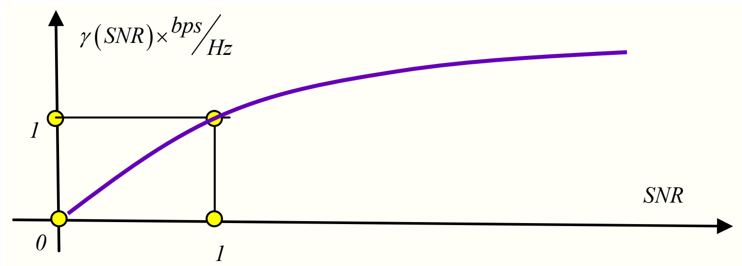


**Fig.3 – Spectral efficiency as function of signal-to-noise ratio**

## 2. Deterministic model of telecommunication system

The Shannon-Hartley formula gives a theoretical insight on the telecommunication system ability. However, it is not a simple task to estimate relevant figures of real channel potential capacity. Besides, the scope of relevance of the so called Shannon limit derived in terms of spectral-to-energy channel efficiency is presently one of the largely disputable issues among the specialists [6]. In fact, the classical industrial communication systems utilize the more pragmatic and transparent criterion of channel productivity; this is a required and adoptable bit error rate (BER), or in general case – symbol error rate ( $SER$ ). The $SER$ value uniformly accounts many other impact factors of particular physical and technological aspects, as well as the current state of the channel, distance, attenuation, interference etc. The $SER$ factor requirements can be obtained and proved empirically; again, the current $SER$ can also be dynamically observed and controlled.

Let $STR$ be the symbol transmission rate measured in baud units. We note, that $STR$ is more or less beyond the actual transportation speed of the payload data. That is because of the typical overhead provided on different

OSI layers of a telecom system. If many constructive or technological properties of a telecommunication system are predefined, the transmission rate $STR$ can fare the solely mechanism to keep the $SER$ factor in specified range. The $SER$ factor tends to increase while accelerating the symbol transmission rate $STR$ as shown in fig. 4; here, the typical family of four curves presents the symbol error rate $SER$ as functions of $STR$ argument for different channel lengths $L1 < L2 < L3 < L4$. Increasing the length of the channel results in worsening the signal-to-noise ratio and, therefore, increasing the symbol error rate $SER$ while fixed symbol transmission rate $STR$. One can see, that having limitation of symbol error rate bound of $10^{-6}$ value, we can reach maximal symbol transmission rate $R_1$ for the channel length $L3$, next $R_2$ for $L2$ and finally $R_4$ for $L1$. In turn, if taken the fixed symbol transmission rate $R_5$, we observe the $SER$ jumping from approximately $10^{-5}$ up to $10^{-1}$ value.

The classic approach to telecommunication system supposes that symbol error $SER$ rate is rather small value, e.g. $SER \ll 1$. Because of that, mistaken identity of receiving symbols could be neglected. Let $p\left(x_k / x_n\right) \approx SER$ be conditional probability that discrete symbol value $x_n$ identified at the receiving party of a telecommunication system $S$ as $x_k$ value. Take the system $S$ with QPSK phase modulation having the constellation diagram with four states. For this type of the system we construct the matrix $P = p(n,k),\quad p(n,k) = p\left(x_k / x_n\right)$ of conditional probabilities. In classical system $S$ design, we observe that $p\left(x_k / x_n\right) \cong 1,\quad if\;\; n = k$; $p\left(x_k / x_n\right) \cong 0,\quad if\;\; n \neq k$. So, in this case we have deterministic model of transmitted symbol recognition, and $P$ is diagonal matrix:

$$P = p(n,k) = I := 1, n = k; 0, n \neq k\,. \qquad (2)$$

Another to say, from the receiver's point of view, the identified symbol $x_k$ may take only one of the four states in QPSK modulated channel. According to this concept, the identified symbol values do not depend on the receiving party of a telecommunication system, or another to say, the sender on numerical symbols SNS and receiver of symbols RS in fig.1 do not show any entanglement. The deterministic model of telecommunication system hinders acceleration of data transmission because of severe requirements towards probability of mistakes. Therefore, a restricted area of symbol transmission rate in fig.4 can be utilized. To some extent, the $STR$ requirements can be slacken off due to the special type of coding with overhead, i.e. coding and modulation schemes with unequal error protection (UEP), low-density parity-check (LDPC) codes, [7-8]. However, a large area of communication system potentials remains unused in classical deterministic approach, as shown in fig.4.

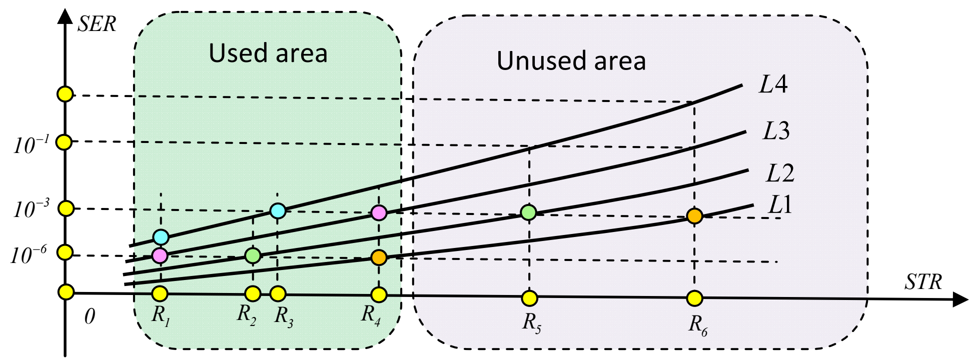


**Fig. 4 – The family of symbol error rate (SER) curves as functions of symbol transfer rate argument (STR)**

### 3. The quantum state uncertainty of a telecommunication system

Suppose a telecommunication system $S$ is accelerated in symbol transmission rate in such a way, that unused area in fig.4 takes place, and therefore, probability of mistaken symbol identity can't be neglected: $p\left(x_k / x_n\right) \neq 0,\quad if\;\; n \neq k$. Let symbol $x_k$ has 4 possible states either at sending or receiving parties of system $S$ (QPSK coding applied). Imagine that 400 symbols have been generated at the sending party, wherein each of four symbol states occurs 100 times. Due to mistaken symbol identity at the receiving party, the following resulting table of symbol identification may take place, fig.5. Each symbol value is correctly received 50 times among 100 (that means $p\left(x_n / x_n\right) \approx 0.5$). The probability of nearest symbol state misunderstood is $p\left(x_{n\pm1} / x_n\right) \approx 0.2$. The opposite symbol states in constellation diagram have the minimal mistakes of identity: $p\left(x_{n\pm2} / x_n\right) \approx 0.1$. In our particular case, both matrices in fig.5 are symmetric towards the main diagonal; so the sum of the elements in a table row $m$ equals the sum of the elements in the column $m$ for any $m = 1,2,3,4$ :

$$\sum_{k=1}^{4} p(m,k) = \sum_{n=1}^{4} p(n,m) \equiv 1 . \tag{3}$$

In general case, these matrices are not symmetric towards main diagonal, though the following relation is fair:

$$\sum_{k=1}^{4} p(n,k) = 1, \quad n = 1,2,3,4 . \tag{4}$$

| Identity cases | Received symbols *(k)* | | | |
|---|---|---|---|---|
| Sent symbols *(n)* | 1 | 2 | 3 | 4 |
| 1 | 50 | 20 | 10 | 20 |
| 2 | 20 | 50 | 20 | 10 |
| 3 | 10 | 20 | 50 | 20 |
| 4 | 20 | 10 | 20 | 50 |

⇔

| $P = p(n,k)$ | Received symbols *(k)* | | | |
|---|---|---|---|---|
| Sent symbols *(n)* | 1 | 2 | 3 | 4 |
| 1 | 0.50 | 0.20 | 0.10 | 0.20 |
| 2 | 0.20 | 0.50 | 0.20 | 0.10 |
| 3 | 0.10 | 0.20 | 0.50 | 0.20 |
| 4 | 0.20 | 0.10 | 0.20 | 0.50 |

**Fig. 5 – Matrices of symbol mistaken identity**

Considering the property (4), matrix $P = p(n,k)$ in fig.5 can be mutually mapped onto the matrix $P_{\Sigma}$ shown in fig.6.

| $P = p(n,k)$ | Received symbols *(k)* | | | |
|---|---|---|---|---|
| Sent symbols *(n)* | 1 | 2 | 3 | 4 |
| 1 | 0.50 | 0.20 | 0.10 | 0.20 |
| 2 | 0.20 | 0.50 | 0.20 | 0.10 |
| 3 | 0.10 | 0.20 | 0.50 | 0.20 |
| 4 | 0.20 | 0.10 | 0.20 | 0.50 |

⇔

| $P_{\Sigma} = p_{\Sigma}(n,k)$ | Received symbols *(k)* | | | |
|---|---|---|---|---|
| Sent symbols *(n)* | 1 | 2 | 3 | 4 |
| 1 | 1 | 0.20 | 0.10 | 0.20 |
| 2 | 0.20 | 1 | 0.20 | 0.10 |
| 3 | 0.10 | 0.20 | 1 | 0.20 |
| 4 | 0.20 | 0.10 | 0.20 | 1 |

**Fig. 6 – Mapping the matrix of symbol mistaken identity**

Let telecommunication system $S$ operates symbols $x_n$ with $N \geq 2$ discrete states. The function

$$p\left( {x_k}/{x_n} \right) = \frac{1}{N} \cdot p_{\Sigma}(n,k), n \neq k \tag{5}$$

determines the probability of symbol $x_n$ mistaken identity. The probability of fair symbol identification is

$$p\left( {x_n}/{x_n} \right) = p_{\Sigma}(n,n) - \sum_{k=1}^{N} \rho^2 \left( {x_k}/{x_n} \right), \quad n \neq k . \tag{6}$$

Relations (5) and (6) reflect the quantum state uncertainty occurred in a telecommunication system $S$ . The following square root function $\psi$ can be defined based on the relation (5) as an abstract magnitude of the probability distribution

$$\psi\left( {x_k}/{x_n} \right) = p^{0.5}\left( {x_k}/{x_n} \right) = \sqrt{\frac{1}{N} \cdot p_{\Sigma}(n,k)} . \tag{7}$$

The function $\psi\left( {x_k}/{x_n} \right)$ is a square matrix which satisfies the following relations:

$$\begin{cases} \psi\left( {x_k}/{x_n} \right) \cdot \psi^*\left( {x_k}/{x_n} \right) = p\left( {x_k}/{x_n} \right) \\ Tr\left( p\left( {x_k}/{x_n} \right) \right) = \sum_{n=1}^{N} p\left( {x_n}/{x_n} \right) = 1 \end{cases} . \tag{8}$$

If $p\left( {x_k}/{x_n} \right)$ is symmetric matrix (symmetric quantum entanglement) it has a geometric interpretation as a linear vector set; the diagonal elements of the matrix are squared lengths of vectors; the non-diagonal elements of the matrix are scalar products of distinct pairs of vectors. To adequately interpret measurement results, the two following conditions are to be met:

a) Series of symbols are to be generated at the sending party of a telecommunication system instead of single ones; based on these series each symbol transaction is shaped as N-dimensional outcome vector $\tilde{x}$ .

b) The receiving party of system $S$ must be aware of probability distribution matrix $p\left( x_k / x_n \right)$.

In case matrix has its inversion, the reconstructed income vector $x$ can be estimated as

$$x \approx \tilde{x} \cdot p^{-1}\left( x_k / x_n \right). \qquad (9)$$

Here, vector $x$ is expected to be near to deterministically shaped, that is to have solely one none-zero value element, while all the others have close to zero values. For example, if N=4, $x = \left[ 0.1; 0.8; 0.05; 0.05 \right]$, then the following decision can be made: $x = \left[ 0.1; 0.8; 0.05; 0.05 \right] \to \left[ 0; 1; 0; 0 \right] \to x_2$ (i.e. symbol $x$ value is identified as 2).

## 4. Conclusion

In contrast to conventional deterministic case, the state of a quantum system $S$ is not described by the outcome of a single case measurement, as its result includes some uncertainty depending on all the components of system $S$ (sender of numerical symbols, digital channel and symbol receiver). The quantum state uncertainty occurs when increasing the symbol transmission rate and/or signal-to-noise ratio decrement. To enable the efficient acceleration and productivity increase of a telecommunication system, the quantum state uncertainty statistics must be empirically predetermined.

## References

1. Stallings W. “Data and computer communications”, 10-th edition. Prentice Hall, New Jersey, 2013. – 912 pp.

2. Couch L.W. “Digital and analog communication systems”, 8-th edition, 2013. − 789 pp. Available at http://www.slideshare.net/ovalence/digital-analog-communication-systems-8th-edition.

3. "An Introduction to LTE". 3GPP LTE Encyclopedia, 2010. – Available at https://sites.google.com/site/lteencyclopedia/home.

4. Jean-Paul M.G. “Multi-Carrier Transmission over Mobile Radio Channels”. – Available at http://www.wirelesscommunication.nl/reference/ppt/ofdm_mimo4mm_edition.pdf.

5. “Shannon’s equation. Amount of information a channel can carry”. – Available at https://www.st-andrews.ac.uk/~www_pa/Scots_Guide/iandm/part8/page1.html.

6. Patel M.R. “New Channel Coding Technique to Achieve The Ultimate Shannon Limit”, National Conference on Recent Trends in Engineering & Technology, 2011. − Available at http://www.bvmengineering.ac.in/misc/docs/published-20papers/etel/etel/401031.pdf .

7. Deetzen N, Sandberg S. “On the UEP capabilities of several LDPC construction algorithms,” IEEE Trans. Commun., vol. 58, no. 11, Nov. 2010, pp. 3041–3046,.

8. Neto H., Henkel W., Rocha V. “Multi-edge type unequal error protecting low-density parity-check codes,” Proc. IEEE Information Theory Workshop (ITW’11), Paraty, Brazil, Oct. 2011, pp. 335–339.

Рецензія/Peer review : 29.9.2016 р. Надрукована/Printed : 11.11.2016 р.
Стаття рецензована редакційною колегією