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THE MATHEMATICAL OPTIMIZATION MODEL OF MOBILE RADIO SYSTEM VIA CONNECTIVITY FIGURE

In this paper it is proposed the mathematical optimization model of mobile radio system subject to prediction enemy activity. Optimization conducts via connectivity figure which reflects signal quality between mobile radio terminal-retransmitters.

Keywords: *mobile radio system, connectivity figure, radio terminal-retransmitters, optimization model.*

Statement of the problem

For the purpose of achieving information superiority leading countries continue to create and develop communication systems that combine dispersed government, troops (forces), intelligence by means of data on military conditions between different subscribers in-time, near real. The value of communication in the conduct of modern combat is very high, because of their proper and fast functioning depending on the success of military operations and transactions. In this case, the radio remains one of the main sorts of communication during the organization of command and control at the tactical level. It is fast setting, flexible and practically indispensable in difficult maneuvering conditions of warfare.

Due to the fact that military developed countries have in service modern tools and systems for signal monitoring radio intelligence [1-7], which are able to quickly expose radio systems that are deployed in the interests of tactical management of the Armed Forces of Ukraine, so the extremely important problem is a reducing the effectiveness of enemy radio intelligence.

To protect the radio for the entire depth of defense against electronic radio intelligence and Electronic Warfare (EW) of rival the most effective is radio stations for small facilities. The most promising for this method of communication is mobile radio communication system (RCS), in which stations have the same status and interact with each other in the area of line-of-sight or via relaying messages through other radio stations. In this case, the spatial scope of RCS is limited only by the number of relay stations and the limiting distance of propagation of radio signals.

However, under the action of electronic radio intelligence and electronic warfare of enemy and dynamics of changing in tactical management level of the Armed Forces of Ukraine during the operation of RCS is necessary regarding the prediction of possible variants of its structure.

Therefore, the problem of optimization the structure of the RCS in terms of connectivity is of considerable practical interest and is an urgent problem for the branch of military control and communications.

Analysis of recent achievements and publications

Analysis of researches of this problem suggests that the solution to the problem of optimization the RCS structure is paid enough attention. The vast majority of authors on the subject of quality indicators to RCS, which are used for optimization of RCS include cost, reliability and average time delay between relay stations. Thus, the problem of optimization the RCS structure are related to problems of multicommodity streams [8].

As a classical problem of multicommodity streams the structure of RCS assumed given and, for example, the best routes of information transfer for each pair of relay stations are found. However, this optimization is achieved through the solution of combinatorial problems as a result of which labour-intensity increases significantly depending on the number of radio-transmitters in the RCS.

The most common RCS optimization is in terms of throughput capacity. For the first time this problem was considered by Kleinrock L. [10], where the methods of solution for some classes of functions were proposed. Also were developed many heuristic methods, replacing of branches, removal of branches, busy chopping. But the proposed methods are in a narrow range of parameters (cost and throughput capacity) and do not always give accurate results of optimization [9].

Domestic authors consider optimization of RCS's structure by addressing the problem of Steiner for the appropriate number of nodes of the graph. The solution to this problem is putting into network the additional node that provides the best connection with the closest nodes. In addition, for solving the problem of Steiner are used the approximate algorithms by Zlotow A.V.- Khachaturov V.R. [12], heuristic algorithms by Lotaryov D.T. [13] and asymptotically optimal algorithm by Yerzin AI [14].

Thus, there are many models and optimization algorithms of RCS at present. But their purely technical direction does not consider the possibility of enemy action on RCS.

Statement of the problem and its solution

To take into consideration proposed shortcomings the purpose of the article is to create a mathematical model of optimization of the mobile radio system structure in terms of connectivity.

Emphasizing of unsolved aspects of the problem, what the article are about

To date, the optimization problem of RCS is quite far from complete solution. A prospecting direction is a conflict of discrete optimization set of RCS/electronic radio intelligence, electronic warfare [15]. It allows increasing or preserving the effectiveness of the disputed parts in terms of deliberate counter by rational choice of their actions.

For optimization of the RCS is proposed to create a game-theoretic model using connectivity index. Connectivity is the connection between any radio terminal-retransmitters of RCS in a given period of time. This figure is an integrated measure of timeliness, accuracy, secrecy of connection and its quality.

The main material research and complete explanation of scientific results

The feature of the solution of optimization problem of RCS in terms of electronic intelligence and electronic warfare is that the connectivity of individual areas of communication $K_{зв\,i}$ and RCS in general $K_{зв}$ will depends on decisions (strategies), what are accepted by both warring parties. That is the optimization criteria should be presented in the form of functional $K_{зв\,i}(S_j, R_i)$ and $K_{зв}(S_j, R_i)$ defined on the set $S \times R$ where $S = \{S_i\}$- is the set of solutions (strategies) for creating RCS, $R = \{R_i\}$ - is the set of strategies for creating electronic warfare. Obviously, the purpose of optimization of RCS is to maximize the connectivity of the system $K_{зв} = \max_S K_{зв}(S_j, R_i)$, and the purpose of the opposing party (electronic intelligence and electronic warfare) is a minimizing of connectivity of RCS $K_{зв} = \min_R K_{зв}(S_j, R_i)$.

In conditions of the conflicting interests the legitimate is an attempt of each part to get any guaranteed results. In this case it is advisable to use the minimax approach to problem solving of optimization by criterion $K_{зв} = \max_S \min_R K_{зв}(S_j, R)$

To solve the optimization problem by this criterion it is appropriate to use the methods of game theory as a conflict of interest (RCS / electronic intelligence and electronic warfare) is to enhance or preserve the efficiency in terms of deliberate counteraction of parties by rational choice of their action.

Due to a number of sets of strategies S and R, their finite, limited resources allocated for the organization of communication, electronic intelligence and electronic warfare, the index of effectiveness of RCS can be expressed by ultimate connectivity matrix $\{K_{зв}\}$, and the game itself is classified as antagonistic size of matrix games $S \times R$ which decisions are to find the saddle point:

$$\max_S \min_R K_{зв}(S_i, R_j) = \min_R \max_S K_{зв}(S_i, R_j) \quad (1)$$

The greatest difficulties in the optimization of RCS by the criterion (1) is due to the so-called "curse of dimensionality", i.e. avalanche-like increase of matrix dimension $K_{зв}(S_j, R_i)$ by increasing the number of admissible strategies S_j and R_i. This makes the optimization problem difficult to solve RCS with increasing number of areas due to two-digit number.

The solution of this situation can be found if consider the final number of options electronic intelligence and electronic warfare and related schemes of communication (Table. 1).

Let some final number of options system electronic intelligence and electronic warfare R - and options for communication S. Then, for each pair of options $S_i \in S$ and $R_i \in R$ can be calculated connectivity of the RCS as $K_{зв}(S_i, R_j)$. By analyzing various combinations of pairs we get the matrix connectivity of RCS:

$$\{K_{зв}\}_{S\times R} = \begin{vmatrix} K_{зв}(S_1, R_1), K_{зв}(S_1, R_2), ..., K_{зв}(S_1, R_R) \\ K_{зв}(S_2, R_1), K_{зв}(S_2, R_2), ..., K_{зв}(S_2, R_R) \\ .... \\ K_{зв}(S_S, R_1), K_{зв}(S_S, R_2), ..., K_{зв}(S_S, R_R) \end{vmatrix} \quad (2)$$

According to the terminology adopted in the game theory, this will be the solution to the "numerical strategies», which clearly defines the actions of the opposing sides. However, the real matrix (2) often cannot have a saddle point, which is a decision to a "pure strategies" does not exist. In this case, the solution can be obtained in a "mixed strategy" in which each of the options of RCS - electronic intelligence and electronic warfare can attribute defined probability $\sum_{i=1}^{S} P_{iS} = 1, \ \sum_{i=1}^{R} P_{jR} = 1.$

That is proven in game theory that for final-dimensional matrix gaming solutions in mixed strategies always exists. Therefore, when a large number of options are considered, to simplify the solution can be used special techniques to reduce the dimension table.

Formation of the initial data for solving game problem

Formation of matrix connectivity $\{K_{зв}(S_j, R_i)\}_{S\times R}$ and solution of play in the net or mixed strategies where $S = \{S_j\}$ - is the set of construction strategies of RCS, $R = \{R_i\}$ is the set of electronic intelligence systems strategy.

A criterion of optimization the structure of the RCS $\max_S \min_R K_{зв}(S_i, R_j) = \min_R \max_S K_{зв}(S_i, R_j)$.

The cost of the game $\alpha \le \nu \le \beta$: where $\alpha = \max_S \min_R K_{зв}(S_i, R_l)$ and $\beta = \min_R \max_S K_{зв}(S_i, R_j)$.

if $\alpha = \beta$, then

$$\{K_{зв}\}_{S\times R} = \begin{vmatrix} K_{зв}(S_1, R_1), K_{зв}(S_1, R_2), ..., K_{зв}(S_1, R_R) \\ K_{зв}(S_2, R_1), K_{зв}(S_2, R_2), ..., K_{зв}(S_2, R_R) \\ .... \\ K_{зв}(S_S, R_1), K_{зв}(S_S, R_2), ..., K_{зв}(S_S, R_R) \end{vmatrix}. \quad (3)$$

Table 1

Theoretical Game Model of Conflict

Part/side		Variants of the enemy's actions R_R				
		R_1	R_2	..	…	R_R
Variants of RCS's structure S_S	S_1	$K_{\text{зв}}(S_1, R_1)$	$K_{\text{зв}}(S_1, R_2)$	…	…	$K_{\text{зв}}(S_1, R_R)$
	S_2	$K_{\text{зв}}(S_2, R_1)$	$K_{\text{зв}}(S_2, R_2)$	…	…	$K_{\text{зв}}(S_2, R_R)$
	…	…	…	…	…	…
	…	…	…	…	…	…
	S_S	$K_{\text{зв}}(S_S, R_1)$	$K_{\text{зв}}(S_S, R_2)$	…	…	$K_{\text{зв}}(S_S, R_R)$

For distribution $P_{iS} = (P_{1S}, P_{2S}, ..., P_{SS})$ $P_{iS} \geq 0$ to a mixed strategy $j = \overline{1, R}$ connectivity is defined as $\overline{K}_{\text{зв}} = K_{\text{зв}1j} P_{1S} + K_{\text{зв}2j} P_{1S} + ... + K_{\text{зв}\, jS} P_{SS}, \quad \forall j \in R$, where $\sum_{i=1}^{S} P_{iS} = 1$.

For distribution $P_{jR} = (P_{r1}, P_{r2}, ..., P_{SR})$ $P_{jR} \geq 0$ to a mixed strategy $i = \overline{1, S}$ connectivity is defined as $\overline{K}_{\text{зв}} = K_{\text{зв}1j} P_{r1} + K_{\text{зв}2j} P_{r2} + ... + K_{\text{зв}\, jR} P_{rR}, \quad \forall i \in S$, where $\sum_{i=1}^{R} P_{jR} = 1$.

If, $\alpha \neq \beta$ then, is formed a system of inhomogeneous linear equations with unknowns $P_{SS,}\ P_{rR,}\ \overline{K}_{\text{зв}}$:

$$\begin{cases} K_{\text{зв}1j} P_{1S} + K_{\text{зв}2j} P_{1S} + ... + K_{\text{зв}Sj} P_{SS} = \overline{K}_{\text{зв}}, & j = \overline{1, R}, \\ K_{\text{зв}i1} P_{r1} + K_{\text{зв}i2} P_{r2} + ... + K_{\text{зв}\, i\, R} P_{rR} = \overline{K}_{\text{зв}}, & i = \overline{1, S}, \\ P_{1S} + P_{2S} + ... + P_{SS} = 1, & \\ P_{r1} + P_{r2} + ... + P_{rR} = 1 & \end{cases}$$

For example, consider the game dimension of 3x4, which was replaced by the equivalent game of smaller dimension 2x2 (Fig. 1).

R / S	R_1	R_2	R_3	R_4
S_1	0,6	0,4	0,5	0,8
S_2	0,5	0,6	0,9	0,7

⇒

R / S	R_1	R_2
S_1	0,5	0,6
S_2	0,8	0,5

Fig. 1- Replacing the original game by equivalent game

Check the game (Fig. 1b) for the saddle point $\max_S \min_R K_{\text{зв}ij} = 0{,}6 \neq \min_R \max_S K_{\text{зв}ij} = 0{,}8$. That is the solution in pure strategies does not exist.

To find the optimal mixed strategies within the system of equations:

For RCS:

$$\begin{cases} 0{,}5P_{2S} + 0{,}6P_{2S} = \overline{K}_{зв} \\ 0{,}8P_{3S} + 0{,}5P_{3S} = \overline{K}_{зв} \\ P_{2S} + P_{3S} = 1 \end{cases}$$

For electronic intelligence and electronic warfare:

$$\begin{cases} 0{,}5P_{R1} + 0{,}8P_{R1} = \overline{K}_{зв} \\ 0{,}6P_{R2} + 0{,}5P_{R2} = \overline{K}_{зв} \\ P_{R1} + P_{R2} = 1 \end{cases}$$

The solution of these systems of equations will give a result P_{2S} = 0.542, P_{3S} = 0.458, P_{R1} = 0.542, P_{R2} = 0.458, $\overline{K}_{зв}$ = 0.546.

So, the rational option of communication for RCS will be an option with P_{2S} = 0.542 and P_{3S} = 0.458. The use of the first variant of organization of communication should be avoided as irrational.

For electronic intelligence and electronic warfare from the enemy's point of view the perspective will most rational P_{R1} = 0.542 and P_{R2} = 0.458. The use of the third and fourth options the opponent refuses both as the irrational.

Conclusions

The given mathematical model structure of optimization of RCS allows to determine the most optimal structure for organizing the communication system. The novelty of this methodology is the optimization in terms of connectivity, which can be considered as an integrated indicator index of the quality of RCS communication. The provided example allowed to get a corresponding numerical value by which was selected the most rational structure of the RCS, electronic radio intelligence and electronic warfare. However, it should be noted, that the final conclusion about the degree of rationality of derived structures in a battle takes the appropriate official. It can either agree with the results of the optimization or additionally formulate other options of RCS, electronic intelligence and electronic warfare and propose a new game for a new solution.

Prospects for further research

Directions for further research is considered the development of a mathematical model of evaluation connectivity between separate radio stations repeaters in conditions of dynamic changes in the RCS structure, which will allow to solve specific tactical tasks in military management and communication.

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МАТЕМАТИЧНА МОДЕЛЬ ОПТИМІЗАЦІЇ СТРУКТУРИ РУХОМОЇ СИСТЕМИ РАДІОЗВ'ЯЗКУ ЗА ПОКАЗНИКОМ ЗВ'ЯЗАНОСТІ

Д.А. Бухал, А.Ю. Коркін

У статті запропоновано математичну модель оптимізації структури рухомої системи радіозв'язку залежно від прогнозу дій противника. Оптимізація проводиться за показником зв'язаності, який відображає якість зв'язку між рухомими радіостанціями-ретрансляторами.

Ключові слова: *рухома система радіозв'язку, показник зв'язаності, радіостанції-ретранслятори, модель оптимізації.*

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ ОПТИМИЗАЦИИ ПОДВИЖНОЙ СИСТЕМЫ СВЯЗИ ПО ПОКАЗАТЕЛЮ СВЯЗНОСТИ

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В статье предложена математическая модель оптимизации структуры подвижной системы радиосвязи в зависимости от прогноза действий противника. Оптимизация проводиться по показателю связности, который отображает качество связи между подвижными радиостанциями-ретрансляторами.

Ключевые слова: *подвижная система связи, показатель связности, радиостанции-ретрансляторы, модель оптимизации.*