ВІСНИК ПРИАЗОВСЬКОГО ДЕРЖАВНОГО ТЕХНІЧНОГО УНІВЕРСИТЕТУ
2009 р. **Вип. №19**

УДК 621.311

Khalil T.M.[1], Gorpinich A.V.[2]

THE EFFECT OF MUTUAL COUPLING, LOAD UNBALANCE AND HARMONICS ON CAPACITOR PLACEMENT IN DISTRIBUTION NETWORKS

Рассмотрено влияние высших гармоник, несимметрии и взаимоиндукции на выбор мощности и мест установки батарей конденсаторов в распределительных электрических сетях. Предложен бинарный метод оптимизации, основанный на моделировании скоплений частиц (по аналогии с биологическими системами). При составлении целевой функции учитывались стоимость батарей конденсаторов и потерь мощности, типоразмеры батарей конденсаторов, ограничения по уровню напряжения и его искажений в узлах электрической сети.

Introduction

The problem of capacitor placement for loss reduction in electric distribution systems has been extensively researched over the past decades. The objective of capacitor placement is to achieve the loss reduction weighted against capacitors costs keeping the operational and power quality constraints within required limits. In reality, distribution networks are unbalanced systems due to mutual coupling between phase conductors and unbalanced loading on different phases. Moreover, a considerable amount of harmonic distortion exists in distribution system.

Most of the capacitor placement techniques assume the distribution system to be balanced and the supply as sinusoidal. Limited publications have taken into account system unbalance and the presence of harmonics [1-3] when solving the capacitor placement problems. Consideration of three-phase system and harmonics complicate the capacitor placement problem compared with the balanced sinusoidal case. In this paper, the work reported in [3] has been extended to include the effect of mutual coupling and load unbalance on capacitor placement in distribution system.

Problem formulation

The objective function is to minimize the total annual costs due to capacitor placement and power losses [3] with constraints that include limits on voltage, total harmonic distortion and size of installed capacitors (see equations (1)-(5)).

$$P_{loss} = \sum_{h=1}^{H} \left(\sum_{i=0}^{m-1} P^{h}_{loss(i,i+1)} \right); \qquad (1)$$

$$Q^{c}_{\max} = L Q^{c}_{0}; \qquad (2)$$

$$F = K^{p} P_{loss} + \sum_{j=1}^{J} K^{c}_{j} Q^{c}_{j}; \qquad (3)$$

$$THD_{i} \le THD_{\max}; \qquad (4)$$

$$V_{\min} \le |V_{i}| \le V_{\max}, \qquad (5)$$

where P_{loss} – total power losses; $Q^{c}_{\max}$ – maximum allowable capacitor size to be placed; L – integer; Q^{c}_{0} – smallest capacitor size; F – total annual cost function; K^{p} – equivalent annual cost per unit of power losses; K^{c}_{j} – capacitor annual cost/kvar; Q^{c}_{j} – shunt capacitor size placed at bus *j*; $V_{\min}$, $V_{\max}$ – minimum and maximum permissible rms voltage; $THD_{\max}$ – maximum permissible total harmonic distortion; *J*, *m* – shunt capacitor buses and number of buses.

Considering investment costs, there are a finite number of standard capacitor sizes that are integer multiples of smallest size. The cost per kilovar varies from one size to another. Generally, large sizes are cheaper than smaller ones (see table 1).

---

[1]Pryazovskyi State Technical University, Master of Science, Eng., Ph.D. student
[2]Pryazovskyi State Technical University, Candidate of Science, Eng.

Table 1 – Yearly cost of fixed capacitor size

ID	1	2	3	4	5	6	7	8	9
Capacitor size (kvar)	150	300	450	600	750	900	1050	1200	1350
Capacitor cost ($/kvar)	0.5	0.35	0.253	0.22	0.276	0.183	0.228	0.17	0.207
ID	10	11	12	13	14	15	16	17	18
Capacitor size (kvar)	1500	1650	1800	1950	2100	2250	2400	2550	2700
Capacitor cost ($/kvar)	0.201	0.193	0.187	0.211	0.176	0.197	0.17	0.189	0.187
ID	19	20	21	22	23	24	25	26	27
Capacitor size (kvar)	2850	3000	3150	3300	3450	3600	3750	3900	4050
Capacitor cost ($/kvar)	0.183	0.18	0.195	0.174	0.188	0.17	0.183	0.182	0.179

System model at fundamental and harmonic frequencies

The distribution system has been modeled considering mutual coupling effect between phases. A direct approach for unbalanced three-phase distribution load flow solutions which presented in [4] has been used. In this approach, the special topological characteristics of distribution networks have been fully utilized to make the direct solution possible. Two developed matrices (the bus-injection to branch-current matrix and the branch-current to bus-voltage matrix) and a simple matrix multiplication are used to obtain load flow solutions. Due to the distinctive solution techniques of the proposed method, the time-consuming decomposition and forward/backward substitution of the Jacobian matrix or admittance matrix required in the traditional load flow methods are no necessary. For the harmonic flow study, linear load is represented by parallel combination of resistance and inductance to account for the respective active and reactive power at fundamental frequency.

Particle swarm optimization

The Particle Swarm Optimization (PSO) method was first introduced by Kenney and Eberhart [5] in 1995. It was developed through simulation of a simplified social system, and has been found to be robust in solving continuous nonlinear optimization problems [5-8]. One of reasons that PSO is attractive is that there are very few parameters [9]. There are different versions of PSO that aim to widen its applicability. Kennedy and Eberhart [10] proposed the first discrete version.

In PSO algorithm, each member is called "particle", which represents a candidate solution to the problem at hand, and each particle flies around in the multi-dimensional search space with a velocity, which is constantly updated by the particle's own experience and the experience of the particle's neighbors. The basic PSO technique is the real valued PSO, whereby each dimension can take on any real valued number. On the other hand, in binary PSO each dimension of the particle can only take on the discrete values of 0 or 1.

Basic particle swarm optimization

Particle Swarm Optimization is a stochastic optimization algorithm that simulates the social behaviors of bird flocking or fish schooling and the methods by which they find roosting places, foods sources or other suitable habitat. The PSO algorithm searches in parallel using a group of individuals.

In the basic PSO technique, suppose that the search space is d-dimensional [3].

1. Each member is called *particle*, and each particle (i-th particle) is represented by d-dimensional vector and described as $X_i = [x_{i1}, x_{i2} \ldots x_{id}]$.
2. The set of n particle in the swarm are called *population* and described as $pop = [X_1, X_2 \ldots X_n]$.
3. The best previous position for each particle (the position giving the best fitness value) is called *particle best* and described as $PB_i = [pb_{i1}, pb_{i2} \ldots pb_{id}]$.
4. The best position among all of the particle best position achieved so far is called *global best* and described as $GB=[gb_1, gb_2 \ldots gb_d]$.
5. The rate of position change for each particle is called *the particle velocity* and described as $V_i = [v_{i1}, v_{i2} \ldots v_{id}]$.

At iteration k the velocity for d-dimension of i-th particle is updated by:

$$v_{id}^{k+1} = wv_{id}^{k} + c_1 r_1 (pb_{id}^{k} - x_{id}^{k}) + c_2 r_2 (gb_{d}^{k} - x_{id}^{k}), \quad (6)$$

where $i = 1,2 \ldots n$; n – size of population; w – inertia weight; c_1 and c_2 – acceleration constants; r_1 and r_2 – two random values in range [0,1].

The i-th particle position is updated by

$$x_{id}^{k+1} = x_{id}^{k} + v_{id}^{k+1}. \quad (7)$$

Binary particle swarm optimization

In 1997, Kennedy and Eberhart [10] have adapted the PSO to search in binary spaces by applying a sigmoid transformation to the velocity component to squash the velocities into a range [0,1], and force the component values of the locations of particles to be 0's or 1's (see equation (8)). The equation for updating positions (equation (7)) is then replaced by equation (9).

$$sigmoid\,(v_{id}^{k+1}) = \frac{1}{1+\exp(-v_{id}^{k+1})}; \tag{8}$$

$$x_{id}^{k+1} = \begin{cases} 1, \text{if } rand < sigmoid\,(v_{id}^{k+1}) \\ 0 \quad \text{otherwise} \end{cases}. \tag{9}$$

For the capacitor placement problem, a binary PSO will be used as follows. To select the optimal capacitor size Q_j^c to be placed at bus *j* choose a combination of capacitor sizes (*R*-size) from table 1 as an example

$$Q_j^c = b_1 \cdot sz_1 + b_2 \cdot sz_2 + \ldots + b_r \cdot sz_r + \ldots + b_R \cdot sz_R, \tag{10}$$

where $j \in J$; *J* – set of candidate buses to capacitors placement; $b_r = \{0,1\}$; sz_r – capacitor size from table 1; *R* – number of chosen capacitor sizes; $Q_j^c \le Q_{\max}^c$; $Q_{\max}^c$ – maximum allowable capacitor size to be placed at any bus.

Thus, the candidate buses are *J*-buses, and the capacitor Q^c placed at candidate bus *j* consists of small capacitor sizes (*R*-size according to equation (10)).

The population of *n* particles at iteration *k* represented by:

$$pop^k = [X_1^k, X_2^k, \ldots, X_i^k, \ldots, X_n^k].$$

Each particle *i* represented in *J*-dimensional by:

$$X_i^k = [x_{i1}^k, x_{i2}^k, \ldots, x_{ij}^k, \ldots, x_{iJ}^k].$$

Each dimension *j* represented in *R*-dimensional by:

$$x_{ij}^k = [x_{ij1}^k, x_{ij2}^k, \ldots, x_{ijr}^k, \ldots, x_{ijR}^k].$$

Therefore, each particle *i* represented in (*J*, *R*) dimensions

$$X_i^k = \begin{bmatrix} x_{i11}^k & x_{i12}^k & \ldots & x_{i1r}^k & \ldots & x_{i1R}^k \\ x_{i21}^k & x_{i22}^k & \ldots & x_{i2r}^k & \ldots & x_{i2R}^k \\ . & . & \ldots & & \ldots & . \\ x_{ij1}^k & x_{ij2}^k & \ldots & x_{ijr}^k & \ldots & x_{ijR}^k \\ . & . & \ldots & & \ldots & . \\ x_{iJ1}^k & x_{iJ2}^k & \ldots & x_{iJr}^k & \ldots & x_{iJR}^k \end{bmatrix}.$$

For particle *i*, the capacitor size at bus *j* and iteration *k* represented by:

$$Q_{ij}^{c(k)} = x_{ij1}^k \cdot sz_1 + x_{ij2}^k \cdot sz_2 + \ldots + x_{ijr}^k \cdot sz_r + \ldots + x_{ijR}^k \cdot sz_R.$$

The dimension x^k_{ijr} indicates whether capacitor size sz_r to be placed at bus *j* and iteration *k* for particle *i* or not. In other words, x^k_{ijr} is a binary value such that $x_{ijr}^k = 1$ if capacitor size sz_r is placed at bus *j* at iteration *k* for particle *i*, and $x_{ijr}^k = 0$ if it is not placed.

The particle best, global best and the particle velocity are represented also in (*J*, *R*) dimensions. It should be noted that according to above mentioned method each bus *j* capacitor sizes will be same for all three phases.

Numerical example

A 9-bus simple feeder [1] as shown in figure 1 is selected for computer simulation to demonstrate the effect of mutual coupling, load unbalance and supply harmonics on the capacitor placement in distribution system. The loads at different buses for balanced loading conditions and branch data are listed in table 2 and table 3. Line impedances are calculated considering the effect of grounding as (3×3) matrix and the impedance values are shown in table 4.

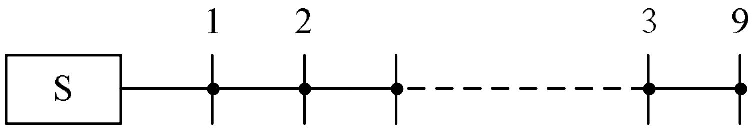

Fig. 1 – Nine-bus test feeder

Table 2 – Bus data

Bus no.	1	2	3	4	5	6	7	8	9
P (kW per phase)	1840	980	1790	1598	1610	780	1150	980	1640
Q (kvar per phase)	460	340	446	1840	600	110	60	130	200

Table 3 – Brunch data

From bus no.	0	1	2	3	4	5	6	7	8
To bus no.	1	2	3	4	5	6	7	8	9
Length (mile)	0.63	0.88	1.7	0.84	2.3	1.05	1.50	3.5	3.9

Table 4 – Impedance matrix including mutual coupling (Z (Ω/mile))

Z_{aa}	0.7433+j1.2092	Z_{ab}	0.1566+j0.4790
Z_{bb}	0.7526+j1.1758	Z_{ac}	0.1536+j0.3865
Z_{cc}	0.7472+j1.1959	Z_{bc}	0.1587+j0.4370

K^p was selected to be 168 \$/kW, and the voltage limits on the rms voltage were selected as $V_{\min} = 0.9$ p.u., and $V_{\max} = 1.1$ p.u. It was assumed that the substation voltage contains 3 % and 2 % of 5-th and 7-th harmonic, respectively. Commercially-available capacitor sizes with real costs/kvar were used in the analysis. It was decided that the largest capacitor size $Q^c_{\max}$ should not exceed the total reactive load, i.e., 4186 kvar. The yearly costs of capacitor sizes are shown in table 1.

Optimum shunt capacitor sizes have been evaluated for the following cases.

1. Harmonic frequencies, mutual coupling and load unbalance are ignored.
2. Harmonic frequencies are considered (maximum *THD* limit is equal to 5 %), mutual coupling and load unbalance are ignored.
3. Harmonic frequencies are ignored, mutual coupling is considered and load unbalance is ignored.
4. Harmonic frequencies are considered (maximum *THD* limit is equal to 5 %), mutual coupling is considered and load unbalance is ignored.
5. Harmonic frequencies and mutual coupling are ignored and load unbalance is considered (upper limit is equal to 5 %).
6. Harmonic frequencies are considered (maximum *THD* limit is equal to 5 %), mutual coupling is ignored and load unbalance is considered (upper limit is equal to 5 %).
7. Harmonic frequencies are ignored, mutual coupling and load unbalance are considered (upper limit is equal to 5 %).
8. Harmonic frequencies (maximum *THD* limit is equal to 5 %), mutual coupling and load unbalance are considered (upper limit is equal to 5 %).
9. Calculation of the rms voltage, *THD*, power losses and benefits for the test feeder with capacitor sizes and places obtained in case 1.

Capacitive kvar required for the above cases are summarized in table 7. In the reported results, a 5 % unbalance indicates that load at each load node of phase B is 5 % higher than the load of phase A and load at phase C is lower by same amount, thus keeping the total three-phase loads as in the balanced loading situation [1].

Table 5 – Summary for Different Cases (0 = No & 1 = Yes)

Case no.	1	2	3	4	5	6	7	8	9
Considering harmonics ($THD_{\max} = 5\%$)	0	1	0	1	0	1	0	1	1
Considering the mutual coupling	0	0	1	1	0	0	1	1	1
Considering unbalance (5%)	0	0	0	0	1	1	1	1	1

Table 6 – Results summary

Accounted parameters	Power losses (%)	Benefits (%)
Harmonics ($THD_{\max} = 5\%$)	Increased	Decreased
Mutual coupling without harmonics	Decreased	Increased
Mutual coupling with harmonics	Increased	Decreased
Unbalance (5%)	Increased	Decreased

Table 7 – Capacitive kvar required for different cases

Cases		1	2	3	4	5	6	7	8
	Q_1^c	0	450	0	450	450	150	0	0
Capacitor	Q_2^c	1200	450	450	450	1500	150	1350	0
bank	Q_3^c	900	750	1350	300	1350	1650	600	1350
placement	Q_4^c	2400	1950	2400	1500	2400	1050	2700	1800
at	Q_5^c	1200	1200	750	1350	0	1200	450	900
each	Q_6^c	0	1650	900	600	750	900	0	450
phase	Q_7^c	450	0	450	0	0	600	750	0
	Q_8^c	450	750	450	450	750	1350	900	450
	Q_9^c	600	0	600	1500	1200	300	450	1650
Total capacitor sizes (3-phase)		21600	21600	22050	19800	25200	22050	21600	19800

Table 8 – Results when ignoring harmonics (BCP – before capacitor placement & ACP – after capacitor placement)

ID	case 1		case 3		case 5		case 7	
	BCP	ACP	BCP	ACP	BCP	ACP	BCP	ACP
$V_{\min}$ (3-phase, p.u.)	0.8032	0.9011	0.69281	0.9001	0.79022	0.90014	0.7036	0.90016
$V_{\max}$ (3-phase, p.u.)	0.9794	0.9894	0.99412	0.9968	0.98053	0.99247	0.9927	0.99542
P_{loss} (3-phase, kW)	3636.8	2880.1	3591	2826.8	3651.4	2910.1	3555	2825.3
Total costs (3-phase, $/year)	610987	487880	603290	479499	613429	494060	597232	479301
Benefits (3-phase, $/year)		123107		123791		119369		117931
Benefits (%)		20.149		20.519		19.459		19.746

Table 9 – Results when considering harmonics (BCP – before capacitor placement & ACP – after capacitor placement)

ID	case 2		case 4		case 6		case 8		case 9
	BCP	ACP	BCP	ACP	BCP	ACP	BCP	ACP	ACP
$V_{\min}$ (3-phase, p.u.)	0.8035	0.9012	0.6935	0.9173	0.79046	0.9003	0.70435	0.9269	0.8883
$V_{\max}$ (3-phase, p.u.)	0.98	0.9899	0.9948	0.9973	0.98113	0.9921	0.99328	0.9959	0.996
$THD_{\max}$ (3-phase, %)	3.4821	4.9285	5.5297	4.9953	3.4855	4.9582	5.5726	4.9878	6.018
P_{loss} (3-phase, kW)	3639.3	2924.5	3609.1	2990.7	3653.9	2958.9	3571.5	3011.3	2922.1
Total costs (3-phase, $/year)	611407	496046	606320	506823	613849	501994	600006	510122	494936
Benefits (3-phase, $/year)		115361		99497		111855		89885	105070
Benefits (%)		18.868		16.41		18.222		14.981	17.512

From the simulation results showed in Tables 6 to 9 it follows.

1. When harmonic frequencies are considered (maximum *THD* limit is equal to 5%) the power losses increased and benefits decreased (cases 1, 3, 5 and 7).
2. When mutual coupling is considered and ignoring the harmonic frequencies, the power losses decreased and the benefits (%) increased (cases 1 and 5). When mutual coupling is considered and harmonic frequencies are ignored the power losses decreased and benefits (%) increased (cases 1 and 5). When mutual coupling is considered and harmonic frequencies are considered (maximum THD limit is equal to 5%) the power losses increased and benefits decreased (cases 2 and 6).
3. When load unbalance is considered the power losses increased and benefits decreased (cases 1, 2, 3 and 4).

From comparing simulation results for case 1 and case 9 may be concluded.

1. Power losses and benefits ($/year) in case 9 less than in case 1.
2. Minimum rms voltage in case 9 less than minimum rms voltage limit.
3. *THD* in case 9 more than maximum *THD* limit.

Conclusions

The necessity considering harmonics, mutual coupling and load unbalance in capacitor placement problem modeling was investigated in this paper. Test results indicated that ignoring supply harmonics, load unbalance and mutual coupling can cause power quality degradation (for example, in case 9 $THD_{\max} = 6\,\%$ and $V_{\min} = 0.89\,\text{p.u.}$). A binary particle swarm optimization was used for discrete optimization of capacitor placement. The objective function was to minimize the total annual costs and power losses due to capacitor placement with constraints including limits on voltages, total harmonic distortion and sizes of installed capacitors. Future work will involve more realistic representation of large-scale distribution system with time-varying harmonics.

References

1. *Ghose T.* Effects of unbalances and harmonics on optimal capacitor placement in distribution system */ T. Ghose, S.K. Goswami* // Electric Power Systems Research. – 2004. – № 3. – P. 167 – 173.
2. *Жежеленко И.В.* Высшие гармоники в системах электроснабжения промпредприятий. – 5-е изд., перераб. и доп. / *И.В. Жежеленко*. – М.: Энергоатомиздат, 2004. – 375 с.
3. *Khalil T.M.* A binary particle swarm optimization for optimal placement and sizing of capacitor banks in radial distribution feeders with distorted substation voltages */ T.M. Khalil, H.K.M. Youseef*, *M.M. Abdel Aziz* // Proc. AIML 06 International Conference. – Sharm El-Sheikh (Egypt), 2006 – P. 129 – 135.
4. *Teng J-H.* A direct approach for distribution system load flow solutions / *J-H. Teng* // IEEE Trans. on Power Delivery. – 2003. – № 3. – P. 882 – 887.
5. *Kennedy J.* Particle swarm optimization / *J. Kennedy, R. Eberhart* // Proc. IEEE Int. Conf. on Neural Networks. – Perth (Australia), 1995 – P. 1942 – 1948.
6. *Shi Y.* A modified particle swarm optimizer / *Y. Shi, R. Eberhart* // Proc. IEEE Int. Conf. Evol. Comput. – Anchorage (AK, USA), 1998. – P. 69 – 73.
7. *Shi Y*. Empirical study of particle swarm optimization / *Y. Shi, R. Eberhart* // Proc. IEEE Int. Conf. Evol. Comput. – Washington (DC, USA), 1999. – P. 1945 – 1950.
8. *Eberhart R.* Comparison between genetic algorithms and particle swarm optimization / *R. Eberhart, Y. Shi* // Proc. IEEE Int. Conf. Evol. Comput. – Anchorage (AK, USA), 1998. – P. 611 – 616.
9. *Shi Y.* Parameter selection in particle swarm optimization / *Y. Shi, R. Eb*erhart // Proc. 7th Ann. Conf. Evolutionary Program. – 1998. – P. 591 – 600.
10. *Kennedy J.* A discrete binary version of the particle swarm algorithm / *J. Kennedy, R. Eberhart* // Proc. of Conf. on Systems, Man, and Cybernetics (SMC 97). – 1997. – P. 4104 – 4109.

Рецензент: Ю.Л. Саенко
д-р техн. наук, проф., ПГТУ

Статья поступила 09.02.2009