ТЕХНІКА ТА МЕТОДИ ЕКСПЕРИМЕНТУ

УДК 53.088.23

I. S. Timchenko, A. Yu. Buki

National Science Center "Kharkov Institute of Physics & Technology", Institute of High-Energy Physics and Nuclear Physics, Kharkov

APPLICATION OF THE EQUIVALENT RADIATOR METHOD FOR RADIATIVE CORRECTIONS TO THE SPECTRA OF ELASTIC ELECTRON SCATTERING BY NUCLEI

For calculating the radiative tails in the spectra of inelastic electron scattering by nuclei, the approximation, namely, the equivalent radiator method (ERM), is used. However, the applicability of this method for evaluating the radiative tail from the elastic scattering peak has been little investigated, and therefore, it has become the subject of the present study for the case of light nuclei. As a result, spectral regions were found, where a significant discrepancy between the ERM calculation and the exact-formula calculation was observed. A link was established between this phenomenon and the diffraction minimum of the squared form-factor of the nuclear ground state. Varieties of calculations were carried out for different kinematics of electron scattering by nuclei. The analysis of the calculation results has shown the conditions, at which the equivalent radiator method can be applied for adequately evaluating the radiative tail of the elastic scattering peak.

Keywords: electron scattering, radiative corrections, elastic equivalent radiator method, diffraction minimum of form-factor.

Introduction

The experimental spectra of electrons scattered by nuclei are distorted by a variety of physical effects, among which we mention the radiation losses manifested in all the measurements. The currently adopted method of estimation of this effect (subsequently referred to as the radiative correction to the spectrum) has been described in references [1 - 4].

Of all the operations that the radiative correction includes, the calculation of radiative tails from the peaks and differential cross sections of the continuum spectrum is the most complicated. For calculation simplicity, several approximation techniques have been developed [1, 5 - 7], one of them being the equivalent radiator method (ERM) [1]. This method is considered to be a good alternative to exact calculations (see refs. [4, 8, 9]), and it is commonly used in the radiative correction to inelastic scattering spectra.

In papers [1, 8], and in the recent work [9] focused on the treatment of JLab measurements [10], it was proposed to use the ERM for evaluating the radiative tail from the elastic scattering peak. The proposal was based on the results of comparison between the ERM calculations of the radiative tail and its calculations by exact equations. The comparison was given in refs. [1, 8] for electron scattering by ^{1}H nuclei at the angles $\theta = 5°$ and 14°, and also, for one spectrum of scattering by ^{12}C nuclei at $\theta = 14°$ and at initial electron energy $E_s = 800$ MeV [8].

The undertaken verifications of the ERM approximation seem insufficient, because they were carried out only at small scattering angles, and in the ^{12}C case – only for one E_s value. Besides, consideration was given to scattering by two nuclei, of which ^{1}H is a peculiar nucleus, and its characteristics are not typical of other nuclei.

The present paper deals with the applicability of the ERM approximation for calculating the radiative tail from the elastic scattering peak.

The equivalent radiator method

The differential radiative tail cross-section $\frac{d\sigma_{0,\,t+r}\left(E_s, E_p, T\right)}{d\Omega \cdot dE_p}$ for scattered electron energies E_p and the target of thickness T can be represented as

$$\frac{d\sigma_{0,\,t+r}\left(E_s, E_p, T\right)}{d\Omega \cdot dE_p} =$$

$$= \frac{d\sigma_{0,\,t}\left(E_s, E_p, T\right)}{d\Omega \cdot dE_p} + \frac{d\sigma_{0,\,r}\left(E_s, E_p\right)}{d\Omega \cdot dE_p}. \quad (1)$$

Here the first term $\frac{d\sigma_{0,\,t}\left(E_s, E_p, T\right)}{d\Omega \cdot dE_p}$ describes the so-called external energy losses occurring as the electron passes through the target substance. These are the bremsstrahlung losses and the atomic ionization losses on the electron trajectory in the target. The second term $\frac{d\sigma_{0,\,r}\left(E_s, E_p\right)}{d\Omega \cdot dE_p}$ takes into account the energy decrease due to photon emission by the

electron during its scattering by the nucleus. These losses are usually called internal. The equation for exact calculation of internal losses is very cumbersome, and its solution calls for the data measured at lower initial electron energies (see Eq. (B5) in ref. [2]. p. 230; or Eq. (A24) in ref. [4], p. 1914). To avoid solving this equation, the ERM approach assumes that the internal bremsstrahlung is equivalent to the bremsstrahlung in two radiators, each of thickness t_r, arranged on the path of the electron before and after the scattering event. In this case, the expression for the radiative tail takes on the form

$$\frac{d\sigma_{0,\,t+r}}{d\Omega \cdot dE_p}\left(E_s, E_p, T\right) = \frac{d\sigma_{0,\,t}\left(E_s, E_p, T + 2t_r\right)}{d\Omega \cdot dE_p}. \quad (2)$$

Here $t_r = \frac{\alpha}{\pi b}\left(\ln\frac{Q^2}{m^2} - 1\right)$ is the thickness of the equivalent radiator, $Q^2 = 4 \cdot E_s \cdot E_p \cdot \sin^2\left(\theta/2\right)$ is the transfer four-momentum; m is the mass of the electron; b is the quantity, which is weakly dependent on the target material performance and is taken to a good accuracy to be $b \approx 4/3$. As mentioned in the Introduction, the results of ERM calculations and the calculations by exact equations for the ^{1}H and ^{12}C nuclei were published in refs. [1, 8]. To verify our computational programs, we have repeated those calculations and obtained coincidence in four mantissa signs with the calculation data of refs. [1, 8]. For convenience in comparison between the ERM calculations (R_{ERM}) and the exact-equation calculations of ref. [2] (R), we have used the function $U(E_\mathrm{p}) = (R_{\mathrm{ERM}}(E_\mathrm{p})/R(E_\mathrm{p}) - 1) \cdot 100$, which shows the percentage divergence between these calculations.

The effect of the diffraction minimum of nuclear form-factor on the ERM calculation

Fig. 1, *a* shows the functions $U(E_\mathrm{p})$ calculated for the radiative tails of peaks of elastic scattering by the ^{1}H and ^{4}He nuclei at the energies E_s = 260 MeV, 100 MeV; the scattering angle $\theta = 60°$ and the target thicknesses T = 0.2 % radiative length[1]. As is obvious from the figure, in the case of ^{1}H, the functions $U(E_\mathrm{p})$ increase monotonically with a decreasing E_p, but in the case of E_s = 1000 MeV the function $U(E_\mathrm{p})$ of the ^{4}He nucleus has a deep minimum at E_p = 600 MeV (we shall denote the E_p values corresponding to the minimum as $E_{\mathrm{p,\,m}}$). This strong difference between the functions under consideration should be due to the difference in some of the characteristics of the nuclei. This characteristic must be present in the

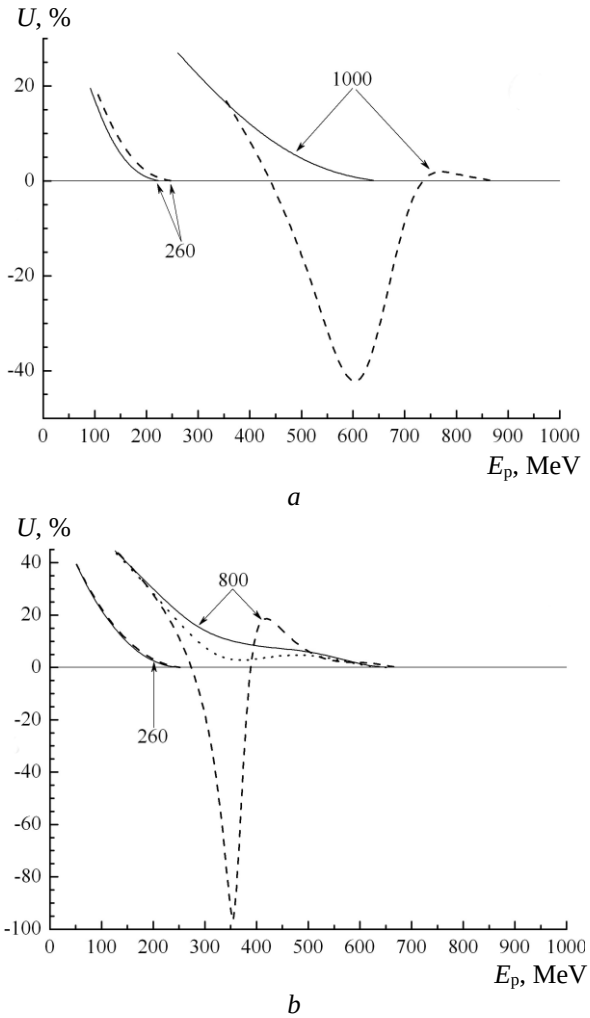

Fig. 1. Calculated $U(E_\mathrm{p})$ for the nuclei ^{1}H, ^{4}He, ^{7}Li and ^{12}C at the scattering angle $\theta = 60°$ and target thicknesses T = 0.2 % rad. length. The arrows show the initial electron energies in MeV units. *a* – solid lines – ^{1}H; dashed lines – ^{4}He; *b* – solid and dotted lines represent the calculations for ^{7}Li with the full form-factor and only with the charge form-factor, respectively; the dashed curves are for ^{12}C.

radiative tail calculations and be different for the ^{1}H and ^{4}He nuclei at $Q > 1$ fm^{-1}, but not at $Q \approx 1$ fm^{-1}, where according to the calculation for E_s = 260 MeV, the functions $U(E_\mathrm{p})$ of the both nuclei considered are similar to each other. The only nuclear characteristic that corresponds to the mentioned requirements is the squared form-factor of the nuclear ground state $F_{\mathrm{el}}^2(Q)$ (hereafter, more simply the form-factor). So, the function $F_{\mathrm{el}}^2(Q)$ of the ^{1}H nucleus decreases monotonically with increasing Q at all its values. As for the case of the ^{4}He nucleus, at Q = 3.15 fm^{-1}, its form-factor has the diffraction minimum (Fig. 2). Further on, the transfer momentum corresponding to the first diffraction minimum of the form-factor will be denoted as $Q_{\mathrm{m,\,1}}$. To verify the found relationship between the minima of the functions $U(E_\mathrm{p})$ and $F_{\mathrm{el}}^2(Q)$, we consider the behavior of $U(E_\mathrm{p})$ for the ^{7}Li and ^{12}C nuclei. Owing to the peculiarities of

[1] All further calculations were carried out for the targets of this thickness and by using equations of Ref. [2].

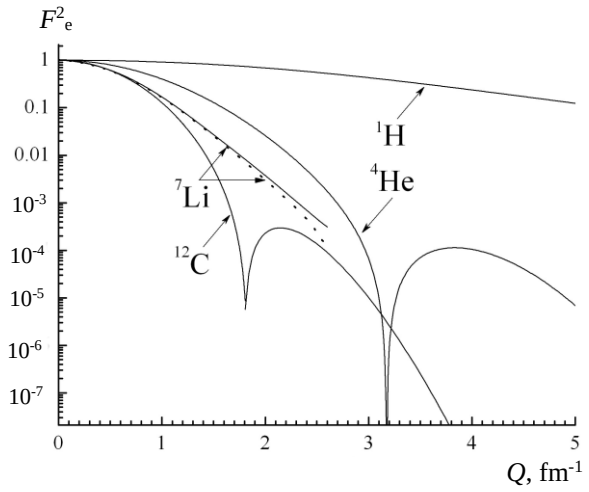

Fig. 2. Squared ground-state form factors of ^{1}H, ^{4}He, ^{7}Li and ^{12}C (solid lines). In the case of ^{1}H and ^{7}Li nuclei, the form-factors were calculated for $\theta = 60°$. The dotted line shows the charge form-factor of the ^{7}Li nucleus.

charge density distribution in the ^{7}Li nucleus, its form-factor has no diffraction minimum at $Q < 2.7$ fm^{-1}, while at the given transfer momenta we have a well-marked minimum in the form-factor of the ^{12}C nucleus (see Fig. 2). So, in the nature of the functions $F_{el}^2(Q)$, the pair of ^{7}Li and ^{12}C nuclei is similar to the pair of ^{1}H and ^{4}He. The calculation of $U(E_p)$ for ^{7}Li and ^{12}C is given in Fig. 1, *b*. It can be seen that the function $U(E_p)$ of the ^{7}Li nucleus has no minimum, while in the ^{12}C case the function $U(E_p)$ shows the minimum at $E_{p,m} = 360$ MeV. In other words, Fig. 1, *b* for $U(E_p)$ of the nuclei ^{7}Li, ^{12}C is similar to Fig. 1, *a* for $U(E_p)$ of the nuclei ^{1}H, ^{4}He, and this is in agreement with the above-said assumption about the reason for minimum appearance in the function $U(E_p)$.

The emergence of the diffraction minimum of the form-factor in the function $U(E_p)$ can be attributed to the fact that the form-factor is differently involved in the exact-formula/ERM calculations of the radiative tail. Thus in the exact calculation, the form-factor is averaged over some range of transfer momenta, and this smoothes the calculated function of the radiative tail, while in the ERM approach the form-factor enters into the calculation as a multiplying factor. So, the difference between the two calculations is the more, the greater is the second derived function of the form-factor, in other words, at Q's somewhat lower than in the diffraction minimum and in the minimum itself.

It should be noted that the equation for the radiative tail of the elastic scattering peak includes the contributions of both the longitudinal and transverse form-factors. The contribution from the transverse form-factor is absent in the form-factors of the ^{4}He and ^{12}C nuclei. It can be assumed that the minima in the functions $U(E_p)$ of these nuclei are due to the absence of the transverse form-factor contribution in them. For checking this assumption, we have calculated the function $U(E_p)$ of the ^{7}Li nucleus with the form-factor, whereof the transverse form-factor contribution was excluded. That is, the calculation was done only for the charge form-factor of this nucleus, as is the case with ^{4}He and ^{12}C nuclei. It can be seen in Fig. 1, *b* that the function $U(E_p)$ calculated in this way is little different from the function having the contribution from the transverse form-factor. From this it can be concluded that there is no connection between the transverse form-factor of the nucleus and the clearly marked minimum in the function $U(E_p)$.

The practical consequence of revealing the minimum in the function $U(E_p)$ is the fact that this minimum restricts the energy range (E_p) that permits the usage of the ERM approximation (Fig. 3). For a more detailed consideration of the conditions for the appearance of this restriction, a number of calculations were performed.

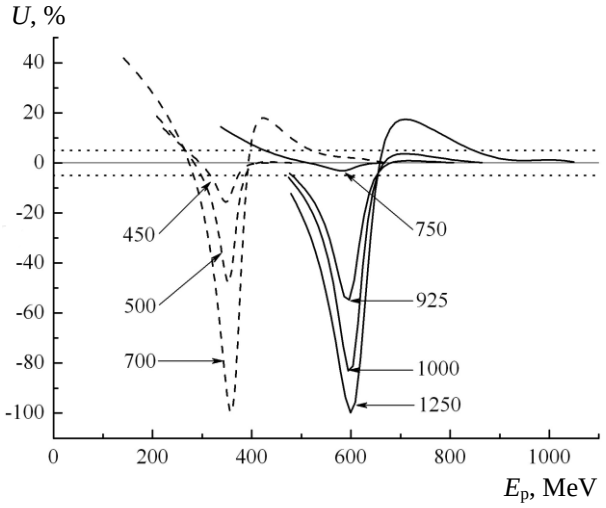

Fig. 3. Functions $U(E_p)$ of the nuclei ^{4}He (solid lines) and ^{12}C (dashed lines) at $\theta = 60°$ for different initial energies E_s of MeV values indicated by arrows. The dotted lines show $\Delta U = \pm 5$.

The minima of functions $U(E_p)$ at different calculation conditions

The calculations of the functions $U(E_p)$ for different kinematic conditions of elastic electron scattering by nuclei must show the restrictions on the applicability of the ERM. Carrying-out of these calculations for a few nuclei may point to common properties and differences of the functions $U(E_p)$, which are typical of these nuclei, and thus may be helpful in generalizing the ERM applicability conditions for some group of nuclei.

Fig. 3 shows the calculated functions $U(E_p)$ for the nuclei ^{4}He and ^{12}C at a fixed scattering angle and at different initial energies. Fig. 4 gives the calculated functions $U(E_p)$ for the ^{12}C nucleus at different θ angles. It is evident from the figures that:

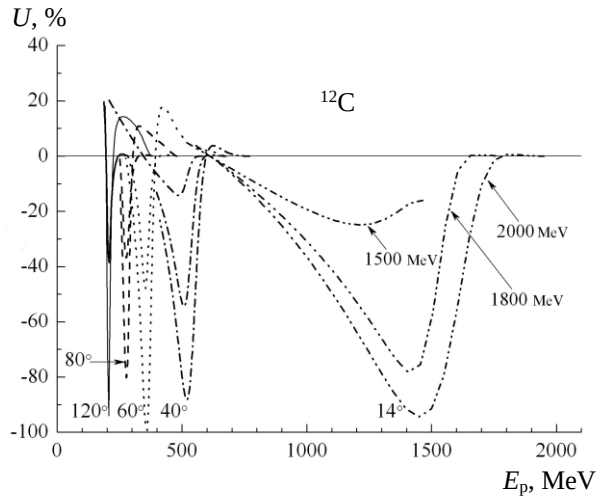

Fig. 4. Functions $U(E_p)$ of the nucleus ^{12}C at different scattering angles θ. The functions $U(E_p)$ corresponding to the same θ value but to different energies E_s are shown by the same-type lines. For $U(E_p)$ at θ = 60° the energy E_s values are indicated by arrows.

1) for the given nucleus and a fixed scattering angle, the position of the minimum of the function $U(E_p)$ with respect to the scattered electron energy ($E_{p,1}$), is almost independent of the initial energies E_s;

2) with decrease in the energy E_s, the depth of the minimum of the function $U(E_p)$ decreases and becomes close to zero at $E_s \approx E_{p,m}$;

3) the $E_{p,1}$ value increases with a decrease in the scattering angle θ.

Fig. 5 shows $E_{p,1}$ as a function of θ, along with its calculation by the empirical formula

$$E_{p,m} = Q_{m,1} \cdot 100 / \sin(\theta/2) + \Delta, \qquad (3)$$

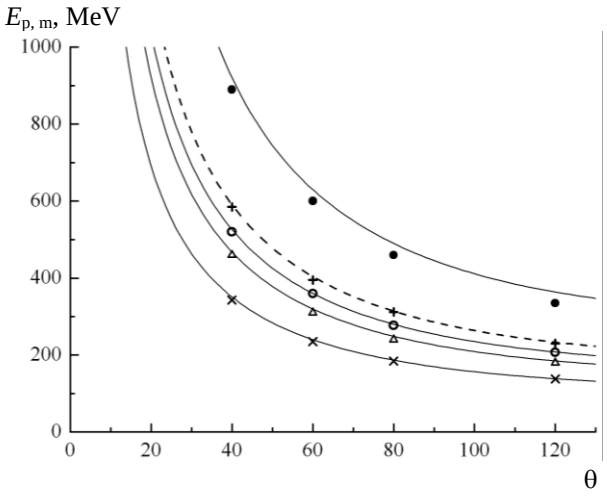

Fig. 5. Scattered electron energies $E_{p,1}$ versus θ. The energies $E_{p,1}$ correspond to the minimum of the function $U(E_p)$ for the nuclei ^{4}He (points), ^{12}C (circles), ^{16}O (triangles), ^{40}Ca (sidelong crosses). The solid lines represent the calculation by formula (3). Straight crosses show the $E_{p,2}$ values of ^{40}Ca, related to the second minimum of the form-factor. The corresponding calculation is shown by the dashed line.

where the four-momentum $Q_{m,1}$ is in units of fm^{-1}, Δ is the parameter invariable for each given nucleus and amounting to several MeV. For plotting the curves in Fig. 5, the Δ value was taken as zero.

With substitution of appropriate $Q_{m,1}$ values, formula (3) can be also used for higher differential minima of the nuclear ground-state form-factor. As an example, Fig. 5 shows the $E_{p,2}$ values for the minimum of the function $U(E_p)$ related to the second diffraction minimum of ^{40}Ca nuclear form-factor. Alongside, the same figure illustrates $E_{p,2}$ as a function of θ, calculated by formula (3), where the momentum $Q_{m,1}$ was replaced by the momentum $Q_{m,2} = 2.02$ fm^{-1}.

We put the acceptable deviation of the ERM calculation from the exact-formula calculation to equal 5 %, that is, when $|U(E_p)| < 5$, and denote $|U(E_p)| = 5$ as ΔU. The experimental spectrum begins from the elastic peak energy E_{el}, where $U(E_p) = 0$. With a decrease in the scattered electron energy, the $|U|$ value increases, and at a certain $E_{p,u}$ it reaches the ΔU value. We denote the spectral region, where the ERM is applicable for calculating the radiative tail of the elastic scattering peak, by $\varepsilon_{max} = E_{el} - E_{p,u}$.

From Figs. 1, 3, 4 it is evident that in some cases the $E_{p,u}$ value determines the drop of the function $U(E_p)$ to the minimum, and in the other cases, it specifies a small rise of the function on the high-energy side.

At the energies $E_s \le E_{p,m}$, the $E_{p,u}$ value is determined by the growth rate of the function $U(E_p)$ as E_p decreases (see Fig. 1). The calculated data on ε_{max} for the nuclei ^{4}He, ^{12}C, ^{16}O, ^{40}Ca at θ = 40° ÷ 80°

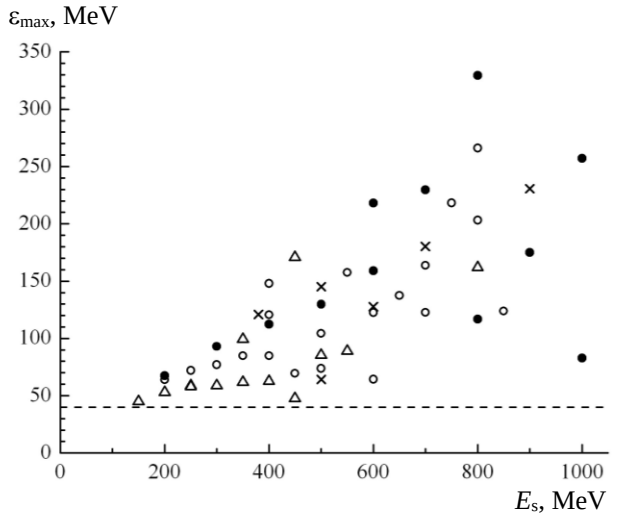

Fig. 6. The nuclear excitation energy ε_{max}, up to which the ERM-calculated rad. tail of the elastic scattering peak differs within 5 % from the rad. tail value calculated by exact equations. The calculation is made for the scattering angles θ = 40° ÷ 80°. The notation of the nuclei, for which the ε_{max} were found, is the same as in Fig. 5.

have shown (Fig. 6) that up to 600 MeV ($Q < 3.8$ fm^{-1}), we have $\varepsilon_{max} = 40$ MeV. With a decrease in the angle θ, the ε_{max} values become to vary greatly and fail to be systematized. The increase in the angle θ reduces the ε_{max} value down to $\varepsilon_{max} < 16$ MeV at $\theta = 120°$.

Conclusions

Consideration has been given to the possibility of calculating the radiative tail of the elastic scattering peak using the equivalent radiator method.

In some spectral regions, the analysis has revealed great differences between the ERM calculations and the calculations by exact formulae.

Links have been established between the spectrum regions, where the ERM calculation appears invalid, and the diffraction minima of the nuclear ground-state form-factors.

The excitation energies ε_{max}, up to which the ERM is applicable, have been determined for the nuclei ^{4}He, ^{12}C, ^{16}O, ^{40}Ca.

The ERM calculations for various nuclei at different scattering angles and initial electron energies just led to finding out a few regularities in the deviation of the ERM calculations from the exact-formula calculations. These regularities have enabled us to extend the results obtained to a group of light nuclei ($A \leq 40$).

The practical outcome of the present work lies in the determination of the conditions, at which the ERM approximation can be applied for calculating the radiative tail of the elastic scattering peak. According to the analysis results, the ERM calculation of the radiative tail of the elastic scattering peak can be used in the studies of excited states of light nuclei at electron scattering angles $\theta = 40° \div 80°$ up to excitation energies of 40 MeV. As for the studies of quasielastic electron scattering and electroproduction processes, the ERM approximation for calculating the radiative tail of the elastic scattering peak does not appear sufficiently exact.

REFERENCES

1. *Mo L.W., Tcai Y.S.* Radiative Corrections to Elastic and Inelastic *ep* and μp Scattering // Rev. Mod. Phys. - 1969. - Vol. 41. - P. 205 - 235.
2. *Tcai Y.S.* Radiative Corrections to Electron Scattering // California, U.S.A. - 1971. - 66 p. - (SLAC-PUB-848).
3. *Whitney R.R., Sick I.* Quasielastic Electron Scattering // Phys. Rev. - 1974. - Vol. C9, No. 6. - P. 2230 - 2235.
4. *Stein S., Atwood W. B., Bloom E. D. et.al.* Electron Scattering at 4° with Energies of 4.5 - 20 GeV // Phys. Rev. - 1975. - Vol. D12, No. 7. - P. 1884 - 1919.
5. *Hand L.N.* Experimental Investigation of Pion Electroproduction // Phys. Rev. - 1963. - Vol. 129, No. 4. - P. 1834 - 1847.
6. *Allton E.A.* Bremsstrahlung in Electron-Proton Scattering // Phys. Rev. - 1964. - Vol. B135. - P. 570 - 576.
7. *Bjorken J.D.* Radiative Corrections to Inelastic Electron Scattering // Ann. Phys. (N.Y.). - 1963. - Vol. 24. - P. 201 - 210.
8. *Gann V.V., Mitrofanova A.V.* Computation programs for radiative corrections to electron scattering by nuclei. - Kharkov, 1974. - 20 c. - (Prepr. / KIPT; 74-41). (In Russian)
9. *Huan Yao,* Precision Measurement of Longitudinal and Transverse Response Functions of Quasielastic Electron Scattering in the Momentum Transfer Range 0.55 GeV/c $\leq |q| \leq 1.0$ GeV/c // PhD Thesis, 2012, Temple University, http://digital.library.temple.edu/cdm/ref/collection/p245801coll10/id/152421.
10. *Precision* Measurement on Longitudinal and Transverse Response Function of Quasi-Elastic Scattering in the Momentum Transfer Range $0.55 \leq |q| \leq 1$ GeV/c // http://hallaweb.jlab.org/experiment/E05-110/exp_home/.

І. С. Тімченко, О. Ю. Буки

Національний науковий центр «Харківський фізико-технічний інститут», Інститут фізики високих енергій та ядерної фізики, Харків

ВИКОРИСТАННЯ МЕТОДУ ЕКВІВАЛЕНТНОГО РАДІАТОРА ДЛЯ РАДІАЦІЙНОГО КОРЕГУВАННЯ СПЕКТРІВ ПРУЖНОГО РОЗСІЯННЯ ЕЛЕКТРОНІВ НА ЯДРАХ

У процедурі радіаційного корегування спектрів непружного розсіяння електронів на ядрах для розрахунку радіаційних хвостів використовується наближений метод – метод еквівалентного радіатора. Однак можливість використання цього методу для оцінки радіаційного хвоста піка пружного розсіяння була мало досліджена і в даній роботі вивчається на прикладі легких ядер. Як результат, було знайдено області спектра, де розрахунки за методом еквівалентного радіатора та точною формулою значно розходяться. Було встановлено зв'язок між цим феноменом і дифракційним мінімумом квадрата формфактора основного стану ядра. Проведено ряд розрахун-

ків для різних кінематичних умов розсіяння електронів на ядрах, а з аналізу результатів було визначено умови, за яких метод еквівалентного радіатора може бути використано для адекватної оцінки радіаційного хвоста для піка пружного розсіяння.

Ключові слова: розсіювання електронів, радіаційні поправки, метод еквівалентного радіатора, дифракційний мінімум формфактора.

И. С. Тимченко, А. Ю. Буки

Национальный научный центр «Харьковский физико-технический институт», Институт физики высоких энергий и ядерной физики, Харьков

ПРИМЕНЕНИЕ МЕТОДА ЭКВИВАЛЕНТНОГО РАДИАТОРА ДЛЯ РАДИАЦИОННОЙ КОРРЕКТИРОВКИ СПЕКТРОВ УПРУГОГО РАССЕЯНИЯ ЭЛЕКТРОНОВ НА ЯДРАХ

В радиационной корректировке спектров неупругого рассеяния электронов на ядрах для расчета радиационных хвостов вместо точного, но трудоемкого расчета применяется приближенный метод эквивалентного радиатора. В настоящей работе рассматривается возможность применения этого метода для расчета радиационного хвоста от пика упругого рассеяния. В процессе исследования обнаружены участки спектра рассеянных электронов, на которых наблюдается большое расхождение расчетов точного и по методу эквивалентного радиатора и установлена связь этого расхождения с дифракционными минимумами квадрата формфактора основного состояния ядра. Из анализа результатов ряда расчетов определены условия, при которых можно применять метод эквивалентного радиатора для расчета радиационных хвостов от пиков упругого рассеяния электронов на легких ядрах.

Ключевые слова: рассеяние электронов, радиационные поправки, метод эквивалентного радиатора, дифракционный минимум формфактора.

Надійшла 20.04.2015
Received 20.04.2015