ПРЕЦИЗІЙНІ ВИМІРЮВАННЯ ТА НОВІТНІ ТЕХНОЛОГІЇ

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THE MAXFLOW PROBLEM ANALYSIS ON FREE-ORIENTED NETWORK GRAPH

Abstract – *A short review is given on the key issues of network Maxflow problem. It is proved, that known approaches to Maxflow study are not always relevant to modern telecoms. Unlike traditional Maxflow task statement, whereby the maximum permissible unidirectional product flow is calculated between emitting and receiving nodes, the bi-directional multi-source and multi-destination data exchange is inherent to telecom system. Again, known algorithms for Maxflow task solution imply directed links between nodes, having fixed capacities in the onward and/or backward directions. Currently, the flexible adaptation of onward/backward channel throughput can be achieved with the use of modern reconfigurable add/drop multiplexors. In this paper, a novel framework of a free-oriented multi-pole open graph (FOG) substantiated for transport system analysis, where the free-oriented linkage is provisioned to simulate the dynamic configuration ability of advanced optical channels. Within this concept, an enhanced formalism and related algorithm determined for maximal flow evaluation in particular case of three-pole free-oriented open graph in terms of the overall circulation flow density between the open poles. The Maxflow formalism, in general case, operates with two types of bidirectional product flow in an open transporting network: unilateral flows that circulate between the network outside and given network via single port; bilateral flows traversing given network via multiple pairs of ports. This work presents the simple case of bilateral product flows. These flows are limited by the border requirements: the total flow in any open link must not succeed its overall capacity with no care about the occurred balance between counter partial flows. The introduced FOG concept extends the scope of bidirectional data transfer scheduling among the multiple ports of a telecom system, as well as the Maxflow algorithm simplifies the Maxflow task statement and provides more comprehensive solutions of the task. Next researches in this direction intend development algorithms of network flow optimization based on given approach.*

Keywords: *Maxflow problem, free-oriented graph, multi pole telecom network, dynamic channel configuration.*

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АНАЛІЗ ПРОБЛЕМИ МАКСИМАЛЬНОГО ПОТОКУ НА ВІЛЬНО-ОРІЄНТОВАНОМУ ГРАФІ МЕРЕЖІ

Анотація – *Надано стислий огляд з ключових питань проблеми максимального потоку. Доведено, що відомі підходи до дослідження максимального потоку не завжди актуальні для сучасних телекомунікацій. Обґрунтовано нову структуру вільно-орієнтованого багатополюсного відкритого графа (FOG) для аналізу транспортної системи, який моделює можливість динамічної конфігурації сучасних оптичних каналів. Представлена концепція FOG розширює можливості для планування двонаправленої передачі даних між декількома портами телекомунікаційної системи. В рамках цієї концепції визначено алгоритм формальної оцінки максимального потоку у конкретному випадку триполюсного вільно-орієнтованого відкритого графа в термінах загальної потужності потоків циркулюючих між відкритими полюсами. Цей формалізм спрощує подання задачі про максимальний потік і забезпечує більш ефективне рішення задачі.*

Наступні дослідження в цьому напрямку передбачають розробку алгоритмів оптимізації мережевого потоку на основі запропонованого підходу.

Ключові слова: *проблема максимального потоку, вільно-орієнтований граф, багатополюсна телекомунікаційна мережа, динамічна конфігурація каналу.*

Introduction

The network models are possibly the most important structures in optimization theory, and general network flow problem (NFP) occupies particular position in theoretical/applied researches where a common scenario of network flow problem arises behind industrial logistics concerns ([1], [2]). This scenario typically implies that some manufactured products must be transferred over logistic network from one or more source terminal nodes to several destination ports being addressed to wholesale consumers. In a simple case, a homogeneous product supposed along with "single source-single target" transporting network model. As a rule, communication links in logistic network graph considered to be one way directed channels having constant weights (denoted "arks"); therefore, related graph is called "directed weighted graph" (DWG). The common objective of logistic task solution is minimizing the overall cost of products supply to meet potential consumer demands (the so called "Min-cost-flow" task, or MCF); herewith, related network transportation model may include capacity restrictions on logistic hubs (network graph vertices) and transportation links (network graph arcs),[3], [4].

There are various approaches to MCF problem. Under certain background (e.g. hypothesis of unlimited manufactures' productivity, along with unbounded node's capacities and consumer's product needs), the general task of minimizing the overall product transfer cost can be reduced to the so called "Maxflow problem" (MFP). Typical examples of "product flow" are consumer goods delivered via logistic infrastructure; gas, oil or water pumped through a pipe system; information data transmission over telecom or computer networks, etc. Though the network transportation model is cast in terms of material streams from source to destination ports, various lateral

applications emerged. Primarily known observations of Maxflow problem do not imply that intermediate network nodes may either accumulate or generate product flow; presumably, they solely transmit the product. It means that the sum of all the input streams must be always equal to the sum of all the output streams for any intermediate network node. The conventional MFP statement involves finding a feasible data stream through a closed two-pole graph with single-source and single-target poles (SS/ST).

The actuality of Maxflow task triggered by logistics in last century, now has taken its new impulse in modern telecommunication sphere. However, common view on Maxflow task not always seems relevant to present IT realm because of particular telecom channel properties, which significantly contradict the legacy transportation model of material product provisioning. The principal aspect of information Maxflow study is bidirectional character of data streams in typical telecommunication channels; from these premises, each open network terminal should be treated as joint source/destination entity. This view is not character to homogeneous product transition in commonly known logistics model. The second distinct aspect of information product supply is dynamic flexibility of modern telecommunication channels with respect to onward/backward conductivity adaptation; it means that the overall channel capacity becomes a fixed property of network graph edges, whereas particular onward/backward channel resource scheduling is optionally possible. This phenomenon requires determination a new type of network graph with free-oriented edges, which simulates dynamic channel scheduling in modern telecommunication systems. The third critical point of data flow analysis is multi-pole scheme of network-to-network interaction, which mistunes known “one source-one target” network graph models. The publications survey indicates that data flow optimization in telecom systems is challenging yet, and more researches needed. *The objective of this work is the enhancement of Maxflow task solution on a free-oriented multi-pole network graph.*

Free-oriented channel as linkage model on a graph

As it was mentioned above, the commonly exhibited models of matter product transportation often relay on the formalism of directed weighted graph (aka digraph, or DWG), where all the graph edges are unidirectional links. Sometimes, however, a digraph is understood as that, where at least one edge is either unidirectional or bidirectional; instead, a graph with all unidirectional edges is called “oriented graph”. Further we will use terms “ark” (or “simplex”) for unidirectional edge in contrast to bidirectional edge noted “bi-ark” (or “complex”). Therefore, a graph with no one directed edge is undirected. However, the term “undirected weighted graph” (UWG) is not uniformly perceived by the specialists. Customary, the undirected weighted graph (UWG) is given as everywhere symmetrically weighted bidirectional graph, or skew-symmetric graph, [5]. This understanding of undirected edge and related graph correlates with full symmetric duplex channels in generic telecom systems, but it is beyond the modern decisions in fiber optic technologies, [6].

Actually, the term of “undirected edge” on a graph is not fully determined in literature for digital communication channel, therefore, more detailed explanation of this entity needed. In fact, advanced telecom technologies support bidirectional data transmission over a single optic fiber link [7]; the data exchange over a single fiber strand is achieved by separating the transmission wavelength of two devices, Fig. 1.

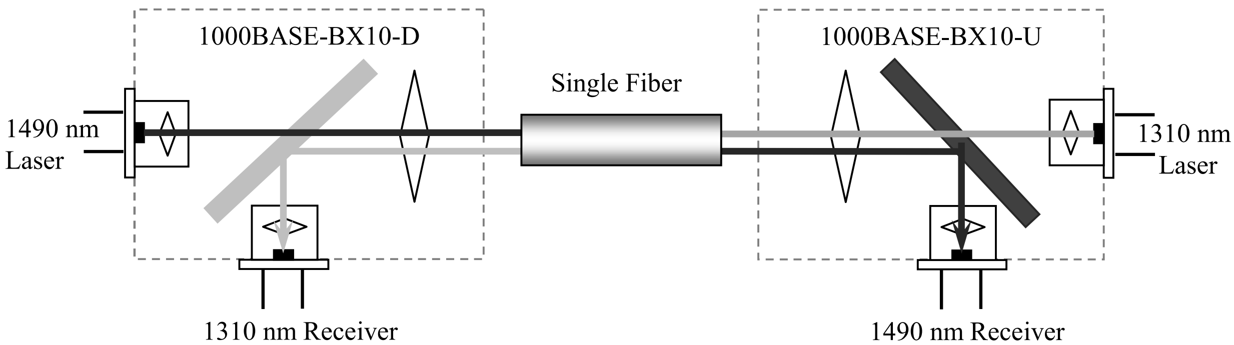


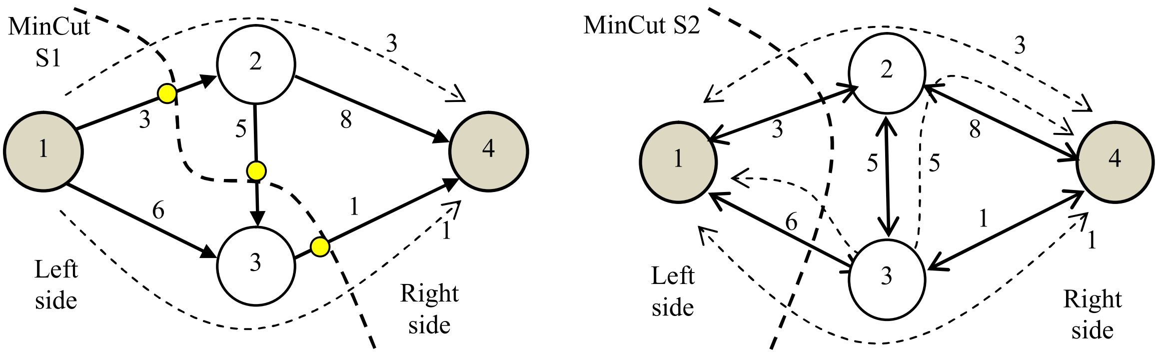
Fig. 1.The principle of bidirectional data transmission over a single optic fiber link

For instance, the 1000BASE-BX10-D network interface transmits at 1490-nm channel and receives a 1310-nm signal, whereas 1000BASE-BX10-U transmits at 1310-nm wavelength and receives a 1490-nm signal. The wavelength-division multiplexing (WDM) splitter is integrated into the Small Form-factor Pluggable module (SFP) to split the 1310-nm and 1490-nm light paths. On the other hand, behind the bidirectional conductivity observed in optic fiber, the flexible adaptation of onward/backward channel throughput can be achieved with the use of modern reconfigurable add/drop multiplexors performing on the base of coherent multi-carrier pumping, [8]. Consider aforesaid, the disputable term of “undirected weighted graph” is overlooked for further interpretation in order to avoid terminology confusions among the specialists. Instead, a new term is introduced for optical bidirectional trunk, namely, “*free-oriented channel*” (FOC). The FOC is defined as that having fixed total capacity P along with dynamic scheduling between the onward channel throughput P+ and backward throughput P−:

$$0 \le (|P^+| / |P^-|) \le 1, \qquad |P^+| + |P^-| = P = const. \tag{1}$$

Similarly, a free-oriented linkage (FOL) is introduced in this paper to simulate a FOC on a graph. Hence, any network graph with entire free-oriented edges treated as "*free-oriented graph*" (FOG). The FOG transporting model seems more adequate to exhibit advanced network technologies, and particular, the modern concepts of network traffic engineering ([9]) and software defined networking ([10]).

The free-oriented linkage (FOL) in network graph model supports dynamic adaptation to flow requirements and improves the overall network resource utilization. This fact illustrates Fig. 2 where two variants of network linkage depicted. Fig. 2-a shows a common directed graph with the source vertex (number 1) and target vertex (number 4). The minimal cut S1 between the source and target vertices easy calculated as 3+1=4 (note, that link "2−>3" of value 5 not included into the S1 collection because the ark "2−>3" traverses the curve S1 in right-to-left direction from target to source vertex, in contrast to arks "1−>2" and "3−>4" traversing the cut S1 from the left to the right side of curve S1, e.g. from source to target vertex).



a) Directed graph:
Maxflow = 3+1 = **4**

b) Free-oriented graph:
Maxflow = 3+1+5= **9**

Fig. 2.Network maximal flow in directed graph ("a") and free-oriented graph ("b")

Figure 2-b shows similar to Fig2-a network graph with the same link capacities, however, the links in Fig.2-b graph are free-oriented, and therefore, do not have fixed directions. To distinguish depicted free-oriented network graph (FOG) from commonly spoken undirected graph (UDG) it is proposed to draw vertex edges of FOG in Fig.2-b by two-arrow links. Similar to previous case in Fig.2-a, it is quite easy to find out the maximal possible flow between the vertices 1 and 4 in Fig.2-b (regardless of the flows distribution in the onward and backward directions); this flow includes three partitions yield 3+1+5=9. This result gained in Maxflow task solution is better than one in previous case, Fig.2-a.

Free-oriented open graph as an enhanced network model

The adoption of aforesaid hypothesis of free-oriented linkage in network graph modeling eliminates necessity in distinct "sources" and "targets" of product flow among the network nodes or related graph vertices (like S- and T-type nodes in common Maxflow problem observation). Again, particular outer vertices still needed as network border ports to generate/consume network flows, in contrast to inner network nodes, which presumably do not emit or absorb product flows but solely switch and put through the product flows in transparent mode.

On these premises, a network graph model is treated as an autonomous closed framework not aware about its outer neighborhood. Probably, this point of view needs conceptual rethinking, as typical transporting networks are open systems or subsystems constituting entire public infrastructure. With this concern, a concept of an *open pole graph* (OPG) is introduced in this work as an open network model. The flowchart in Fig. 3 depicts an open two-pole graph built behind the four-vertex graph prototype in Fig.2-b; however this graph differs in that no particular conservative/emanative vertices exist on the plain.

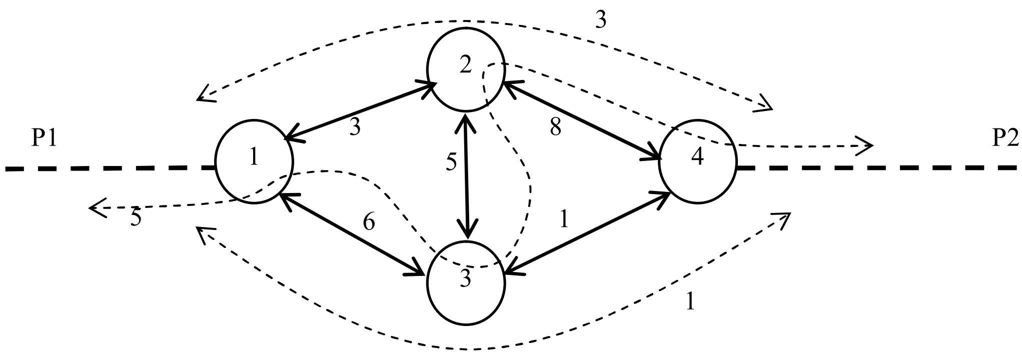


Fig. 3. Free-oriented two-pole open network graph

In this instance, new topological entities added on the open pole graph, i.e. two open network edges with correspondent throughput capacities P1 and P2. The number of open edges in an arbitrary multi-pole free-oriented graph must be equal to the number of open poles. The open network edges on an arbitrary OPG pay a principal role in Maxflow task formalization and related Maxflow algorithms. In general case, the OPG model operates with two types of bidirectional product flow in an open transporting network: unilateral flows that circulate between the network outside and given network via single port; bilateral flows traversing given network via multiple pairs of ports. In this work the simple case of bilateral product flows is considered. For the two-pole open graph on fig.3, the following constraints are to be satisfied in order to prove the previously gained Maxflow result of 9 flow units:

$$(P1, P2) \geq \langle (1+3+5) = 9 \rangle . \quad (2)$$

This question needs more observation in general case of multi-pole network graph study. Further on, a three-pole open network graph studied for Maxflow resolution with open edge constraints.

Consider an open three-pole network graph with unknown or hidden inner structure, which is depicted in general on Fig.4.

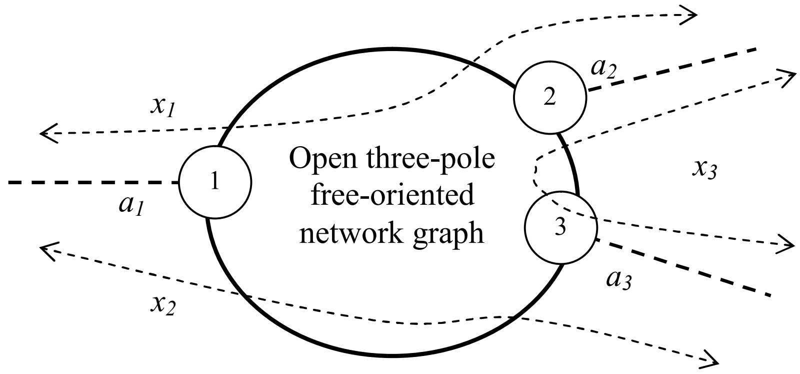


Fig. 4. Free-oriented three-pole open network graph

Suppose open edges have known capacities a_1, a_2 and a_3. Denote sought flows as non-negative real numbers x_1, x_2 and x_3. The following system of inequalities must be satisfied:

$$\begin{cases} x_1 + x_2 \leq a_1 \\ x_1 + x_3 \leq a_2 \\ x_2 + x_3 \leq a_3 \end{cases} . \quad (3)$$

The total network flow we define as the following sum:

$$F_\Sigma = x_1 + x_2 + x_3 . \quad (4)$$

Summing all three inequalities in (3) we yield the following:

$$f_\Sigma = x_1 + x_2 + x_3 \leq (a_1 + a_2 + a_3)/2 . \quad (5)$$

The exact equation $f_\Sigma = (a_1 + a_2 + a_3)/2$ requires that strong equation system occurs in (3):

$$\begin{cases} x_1 + x_2 = a_1 \\ x_1 + x_3 = a_2 \\ x_2 + x_3 = a_3 \end{cases} . \quad (6)$$

Now, we will examine conditions of system (6) accuracy. Reorganize (6) in form

$$\begin{cases} x_1 \cdot 0 + x_2 \cdot 1 + x_3 \cdot 1 = a_3 \\ x_1 \cdot 1 + x_2 \cdot 0 + x_3 \cdot 1 = a_2 \\ x_1 \cdot 1 + x_2 \cdot 1 + x_3 \cdot 0 = a1 \end{cases} . \quad (7)$$

Denote $a = [a_3, a_2, a_1]$; $x = [x_1, x_2, x_3]$; $G = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Present (7) in matrix form

$$x \cdot G = a . \quad (8)$$

The formal solution for linear system equation (8) is

$$x = a \cdot G^{-1} = [a_3, a_2, a_1] \times \begin{bmatrix} -0.5 & 0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 \end{bmatrix} . \quad (9)$$

So, we retrieve the following intermediate result:

$$\begin{cases} x_1 = (a_1 + a_2 - a_3)/2 \\ x_2 = (a_1 - a_2 + a_3)/2 \\ x_3 = (-a_1 + a_2 + a_3)/2 \end{cases}. \tag{10}$$

Now, examine relations (10) on their consistency towards the flows x_1, x_2, x_3. According our initial definitions, all the flow values x_1, x_2, x_3 must be non-negative numbers. Therefore, the following inequalities system required:

$$\begin{cases} a_1 + a_2 \ge a_3 \\ a_1 + a_3 \ge a_2 \\ a_2 + a_3 \ge a_1 \end{cases}. \tag{11}$$

The system (11) obviously presents the well know "triangle rule": any edge capacity a_k among three ones (*k=1, 2, 3*) must be not more that sum of two other ones. Thus, if system (11) valid, then outer maximal network flow limitation defined by relation

$$\max(f_\Sigma = x_1 + x_2 + x_3) = (a_1 + a_2 + a_3)/2. \tag{12}$$

The exact equality in (12) occurs when inner network topology and metrics allow the flows $x = [x_1, x_2, x_3]$ to run between the correspondent pairs of network poles $x_1(1,2)$, $x_2(1,3)$ and $x_3(2,3)$. It is quite evident, that among any three arbitrary non-negative real numbers a_1, a_2 and a_3 only one of them can be more than the sum of two other ones. Consider a case that some of three open edge capacities a_1, a_2 and a_3 violates condition (11), for instance, $a_1 > a_2 + a_3$. For given case $a_1 > a_2 + a_3$ the obvious maximal flows are: $x_1 = a_2$; $x_2 = a_3$; $x_3 = 0$; $\max(f_\Sigma) = \max(x_1 + x_2 + x_3) = a_2 + a_3$.

Conclusion

In this work, known approaches discussed to network flow modeling with particular focus on maximal flow problem statement. Concluded that Maxflow problem is not exhaustively studied in general case, as well as common graph models do not meet the modern networking technologies. To overcome this issue, an original Maxflow vision introduced for telecom data flow analysis based on the concepts of free-oriented channel conductivity and multi-pole open graph model. Formal constraints retrieved for maximal network flow estimation on arbitrary open three-pole free-oriented graph. The next step research in this direction assumes development applicative algorithms of network flow optimization behind the free-oriented multi-pole graph as network model.

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