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THE JUSTIFICATION OF A WAY FOR IMPROVING THE PROTECTION OF SPECIAL BUILDINGS FROM SHOCK EFFECT OF THE PROJECTILE

To increase the protection of engineering structures from the impact of projectiles, it is proposed to use an elastic reinforcement of the outer surface of the coating. It is shown that its use significantly reduces the dynamic deflection of the structure, and, on the contrary, its protective properties. The results are based on an analytical analysis of the constructed mathematical model of dynamics of an elastically reinforced upper part of the protective structure under the condition of a shock effect on it of the projectile. The results obtained simultaneously, under certain constraints, may be the basis for studying the dynamics of the specified structure from the action of a shock wave or a series of strikes of projectiles.

Key words: engineering structure, system of elastic reinforcement, bending oscillations.

The statement of the problem

For the protection of personnel, military equipment against the impact of explosive waves and the impact of projectile used various protective structures (dugouts, trenches, etc.). If the effect of an explosive wave on a protective structure extends, as a rule, along its entire surface, then the projectile - is of a point nature. Despite the fact that the duration of action on the protective structure of these factors is negligible, in many cases it leads to its destruction [1, 2]. Therefore, there is a problem of increasing the protective capacity of these structures. It can be solved by various means, for example, to increase the thickness of the coating; use for its manufacture materials with increased strength characteristics; to make structural changes in its arrangement. If the first two methods are not always acceptable, the latter, in some cases, may prove even more economically advantageous. That is why this work is devoted to the use of the system of adding elastic reinforcement of the outer part of the covering of the protective structure. For this aim, the protective coating is modeled by an elastic body (beam), and in order to reduce the dynamic effect on it, the projectile has been offered an external part of it to elastically tuck up. To justify this in a paper it is constructed a corresponding mathematical model of the dynamics of such a modernized construction taking into account the instantaneous effect of the force factor. Then based on the analytical solution, it has been shown that for a reinforced protective structure, the dynamic deflection can be several times smaller, and from that, the protective ability is greater.

Analysis of basic research and publications

The impact of an explosive wave or projectile on a protective structure, at best, is characterized by a significant deformation [3]. Determine it, and from this, assess the protective ability can be based on the relationships that describe the dynamic deflection of the protective structure [4]. It is described mathematically by boundary-value problems for partial differential equations (systems) [5]. Analytically find it for the simplest physical models of protective structures in the case of a continuous distribution of external load using asymptotic methods of nonlinear mechanics [5, 6]. As for the development of the system-reactive system for instantaneous action, such studies were considered only in individual cases (see, for example, [7-11]) for limitations that are too rigid to analyze the impact of impulse impact of the projectile. In connection with the above mentioned, an attempt was made to obtain analytical dependencies that would be basic for choosing the basic strength characteristics of the reinforcement system of the outer part of the protective structure.

Presenting of the main results

As noted above, in many cases the outer part of the protective structure can be considered as onedimensional elastic bodies (elastic beams). To increase their protective ability, it is proposed to support them with an elastic layer. The latter are modeled by a system of linearly elastic elements with rigidity c.

For simplicity, it is assumed that the ends of the latter are hinged. For the physical model of the object

under study we assume that due to the dynamic effect on it of the shock impulse its mass of the unit of length ρ , the cross-sectional area *S*, the modulus of elasticity of the material *E* and moment of inertia of the cross-section *J* remain unchanged. The central axis of it receives only movement in the direction perpendicular to the undeformed axis. This displacement for a section with a Cartesian coordinate

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x at any given time t we denote by the function u(x,t).

The shock impulse is characterized by the magnitude F and point of application x_0 , and for simplicity, it begins to operate from the initial moment of time. In this case, the differential equation of the bending vibrations of the physical model of the upper layer of the protective structure takes the form

$$\rho \frac{\partial^2 u(x,t)}{\partial t^2} + EJ \frac{\partial^4 u(x,t)}{\partial x^4} + cu(x,t) = \beta (u_t(x,t))^{2r+1} + F_{\begin{vmatrix} x = x_0, \\ t = t_0 \end{vmatrix}}$$
(1)

$$\rho\omega^2 - EJ\kappa^4 - c = 0. \tag{5}$$

By satisfying the boundary conditions (2), for the beginning of the protective structure we obtain: $C_1 = C_2$, $\varphi_0 = -\psi_0$. Similarly, from the boundary conditions for the end of the considered construction, we have $\kappa = \frac{k\pi}{l}$, and from the dispersion relation (5) we find our own frequency of the dynamic process of the undisturbed motion of an elastically reinforced protective structure

$$\omega_k = \sqrt{\frac{EJ}{\rho} \left(\frac{k\pi}{l}\right)^4} + \frac{c}{\rho}.$$
 (6)

Taken together, it is possible to describe a large frequency dynamic process of undisturbed motion as a dependence

$$u_k(x,t) = \sum_{k=1}^{\infty} a_k \left[\sin(\kappa_k x + \omega_k t + \varphi_{0k}) + \sin(\kappa_k x - \omega_k t - \varphi_{0k}) \right], \quad (7)$$

where the parameters a_k , φ_{0k} can be found from the initial conditions. Such a task may be the subject of individual research.

The foregoing allows us to proceed to determine the effect on the oscillations of the reinforced part of the protective structure from the impact of the projectile. The basis for finding this action is the principle of onefrequency oscillations in nonlinear systems with distributed parameters [8] and the propagation of the basic ideas of asymptotic methods of nonlinear mechanics. Consequently, the single-frequency dynamic process of the reinforced part of the structure in the form close to the first form of "dynamic equilibrium" is described by the dependence

$$u(x,t) = a(t) \left[\sin(\kappa_1 x + \overline{\phi}(t)) + \sin(\kappa_1 x - \overline{\phi}(t)) \right],$$

$$\phi(t) = \omega_1 t + \overline{\phi}(t).$$
(8)

The laws of changing parameters a(t) and $\overline{\phi}(t)$ define the impact of the projectile, the force of viscous internal friction in the protective structure and its application point. That is the subject of further research. Differentiating (8) with respect to independent variables we obtain

degree 2r + 1 of displacement, where *r* is an arbitrary number, β - the coefficient of proportionality), and the second – the shock effect of the projectile. For the latter we will assume that $F_{\substack{x=x_0,\t=t_0}} = F_0 \delta(x-x_0) \delta(t-t_0)$, where $\delta(...)$ is the Delta Dirac function, F_0 – the maximum

where the first term of the right-hand side characterizes the real existing resistance forces that accompany the

dynamic process of the protective structure (the strength

of the resistance is proportional to the velocity in the

value of the indicated force, x_0 – characterizes the point of impact of the force, and t_0 – moment of time.

As is known, the dynamical process of systems in addition to external and internal factors is determined additionally by the boundary (initial) conditions, therefore, to the equation (1) we attach boundary conditions that coincide with the fixing of the ends of the outer part of the protective structure, and for simplicity we consider it as fixed cylindrical hinges. In this case, the latter become a form

$$u(x,t)_{|x=0;l} = \frac{\partial^2 u(x,t)}{\partial x^2}_{|x=0;l} = 0,$$
 (2)

where l – the distance between the reference points of the protective structure.

Thus, the problem was reduced to the construction and analysis of the solution of the differential equation (1) under the boundary conditions (2). Taken together, this allows us to use the general ideas of perturbation methods to construct the resulting mathematical model of the process dynamics. Therefore, first, we construct a solution of the so-called unbounded boundary value problem, that is, the equation

$$\rho \frac{\partial^2 u(x,t)}{\partial t^2} + EJ \frac{\partial^4 u(x,t)}{\partial x^4} + cu(x,t) = 0$$
(3)

(4)

under boundary conditions (2).

We will look for it in the form [12, 13]
$$u(x,t) = C_1 \sin(\kappa x + \omega t + \varphi_0) + C_2 \sin(\kappa x - \omega t + \psi_0),$$

where C_1 , C_2 , φ_0 , ψ_0 - are steady (the amplitudes and the initial phases of the direct and reflected waves), and their wave number k and frequency ω are connected by the dispersion relation

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$$\frac{\partial u(t,x)}{\partial t} = a \frac{d\phi(t)}{dt} [\cos(\kappa_1 x + \phi) - \cos(\kappa_1 x - \phi)] + \frac{da}{dt} [\sin(\kappa_1 x + \phi) + \sin(\kappa_1 x - \phi)],$$

$$\frac{\partial^2 u(t,x)}{\partial t^2} = -\frac{da}{dt} \omega_1 [\cos(\kappa_1 x + \phi) - \cos(\kappa_1 x - \phi)] + a\omega_1 \frac{d\phi}{dt} [\sin(\kappa_1 x + \phi) + \sin(\kappa_1 x - \phi)],$$

$$\frac{\partial u}{\partial x} = a(t)\kappa_1 [\cos(\kappa_1 x + \phi) + \cos(\kappa_1 x - \phi)],$$

$$\frac{\partial^4 u}{\partial x^4} = a(t)\kappa_1^4 [\sin(\kappa_1 x + \phi) + \sin(\kappa_1 x - \phi)]$$
(9)

In the work, we assume that:

1. the impact of the projectile on the outer part of the protective structure does not lead to its destruction and from this the latter makes bending vibrations;

2. representation of the solution of perturbed equations in the form (8) has the following basis: the first mode of oscillation plays a dominant role in the

study of nonlinear dynamic processes of systems with distributed parameters.

3. when finding the second derivative in time for the function u(t,x) it takes into account the fact that, according to the Wan der Pol method [14], it is accepted

$$a\frac{d\overline{\phi}(t)}{dt}\left[\cos(\kappa_1 x + \phi) - \cos(\kappa_1 x - \phi)\right] + \frac{da}{dt}\left[\sin(\kappa_1 x + \phi) + \sin(\kappa_1 x - \phi)\right] = 0.$$
(10)

If we substitute in the initial differential equation (1) relations that are consistent with the dependences

(5), (6), (8), (9), then after simple transformations we obtain

$$\frac{da}{dt}\omega_{1}\sin\kappa_{1}x\sin\phi + a\omega_{1}\frac{d\phi}{dt}\sin\kappa_{1}x\cos\phi = \frac{-1}{2\rho}\left[-2a\omega_{1}\sin\kappa_{1}x\sin\phi\right]^{2r+1} - \frac{1}{\rho}F_{\begin{vmatrix} x=x_{0},\\ t=t_{0}\end{vmatrix}}$$
(11)

The last differential equation together with the restriction (10) to the derivatives of the desired

functions determines their laws of change in the form

$$\frac{da}{dt} = \frac{1}{2\rho} \left\{ -\left[-2a\sin\kappa_1 x\sin\phi \right]^{2r+1} - F_0 \delta(x-x_0) \delta(t-t_0) \right\} \sin\kappa_1 x\sin\phi,$$

$$\frac{d\phi}{dt} = \omega_1 + \frac{1}{2\rho} \left\{ -2a\sin\kappa_1 x\sin\phi \right\}^{2r+1} + F_0 \delta(x-x_0) \delta(t-t_0) \right\} \sin\kappa_1 x\cos\phi.$$
(12)

However, in the obtained differential equations, the right-hand sides depend on the linear variable that holds for so-called "long systems" [12], but we are considering "short" reinforcements, and for him the amplitude of the dynamic process varies only in time. This is the basis for the averaging of the right-hand sides of the obtained relationships. Before proceeding to the averaging procedure for a linear variable, use the

following: a system of functions $\{X_k(x)\} = \left\{\sin\frac{k\pi}{l}x\right\}$ that describes the forms of proper oscillations of the protective structure has the property of completeness and orthonormalism [15]. This allows the effect of the shock force on the design to submit in the form

$$F_{\mid x=x_0} = F_0 \delta(t - t_0) \sum_{k=1}^{\infty} \frac{2}{l} \sin \frac{k\pi x_0}{l} \sin \frac{k\pi}{l} x.$$
 (13)

Similarly, we transform the time action of the shock impulse, and from this, after the procedure of averaging equations (12), we have

$$\frac{da}{dt} = \frac{\beta}{2l\rho \, q} \left(\Gamma^2 \left(1 + \frac{2r+1}{2} \right) \Gamma^{-2} \left(\frac{3}{2} + \frac{2r+1}{2} \right) (2a\omega_l)^{2r+1} + F\omega_l \sin \frac{x_0}{l} \begin{cases} \cos(\omega_l t_0 + \bar{\phi}_0), \text{ при } t = t_0, \\ 0, \text{ при } 0 \le t < t_0 - 0 \ i \ t > t_0 + 0 \end{cases} \right)$$

$$\frac{d\phi}{dt} = \omega - \frac{1}{2l\rho \alpha \omega_l} F\omega_l \sin \frac{x_0}{l} \begin{cases} \sin(\omega_l t_0 + \bar{\phi}_0), \text{ при } t = t_0, \\ 0, \text{ при } 0 \le t < t_0 - 0 \ i \ t > t_0 + 0. \end{cases}$$

$$(14)$$

Below, in Fig. 1, the reaction of the reinforced outer part of the protective structure (the law of variation in the amplitude of its bending vibrations) for various values of the rigidity of the additional springload and the various points of the projectile's contact and structure is presented.



Fig.1 Change in time of amplitudes of bending oscillations of the protective structure at different points of impact of a projectile

The obtained results also serve to assess the stresses in the protective structure, which are due to the shock action of the projectile, and thus, the choice of the basic geometric characteristics of its cross-section. Indeed, the maximum stress in the section of the protective structure is determined according to the dependence

$$\sigma_{\max} = \max\left(\frac{EI}{W}\frac{\partial^2 u(x,t)}{\partial x^2}\right),$$
(15)

where *W* is the moment of resistance, which is determined by the dependence $W = \frac{I}{y}$. Taking into account the received law of bending oscillations of the protective structure, the given dependence is transformed to a form $\sigma_{\text{max}} = \frac{EI}{W} 2a \left(\frac{\pi}{l}\right)^2$, which *a* is determined according to differential equations (14).

General conclusions

The obtained theoretical results and the graphic dependences constructed on their basis show:

firstly, an additional elastic reinforcement of the upper part of the protective structure increases the frequency of its own oscillations;

secondly, the value of the deflection of the reinforced part due to the impact action of the projectile assumes the maximum value provided that the point of impact of the projectile on the protective structure is in its middle;

third, in order to increase the protection of special structures, the rigidity of additional reinforcements should be taken more for greater distances from the reference points.

fourth, the dynamic deflection of the reinforced structure is smaller for a greater value of the stiffness of the elastic reinforcement.

It should be noted that the above results can serve as the basis for calculating the protective structure from the impact of a shock wave or other instant force and these methods may be understanded in cases of more complex constructions.

Bibliography

1. Velychko L. Dynamics of a protective structure at impact of a bullet or a fragment of a projectile / L. Velychko, O. Petruchenko, V. Kondrat // Military technical collection. – 2015. – V. 13. – P. 13–19.

2. Petruchenko O. Reduction of effective bullets, shrapnel shells on object protection / O. Petruchenko, O. Khytriak, L. Velychko // Military technical collection. – 2015. – V. 12. – P. 65–69. 3. Albert I. Analysis of the dynamic reaction of constructive-nonlinear mechanical systems / I. Albert, V. Petrov, A. Skvorthova // News VNIIG named after B. Vedeneeva. –2002. – V. 241. – P. 38-59.

4. Babakov I. The theory of oscillations / I. Babakov. – M.: Science, 1965. – 560 p.

5. Mytropolskyy Yu. / Asymptotic solutions of partial differential equations / Yu. Mytropolskyy, B. Moseenkov. – K.: High school, 1976. – 592 p.

6. Vasylenko M. The theory of oscillations and stability of motion / M. Vasylenko. – K.: High school, 2004. – 525 p.

7. Haschuk P. Nonlinear oscillations of the flexible working element of the drive under the influence of impulse forces P. Haschuk, I. Nazar // Dynamics, durability and design of machines and devices. -2007. $-N_{2}$ 588. -P. 20–24.

8. Dzyra B. On the question of justifying the averaging method for the study of single-frequency oscillations excited by instantaneous forces / B. Dzyra // Analytical and qualitative methods for the study of differential and differential-difference equations. – 1977. – P. 34–38.

9. Ulitin H. Shock processes in drilling rigs / H. Ulitin, Yu. Pettyk // Vibrations in engineering and technology. – $2000. - N_{\rm P} 1 (13). - P. 70-74.$

10. Martynthiv M. Influence of pulsed forces on nonlinear oscillations of conservative systems / M. Martyntsiv, B. Sokil, M. Sokil // Scientific herald: Collection of scientific and technical works. – 2003. – V. 13 (1). – P. 72–81.

11. Sokil B. Periodic Ateb-functions in the investigation of nonlinear systems with impulse perturbation / B. Sokil, M. Sokil // Scientific herald: Collection of scientific and technical works. – 2002. – V. 12 (8). – P. 304–311.

12. Mitropol'skii Y. On the application of Ateb-functions to the construction of an asymptotic solution of the perturbed nonlinear Klein-Gordon equation / Y. Mitropol'skii, B. Sokil. – Ukrainian Mathematical Journal. – 1998. – V. 50 (5). – P. 754–760.

13. Sokil B. Transverse vibrations of a nonlinearly elastic medium and the method of D'Alembert in their study / B. Sokil, A. Senyk, I. Nazar, M. Sokil // Dynamics, durability and design of machines and device. $-2004. - N_{\odot} 509. - P. 104-109.$

14. Bogolubov N. Asymptotic methods in the theory of nonlinear oscillations / N. Bogolubov, Yu. Myttropolskyy. – M.: Science, 1974. – 501 p.

15. Koschlyakov N. Partial differential equations / N. Koschlyakov, Ye. Hliner, M. Smirnov. – K.: High school, 1970. – 712 p.

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ОБҐРУНТУВАННЯ СПОСОБУ ПІДВИЩЕННЯ ЗАХИЩЕНОСТІ СПЕЦІАЛЬНИХ СПОРУД ВІД УДАРНОЇ ДІЇ СНАРЯДА

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Досвід ведення воєнних операцій показує існуючі проблеми підвищення захищеності інженерних споруд та техніки від дії уданих хвиль чи снарядів. Вирішити її для інженерних споруд можна різними способами, наприклад, збільшити товщину покриття; використати для її виготовлення матеріали із підвищеними міцнішими характеристиками; зробити конструкційні зміни при її облаштуванні. Якщо перші два способи є не завжди прийнятними, то останній у

деяких випадках може виявитись навіть економічно вигіднішим. Саме обґрунтуванню використання системи додаткового пружного підкріплення зовнішньої частини покриття захисної споруди присвячена дана робота. Зокрема, від ударної дії снарядів запропоновано використовувати додаткове пружне підкріплення зовнішньої частини об'єкта. Для випадку додаткового підкріплення у вигляді пружних балок побудовано математичну модель динаміки захисної конструкції із урахуванням динамічної дії на неї в довільному місці ударної сили снаряда. Математична модель являє собою крайову задачу для нелінійного диференціального рівняння з частинними похідними. За фізично обґрунтованих обмежень щодо основних зовнішніх і внутрішніх чинників, які характеризують об'єкт захисту, показано, що для побудови аналітичного розв'язку вказаної математичної моделі можна використати основні ідеї методів збурень, точніше кажучи, асимптотичні методи нелінійної механіки у поєднанні із принципом одночастотності коливань у нелінійних системах. Із їх використанням отримано аналітичні залежності, що описують визначальні параметри згинальних коливань зовнішньої частини захисної конструкиії. Встановлено: динамічний прогин підкріпленої конструкції є меншим для більших величин жорсткості пружного підкріплення; величина прогину підкріпленої частини, зумовлена ударною дією снаряда, приймає максимальне значення за умови, коли точка удару снаряда об захисну конструкцію знаходиться у її середині. З останнього випливає, що з метою підвищення захищеності спеціальних споруд жорсткість додаткового підкріплення треба брати більшою для більших віддалей від опорних точок. Треба відзначити, що основна ідея роботи може бути використана під час розрахунку, а відтак, надання практичних рекомендацій щодо побудови інженерних споруд з метою захисту їх від дії уданої хвилі.

Ключові слова: інженерна споруда, система пружного підкріплення, згинні коливання.

ОБОСНОВАНИЕ СПОСОБА ПОВЫШЕНИЯ ЗАЩИЩЕННОСТИ СПЕЦИАЛЬНЫХ СООРУЖЕНИЙ ОТ УДАРНОГО ВОЗДЕЙСТВИЯ СНАРЯДА

А.И. Андрухив, Б.И. Сокил, М.Б. Сокил, Н.Н. Гузык

Опыт ведения военных операций показывает существующие проблемы повышения защищенности инженерных сооружений и техники действия в данных волн или снарядов. Решить ее для инженерных сооружений можно разными способами, например, увеличить толщину покрытия; использовать для ее изготовления материалы с повышенными более мощными характеристиками; сделать конструкционные изменения при ее обустройстве. Если первые два способа являются не всегда приемлемыми, то последний в некоторых случаях может оказаться даже экономически выгодным. Именно обоснованию использования системы дополнительного упругого подкрепления внешней части покрытия защитного сооружения посвящена данная работа. В частности, от ударного воздействия снарядов предложено использовать дополнительное упругое подкрепления внешней части объекта. Для случая дополнительного подкрепления в виде упругих балок построена математическая модель динамики ограждающей конструкции с учетом динамического воздействия на нее в произвольном месте ударной силы снаряда. Математическая модель представляет собой краевую задачу для нелинейного дифференциального уравнения в частных производных. По физически обоснованным ограничениям относительно основных внешних и внутренних факторов, характеризующих объект защиты, показано, что для построения аналитических решений указанной математической модели можно использовать основные идеи методов возмущений, точнее говоря, асимптотические методы нелинейной механики в сочетании с принципом одночастотности колебаний в нелинейных системах. С их использованием получены аналитические зависимости, описывающие определяющие параметры изгибных колебаний внешней части ограждающей конструкции. Установлено: динамический прогиб подкрепленной конструкции является меньшим для больших величин жесткости упругого подкрепления; величина прогиба подкрепленной части, обусловлена ударным действием снаряда, принимает максимальное значение при условии, что точка удара снаряда о защитную конструкцию находится в ее середине. Из последнего следует, что с целью повышения защищенности специальных сооружений жесткость дополнительного подкрепления надо брать больше для больших расстояний от опорных точек. Надо отметить, что основная идея работы может быть использована при расчете, итак, предоставление практических рекомендаций по построению инженерных сооружений с целью защиты их от действия у данной волны.

Ключевые слова: инженерное сооружение, система упругого подкрепления, изгибные колебания.