other hand is carried out. The calculation is performed for 25 modes. The results of the calculations which determine the dependence of turbo-mechanisms energy on technical parameters and operating conditions may ten times vary.

To define real power consumption of turbo-mechanisms we should take into account the most rational and possible modes of its operation.To calculate power intensity, defined in the normal turbo-mechanisms operating modes, we must use the minimum and maximum values of static efficiency, static pressure and air volume corresponding to the values of fan parameters performance.In this case we should perform the calculation for the 25 modes.The results of the study prove the dependence of turbo-mechanisms power of technical parameters and operating conditions which may vary by tens of times; therefore, it requires further investigation of equipment automation and devices for its saving.

air-gas flow, turbo-mechanisms, specific power supply, power system, dependence, efficiency

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Modeling of optimal automatic control of the process of biological clearing of polluted waters by fractional order regulators

The problem of modeling the control of the process of biological treatment of polluted waters using fractional $PI^{\lambda}D^{\mu}$ - regulators is considered and solved. Optimum tunings of fractional regulators are obtained, the dynamics of transient processes of control action and the state of the purification system is investigated. Numerical simulation of fractional and classical control is carried out, a higher efficiency of fractional $PI^{\lambda}D^{\mu}$ regulators is shown.

fractional calculus, differentintegrator, optimal control, numerical modeling, bio-purification of waters

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Моделирование оптимального автоматического управления процессом биологической очистки загрязненных вод регуляторами дробного порядка

Решается задача числового моделирования управления процессом биологической очистки загрязненных вод с помощью дробных $PI^{\lambda}D^{\mu}$ - регуляторов. Получены оптимальные настройки дробных регуляторов, исследована динамика переходных процессов управляющего воздействия и состояния очистной системы. Проведено численное моделирование управления дробными $PI^{\lambda}D^{\mu}$ и классическим PID - регулятором, показана более высокая эффективность дробных $PI^{\lambda}D^{\mu}$ регуляторов.

дробное исчисление, диффеинтегратор, оптимальное управление, численное моделирование, биоочистка вод

Introduction. A fractional calculation deals with derivatives and integrals of random order (rational, actual and even complex). Riman, Liouville, Grünwald, Litnik [4,5,8] began in 17 century a fractional calculation to occupy. But especially actively the theory of fractional calculation develops in recent year, and the results of her widely drawn on in the areas of research of chaotic dynamics, dynamic neural networks with fractional orders, constructing (to the synthesis) of regulators of fractional orders in the theory of automatic control and others like that.

Operator, generalizing classical differential and integral operators, called the operator type

$$ {}_aD_t^{\gamma} = \begin{cases} d^{\gamma}/dt^{\gamma}, & \gamma > 0 \\ 1, & \gamma = 0 \\ \int_a^t (d\tau)^{-\gamma}, & \gamma < 0 \end{cases}, \tag{1} $$

where γ – order fractional operator (real number);

a – a constant related to the initial conditions of the dynamic process.

Operator (1) derivativintehrator is so called because it combines two things at once – derivative and integral.

Formulation of the problem. Traditionally, the theory and practice of automatic control is focused on the use of classical differential or integral calculus, it is logical that the development of fractional calculus is needed opportunities to study the application of the laws of fractional fractional management and building controls and control systems identifying characteristics with them.

The purpose of the article. The article is not only a fractional comparison with classic controls and capabilities and efficiency of their application in automatic control, but the numerical simulation of control processes purification of contaminated water.

Presenting main material. This paper considers the problem of numerical modeling of process control biological wastewater treatment using activated sludge - regulators fractional order. Cleaning system (Fig. 1) consists of a bioreactor- aeration tank and clarifier sludge.

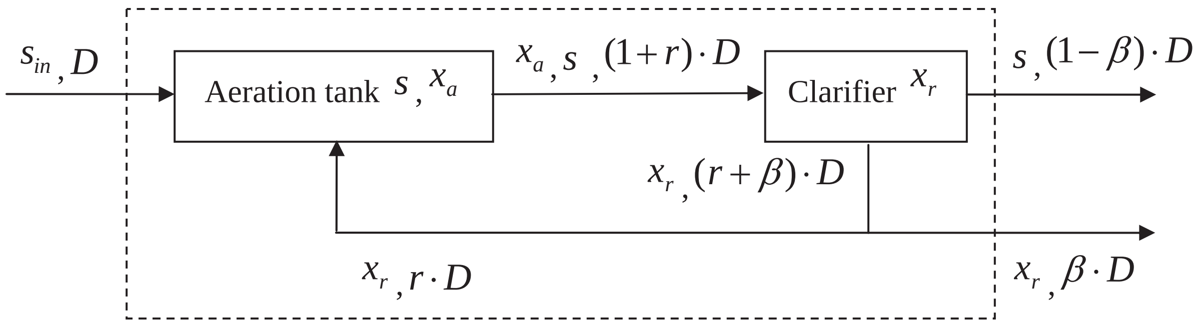

Figure 1 – Wastewater treatment process

A mathematical model describing water treatment for the scheme is obtained from materialbalance for aerator and clarifier as the following system of differential equations.

$$\begin{cases} \dfrac{dx_a(t)}{dt} = \mu(t)x_a(t) - D(t)(1+r)x_a(t) + rD(t)x_r(t), \\ \dfrac{ds(t)}{dt} = -\dfrac{\mu(t)}{Y}x_a(t) - D(t)(1+r)s(t) + D(t)s_{in}(t), \\ \dfrac{dx_r(t)}{dt} = -D(t)(\beta+r)x_r(t) + D(t)(1+r)x_a(t), \end{cases} \qquad (2)$$

where $x_a(t)$, $s(t)$ – according biomass concentration and substrate in the bioreactor;
$x_r(t)$ – recirculation concentration of biomass;
$D(t)$ – dilution, defined as $D(t) = F(t)/V$, where $F(t)$ – volumetric flow rate;
V – volume bioreactor;
$s_{in}(t)$ – substrate concentration in the input stream;
Y – factor output (yield) biomass;
$\mu(t)$ – biomass specific growth rate, which is defined by Mono [3]

$$\mu(t) = \mu_{\max}\frac{s(t)}{k_s + s(t)}, \qquad (3)$$

where μ_{max} – maximum specific growth rate biomass;
k_s – saturation constant, determined experimentally;
r, β – coefficients determined in accordance recirculating flow ratio and flow of waste biomass to the incoming flow;
x_{a0}, s_0, x_{r0} – according biomass concentration, substrate and recirculation of biomass at the initial time t_0;
$t_0 < t \le T$, T – end-time process control.

A value $s(t)$ (the concentration of the substrate in the bioreactor, which determines the quality of water) is selected as an adjustable parameter (output model). The system function dilution $D(t)$ is selected as controlling influence (action).

For the convenience the system (2) is written in vector form

$$\begin{cases} \dfrac{dx(t)}{dt} = f(x(t),u(t)), \qquad t_0 < t \le T, \\ x(t_0) = x^0, \end{cases} \qquad (4)$$

where
$$f(x(t),u(t)) = \begin{pmatrix} f_1(x(t),u(t)) \\ f_2(x(t),u(t)) \\ f_3(x(t),u(t)) \end{pmatrix}, \quad x(t) = \begin{pmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{pmatrix} = \begin{pmatrix} x_a(t) \\ s(t) \\ x_r(t) \end{pmatrix}, \quad u(t) = D(t), \qquad (5)$$

$$f_1(x(t),u(t)) = \mu(x(t))x_1(t) - (1+r)x_1(t)u(t) + rx_3(t)u(t),$$

$$f_2(x(t),u(t)) = -\frac{\mu(x(t))}{Y}x_1(t) - (1+r)x_2(t)u(t) + s_{in}(t)u(t),$$

$$f_3(x(t),u(t)) = -(\beta+r)x_3(t)u(t) + (1+r)x_1(t)u(t),$$

$$\mu(x(t)) = \mu_{\max}\frac{x_2(t)}{k_s + x_2(t)}.$$

Adjustable parameter while written as

$$s(t) = x_2(t) = c^T x(t), \tag{6}$$

where $c = \begin{pmatrix} 0 & 1 & 0 \end{pmatrix}^T$.

Transformed system (4) is linearized in the vicinity of a given nominal control u^* and corresponding vector equilibrium $x^* = (x_1^*, x_2^*, x_3^*)^T$, in which $f(x^*, u^*) = 0$ and which is the solution of systems of nonlinear equations $f(x, u^*) = 0$ on vector *x*. We introduce the notation

$$\Delta x(t) = \begin{pmatrix} \Delta x_1(t) \\ \Delta x_2(t) \\ \Delta x_3(t) \end{pmatrix} = \begin{pmatrix} x_1(t) - x_1^* \\ x_2(t) - x_2^* \\ x_3(t) - x_3^* \end{pmatrix} = x(t) - x^*, \quad \Delta u(t) = u(t) - u^*. \tag{7}$$

Then the system of equations linearized model (4) is represented as

$$\begin{cases} \dfrac{d\Delta x(t)}{dt} = A\Delta x(t) + b\Delta u(t), \\ \Delta x(t_0) = x^0 - x^*, \end{cases} \tag{8}$$

Considering the ratio (6) and symbols (7), the equation for the controlled variable (output model) can be written as

$$\Delta s(t) = c^T \Delta x(t), \tag{9}$$

where $\Delta s(t) = s(t) - c^T x^*$.

Model management (8) has one input and one output. Known methods of stabilization required parameters is to use regulators in the feedback circuit as part of an automated control system. We use fractional $PI^\lambda D^\mu$ - regulator [1,2] and compare its performance with classic *PID* - regulator.

Similar work [7, 9] $PI^\lambda D^\mu$ - regulators represented as

$$\Delta u(t) = k_P\left(\Delta s(t)\right) + k_I\left({}_{t_0}D_t^{-\lambda}\Delta s(t)\right) + k_D\left({}_{t_0}D_t^{\mu}\Delta s(t)\right), \tag{10}$$

where k_P, k_I, k_D – adjustment coefficients regulator;

${}_{t_0}D_t^{-\lambda}\Delta s(t)$ – fractional derivative order λ;

${}_{t_0}D_t^{\mu}\Delta s(t)$ – fractional integral of order μ, moreover λ, μ – arbitrary real number in the interval, ie $\lambda, \mu \in (0,2)$. If $\lambda \ge 2$ or $\mu \ge 2$, then $PI^\lambda D^\mu$ - regulator takes high order, and structure it differs from the classical *PID* - regulator. The controller (10) is a generalized fractional - regulator. At $\lambda=1$ and $\mu=1$ are classic *PID* - regulator, if $\lambda=1$, $\mu=0$, we get *PI* - regulator, if $\lambda=0$, $\mu=1$ have *PD* - control and in $\lambda=0$, $\mu=0$ – *P* - regulator. These types of classic *PID* - regulators are special cases of fractional $PI^\lambda D^\mu$ - regulator (10). However $PI^\lambda D^\mu$ - regulator is more flexible and has the ability to better regulate (adjust) the dynamic properties of control systems. On $P-I-D$ - plane, this means that instead of "hops" between four fixed points (*P*, *PI*, *PD i PID* (Fig. 2)) the plane is the possibility of continuous movement ($PI^\lambda D^\mu$) between them.

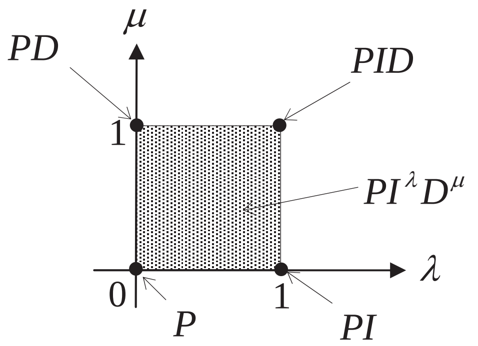

Figure 2 – $P-I-D$ - plane order fractional derivatives and integrals

Fractional derivatives and integrals defined as the limit

$$ {}_{t_0}D_t^{\alpha} f(t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\left[\frac{t-t_0}{h}\right]} (-1)^j \binom{\alpha}{j} f(t - jh), \tag{11} $$

where $\binom{\alpha}{j}$ – binomial coefficients form $\binom{\alpha}{j} = \dfrac{\Gamma(\alpha+1)}{\Gamma(j+1)\cdot\Gamma(\alpha-j+1)}$, in which $\Gamma(x)$ – Euler gamma function view $\Gamma(x) = \int\limits_0^{+\infty} e^{-y} y^{x-1} dy$ (recall that in general, $x = k$ it is $\Gamma(k+1) = k!$);

$[\cdot]$ – floor and ceiling functions;

$h > 0$ – increase temporal coordinates (quantization).

If $\alpha > 0$, then correlation (11) defines a fractional derivative, if $\alpha < 0$, then – fractional integral. Therefore the relation (11) is also often called derivativintehrator as equation (1). Note also that in the entire orders α in (11) will end amount, while fractional α – an infinite number of members of the series.

Considering (9), $PI^{\lambda}D^{\mu}$ - regulator (10) is written as the operator of the state $\Delta x(t)$

$$ \Delta u(t) = c^T \left(k_P \left(\Delta x(t) \right) + k_I \left({}_{t_0}D_t^{-\lambda} \Delta x(t) \right) + k_D \left({}_{t_0}D_t^{\mu} \Delta x(t) \right) \right). \tag{12} $$

and criterion as automatic control system functioning biological treatment –

$$ J_p = \int\limits_{t_0}^{T} \left| \Delta s(t) \right|^p dt = \int\limits_{t_0}^{T} \left| c^T \Delta x(t) \right|^p dt, \tag{13} $$

where $p > 0$ – option, which in practice is considered equal p =1 (module error) or p = 2 (Mean square error).

For the implementation of this numerical problem of optimal regulation do dedicated system (8), fractional $PI^{\lambda}D^{\mu}$ - regulator (12) and criterion (13), breaking time interval $[t_0, T]$ on n parts of step $h = (T - t_0)/n$ (h–during quantization). The points breakdown in $[t_0, T]$ denote t_k, and the state of the system (8) in these times t_k – as $z_k = \Delta x(t_k)$.

Approximate continuous input $\Delta u(t)$ piecewise constant function: $\Delta u(t) = u_k$ at $t_k \le t < t_{k+1}$, $k = 0,1,2,...,n$, using a matrix of linear continuous system (8) and obtain its next discrete analog

$$\begin{cases} z_{k+1} = e^{Ah} z_k + A^{-1}(E - e^{-Ah}) b u_k, & k = 0,1,2,...,n-1, \\ z_0 = x^0 - x^*, \end{cases} \tag{14}$$

where E – identity matrix, e^{Ah} – matrix exhibitor.
Next discrete fractional $PI^{\lambda}D^{\mu}$ - regulator is represented as

$$u_k = c^T \left(k_P(z_k) + k_I \left(h^{\lambda} \sum_{j=0}^{k} w_j^{(-\lambda)} z_{k-j} \right) + k_D \left(h^{-\mu} \sum_{j=0}^{k} w_j^{(\mu)} z_{k-j} \right) \right). \tag{15}$$

Note that when k = 0, then a control signal is

$$u_0 = \left(k_P + k_I h^{\lambda} + k_D h^{-\mu} \right) c^T z_0 . \tag{16}$$

Quality criterion (13) is written in discrete form

$$J_p = \frac{h}{2} \left(\left| c^T z_0 \right|^p + 2 \sum_{j=1}^{n-1} \left| c^T z_j \right|^p + \left| c^T z_n \right|^p \right). \tag{17}$$

Numerical simulation of control system of biological treatment search for the optimal regulator conducted at the following initial data: $s_{in} = 200$ [mg / l], $Y = 0.65$, $\mu_{\max} = 0.15$ [h^{-1}], $k_s = 100$ [mg / l], $r = 0.6$, $\beta = 0.2$, $u^* = 0.05$ [h^{-1}], $t_0 = 0$, $T = 1$ [h], вектор початкового стану системи (8) приймався рівним $x^0 = (x_1^0, x_2^0, x_3^0)^T = (286,\ 17,\ 568)^T$ [mg / l].

The method of exhaustive search with a uniform step to solving the problem of minimizing the criterion relative

$$I_p(\lambda, \mu) = \min_{k_P, k_I, k_D} J_p(k_P, k_I, k_D, \lambda, \mu) \tag{18}$$

parameters λ and μ. The results of the optimization method of exhaustive search criteria (18) are shown in the table 1.

Table 1 – Best shot settings fractional $PI^{\lambda}D^{\mu}$ - and classical PID -regulators

p	λ	μ	k_P	k_I	k_D	$I_p(\lambda,\mu)$
1	1	1	–0.1381	–3.3019	–0.0016	0.0963
1	0.9750	0.750	–0.2231	0.0072	$-1.0847 \cdot 10^{-5}$	0.0854
2	1	1	–0.1294	–3.6445	– 0.0015	0.1184
2	0.9875	0.600	–0.2234	0.0068	$-3.1249 \cdot 10^{-6}$	0.0855

In the pages λ and μ orders are fractional derivatives and integrals regulators, in pages k_P, k_I, k_D – optimal settings of these controls in the last column - the minimum value of the criterion (18). Here are the results for comparison to classical optimization *PID* - regulator with $\lambda = 1$ and $\mu = 1$.

The results show that the objective function value $I_p(\lambda,\mu)$ ($p=1, p=2$) the optimal fractional $PI^{\lambda}D^{\mu}$ - regulator less than the classic *PID* - regulator.

To study surface quality criterion $I_p(\lambda,\mu)$ Fig. 3 shows a graph of the criterion of fractional order derivatives (μ) and integrals (λ), used in fractional $PI^{\lambda}D^{\mu}$ - law regulation (15).

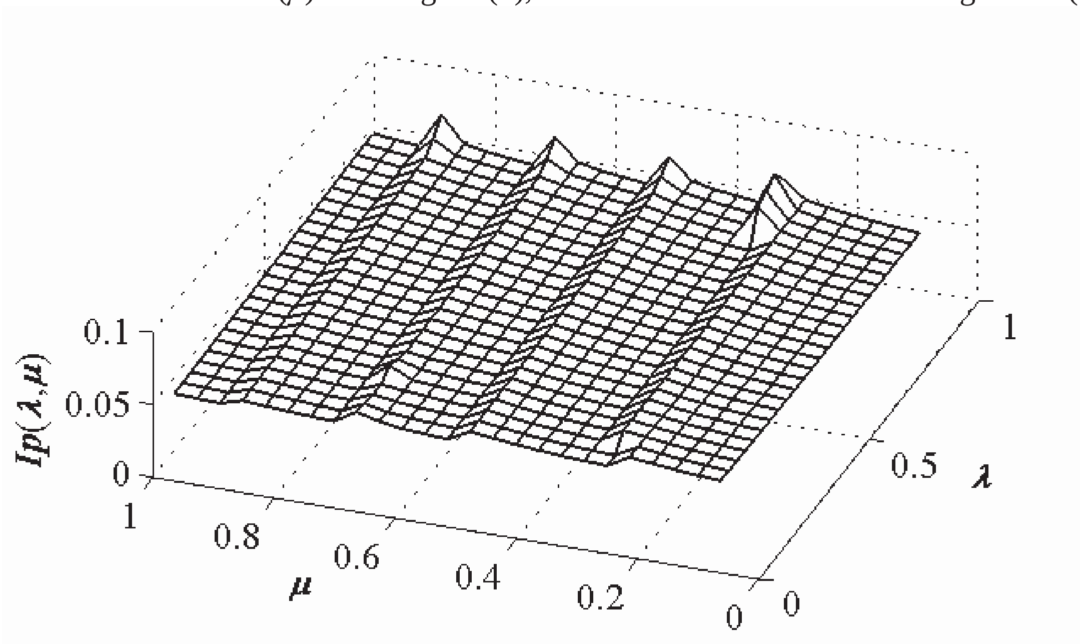

Figure 3 – Schedule surface of objective function $I_p(\lambda,\mu)$

With a package system MATLAB Optimization Toolbox following results were obtained. Figure 4 graphs optimum control functions (dilution rate of fluid flow) of water in biological purification classic *PID* - i $PI^{\lambda}D^{\mu}$ - fractional regulators (15).

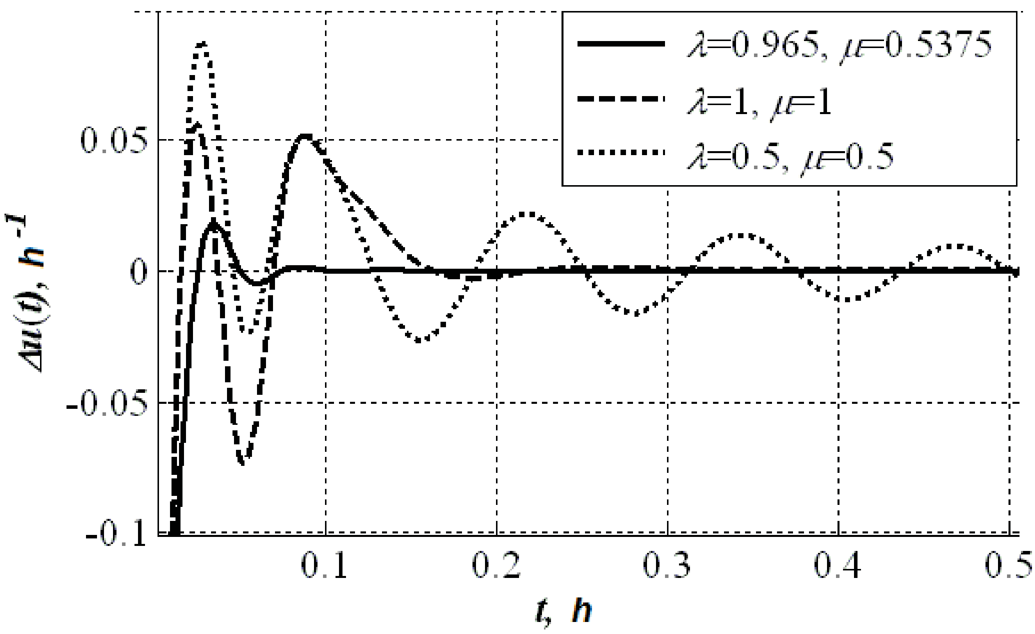

Figure 4 – Dynamics optimal fractional $PI^{\lambda}D^{\mu}$ - controllers (speed dilution fluid flow) and classic *PID*-regulator ($\lambda=1, \mu=1$)

Figure 5 shows respective optimal transient (changing substrate concentration) of the system by criterion $J_2(k_P, k_I, k_D, \lambda, \mu)$.

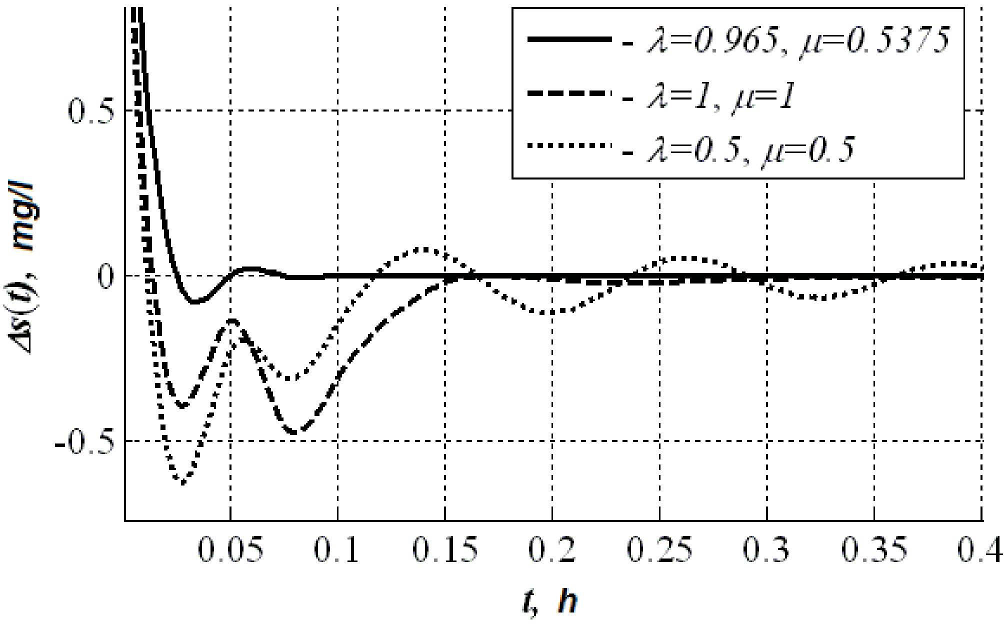

Figure 5 – Optimal transient changes in substrate concentration at different fractional $PI^{\lambda}D^{\mu}$ - and classic PID - regulators

Comparative analysis of transient dynamics shows more speed and quality with optimal damping fractional $PI^{\lambda}D^{\mu}$ - regulator ($\lambda = 0.965, \mu = 0.5375$) compared to the best classic PID - regulator ($\lambda = 1, \mu = 1$). It is seen that the optimal fractional controllers with accurate configuration settings λ (fractional order integral) and μ (fractional order derivative) are more efficient compared to classical PID - regulator.

Conclusions. The degree of fractional efficiency regulators and causes high sensitivity optimality criterion and transients on the order of fractional derivatives and integrals require further research.

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Моделювання оптимального автоматичного керування процесом біологічного очищення забруднених вод регуляторами дробового порядку

Розглядається і розв'язується задача оптимального керування процесом біологічного очищення забруднених вод за допомогою дробових $PI^{\lambda}D^{\mu}$ - регуляторів. Пропонується математичне моделювання процесу біологічного очищення як об'єкта керування, виводиться нелінійна динамічна модель керування та проводиться її лінеаризація. Модель керування має один вхід та один вихід. Вводиться до розгляду оптимальний критерій якості автоматичного керування за допомогою дробового регулятора функціонування біологічної системи очищення води. Отримані оптимальні параметри налаштування дробових $PI^{\lambda}D^{\mu}$ - регуляторів. Досліджена динаміка перехідних процесів керувального впливу і стану системи очищення.

Чисельне моделювання дробового $PI^{\lambda}D^{\mu}$ - і класичного *PID* - керування проведене для підтвердження більш високої ефективності дробових регуляторів, що відображено в результатах досліджень.
дробове числення, диферінтегратор, оптимальне керування, чисельне моделювання, біоочищення вод

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Шляхи модернізації систем автоматики холодильного устаткування з одним терморегулювальним вентилем

Стаття присвячена аналізу тенденцій підвищення ефективності холодильного устаткування за рахунок модернізації системи керування роботою терморегулювального вентиля. Показано, що холодильне обладнання фірми Danfoss підвищує ефективність утворення холоду за рахунок зміни уставки перегріву випарника. Головна ідея модернізації полягає у застосуванні системного підходу до розгляду холодильного устаткування у комплексі з холодильною камерою та продуктами, які зберігаються у ній. Для реалізації зазначеної ідеї у статті розроблена нова структурна схема системи охолодження продуктів як багатовимірної системи слідкування, яка функціонує в умовах зміни температури у холодильній камері, коливаннях тиску та температури хладагенту.
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