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EQUATIONS OF PERIODIC MODES, WHICH TAKE INTO ACCOUNT FEATURES OF THE DYNAMICS OF THEIR COURSE IN NONLINEAR AUTOMATIC SYSTEMS WITH COMPUTERS IN CONTROL SYSTEM

In the article, based on the application of a continuous-discrete approach to the description of periodic modes that are possible in automatic systems with a control computer, equations are obtained that take into account the peculiarities of the dynamics of their flow in nonlinear systems of the noted class and provide an increase in the accuracy of calculating their parameters. This is achieved by taking into account the specifics of the structural diagram of the system under study by replacing the NOT system with harmonic linearization coefficients, which have correspondingly different formula expressions. The use of the proposed equations will make it possible to use the investigated modes more efficiently during the functioning of the system, or vice versa, it is guaranteed to get rid of them.

Keywords: *harmonic linearization, impulse element, nonlinear element, computer, continuous-discrete system.*

Introduction

General Problem Statement. In modern conditions, the problem of increasing the combat capability and combat readiness of the Armed Forces of Ukraine acquires a defining status. This fully applies to the anti-aircraft missile forces (AMF) of the Air Force of the Armed Forces of Ukraine. Given the obvious trend of outpacing the growth of inconspicuous and vulnerable to anti-aircraft missile systems (AMS) manned and unmanned aerial vehicles and their share in the total arsenal of air attack potential enemy [1–3], we can conclude that effective measures are needed to increase combat capabilities AM, which are in service with the AMF of the Air Force of the Armed Forces of Ukraine [4–6].

The needs of the troops constantly dictate the need to increase accuracy, noise protection, adaptability to firing conditions and the overall efficiency of anti-aircraft guided missiles (AAR) [3; 7–8].

The end result of applying AAR against an air target depends on the quality of in-flight operation of all of its major subsystems. Significant results of improving the tactical and technical characteristics (TTC) of AAR, and ie increasing the efficiency of AMS in general, can be achieved through the use of digital PC as part of its onboard equipment. The use of PC as part of the onboard equipment AAR allows you to implement complex algorithms for information processing, significantly increase the noise immunity and accuracy of aiming the missile at the target [9–10]. With the use of PC can be optimized and the process of controlling the operation of AAR combat equipment, and ie the whole process of destruction of the air target. On the basis of PC it is possible to carry out complex digital algorithmizing and optimization of process of functioning of the onboard equipment AAR and as a result – to increase efficiency of application of AAR [11–13].

Analysis of recent research and publications. The inclusion of PC in the on-board equipment AAR opens wide prospects for improving the TTC missile, but the presence of PC control systems can lead to some negative phenomena [4; 11; 14]. For example, in such systems, uncontrolled oscillatory processes can occur unforeseen, which can nullify the process of aiming the AAR at the target [15]. To prevent these phenomena, such systems need to be studied in more detail.

A comprehensive analysis of AAR control systems with PC as an object of study suggests that they belong to the class of nonlinear continuous-discrete automatic control systems (NCDS) with extremely complex dynamic properties [16–17]. A characteristic feature of such systems is the presence of interruption of the signal operating in the system, at one or more points of the circuit while maintaining the continuity of the output signal of the system. The dynamics of functioning of such systems is described by a set of differential and difference equations, some of which may be nonlinear or time-varying coefficients [11–13]. There are no exact methods for solving such equations.

When studying and calculating AAR control systems with PC in the frequency domain, it is advisable to use both the mathematical apparatus of continuous and discrete Laplace transform, which allows to take into account the peculiarities of processes in automatic systems of continuous discrete class [14]. This theory is developed only for linear NCDSs. For the analysis and synthesis of nonlinear NCDS linear model mathematical

description of the processes occurring in the system is not enough. Linear theory does not take into account the influence of non-system on the dynamics of its functioning. To identify the conditions for the emergence of involuntary periodic regimes in nonlinear continuous-discrete automatic control systems, it is advisable to use the method of harmonic linearization [11; 14].

Aim of the Research. In this regard, the aim of the article is to obtain equations that use the method of harmonic balance to describe the dynamics of spontaneous periodic regimes in nonlinear automatic systems with PC in the control loop. To achieve this goal it is necessary to solve the following tasks:

– substantiate the procedure for harmonic linearization of the NOT system, taking into account its placement in the structural scheme of the system relative to the IE and the linear part of the system, as well as the presence or absence of inertial properties in the NOT itself;

– to obtain mathematical (formulaic) dependences for the coefficients of harmonic linearization NOT of all possible variants of structural construction of the system;

– to obtain equations, which using the corresponding coefficients of harmonic linearization NOT describe involuntary periodic regimes in nonlinear automatic systems with PC in the control loop, taking into account the specifics of the structural construction of the system.

This will allow you to use the positive possibilities of the theory of linear NCDS to more accurately describe the process of operation of the control system with a PC, and ie a higher level of achieving the desired end result.

Statement of basic materials

Research the method of harmonic linearization. The method of harmonic linearization for solving automatic control problems was first applied and further developed in detail for nonlinear continuous automatic control systems [14–16]. In accordance with the fact that discrete modes of operation are increasingly used in automatic systems, there have been attempts to apply this method for the study of periodic processes in nonlinear pulsed automatic control systems [16–17]. However, significant results in the application of the method of harmonic linearization to pulse systems were obtained only after the development of the mathematical apparatus of discrete Laplace transform [11–17]. In these works, the authors propose to perform harmonic linearization of inertial NOT under the assumption that the lattice function of the signal at the output of the linear continuous part of the system varies according to the sinusoidal law, while the signal itself may differ significantly from sinusoidal. Nomination of such requirements to the filtering properties of the linear continuous part of the system is explained by the fact that the mathematical description of the processes occurring in the pulsed system using Laplace discrete transformation involves consideration of the original coordinate of the system only at discrete moments

$$t = nT + \varepsilon T, \quad 0 \le \varepsilon \le 1,$$

rigidly related to the n - moments of closure of the IE system. This in turn corresponds to the introduction into the block diagram of a nonlinear pulse system at its output "imaginary" IE (Fig. 1, a).

Reaction NOT $y = F(x)$ on a sinusoidal lattice $x(nT) = A\sin(\omega_1 nT + \varphi)$ the function is also a lattice periodic function, but the curve that encircles it does not correspond to the harmonic law of process change. Such a periodic lattice function, in contrast to a continuous one, can be represented not by a Fourier series but by a trigonometric polynomial with a finite number of members.

$$y(nT) = \sum_{v=0}^{N} \left(a_y \cos v_n \frac{\pi}{N} + b_y \sin v_n \frac{\pi}{N} \right), \quad (1)$$

where

$$N = \begin{cases} \dfrac{M}{2} paredM; & M = \dfrac{\omega}{\omega_1} = \dfrac{T_1}{T} \ge 2; \\ \dfrac{M-1}{2} nonparedM; & M - relative\ period. \end{cases}$$

The coefficients of this polynomial are determined by formulas

$$\alpha_0 = \frac{1}{M} \sum_{k=0}^{M-1} F(x) \quad a_y = \frac{2}{M} \sum_{k=0}^{M-1} F(x) \cos \nu_k \frac{2\pi}{M}, \quad (2)$$

at $\nu = 1, 2, \ldots, N-1$;

$$\alpha_y = \frac{1}{M} \sum_{k=0}^{M-1} F(x) \cos k\pi \quad b_y = \frac{2}{M} \sum_{k=0}^{M-1} F(x) \sin \nu_k \frac{2\pi}{M}$$

at $\nu = N, N+1, \ldots, M$, if $M = 2N + 1$.

The introduction of additional IE allows formally in all cases linearization of NOT system to carry out under the above assumption of the harmonious nature of the lattice function of the input signal NOT, but on the other hand the introduction of "imaginary" IE corresponds to simplification of mathematical description system properties. The loss of information about the dynamic properties of the system is due to the neglect of the continuous-discrete nature of processes in automatic control systems in the transition from continuous description of the process at the output of the system to discrete, ie at the time of introduction of "imaginary" key. The introduction of IE at the output of the system corresponds to the transfer of the system of differential and difference equations to the system of difference equations. As shown in [14], even for linear systems, such a replacement makes it impossible to study some features of the dynamics of NCDS. For a system of nonlinear equations, this remark becomes even more important, because the accuracy of the description of the

input signal NOT depends on the result of linearization. In this regard, the authors of this article in [11] aimed to consider the features of the harmonic linearization of a nonlinear element in the control systems of aircraft with a PC in the control circuit. Thus, it was proved that in linearization it is necessary to consider in interaction both the properties inherent in the continuous part of the system (nonlinearity, nonstationary, many modes, etc.) and the properties caused by the sampling of time control processes occurring in digital PC. It was assumed that the effect of level quantization is instantly taken into account when forming the amplitude of the signal during its sampling in the PC over time. That is, the onboard digital PC is fed in the block diagram of the system only by a pulse element, taking into account only the nonlinearity that is part of the continuous part of the system. It is also shown [11] that when conducting harmonic linearization of NOT in NCDS it is necessary to take into account its placement in the structural scheme of the system relative to IE and the linear part of the system, as well as the presence or absence of inertial properties in NE itself. That is, the application of the method of harmonic linearization to nonlinear NCDS has significant features. They are that, depending on the type of structural scheme of the system (Fig. 1, a, b, c, d), the harmonic linearization of NOT should be carried out by replacing it with coefficients having different formulaic expressions.

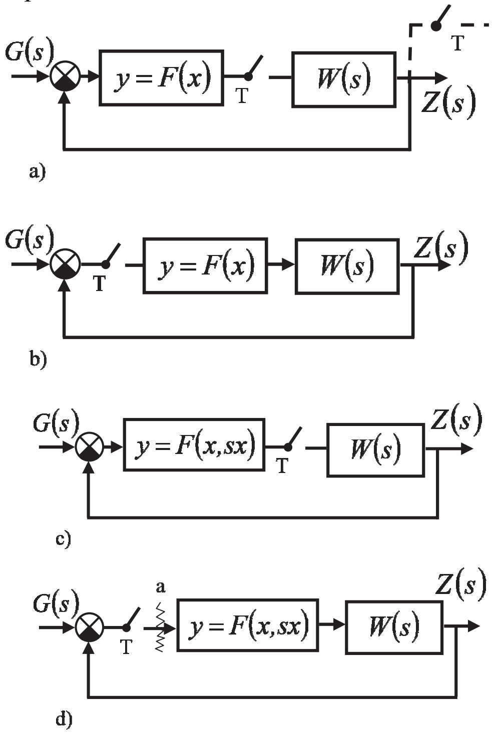


Fig. 1. Possible variants of block diagrams of single-circuit NCDS with one pulse and one nonlinear element

Among all the variety of single-circuit structural circuits of nonlinear NCDS automatic control, containing one NOT and one IE without extrapolator, there are four variants of circuits that differ in the nature of the signal change (continuous, discrete) at the input NOT (Fig. 1, a, b, c, d). The signal at the NOT input belonging to the NCDS will change discretely or continuously depending on the location of the IE in the block diagram of the system. Therefore, the signal at the input NOT when replacing it with an equivalent linear one should be considered as a sinusoidal lattice function only if the IE is located immediately in front of the nonlinear one. This case is illustrated by structural diagrams (Fig. 1, b, d). The nature of the change in the NOT output signal depends on the nature of the change in its input signal, as well as the presence or absence of the inertia property in the NOT system. Thus, in three of the four possible typical block diagrams (Fig. 1, a, c, d) the output signal NOT should be considered as a continuous function of time.

For the correct formation of the coefficients of harmonic linearization it is necessary to determine the expression for the first harmonic of the output signal NOT by representing this signal:

decomposition into a Fourier series, if $y = F(t)$;

trigonometric polynomial, if $y = F(nT)$.

With harmonic linearization NOT $y = F(x)$, belonging to the structural scheme (Fig. 1, a), the harmonic linearization coefficients and the equivalent complex gain NOT are calculated by the formulas [11]:

$$q(A) = \frac{1}{\pi A}\int_0^{2\pi} F(A\sin\omega t)\sin\omega t d\omega t; \quad (3)$$

$$q'(A) = \frac{1}{\pi A}\int_0^{2\pi} F(A\sin\omega t)\cos\omega t d\omega t; \quad (4)$$

$$W(A) = \sqrt{q^2\left(A + q'^2(A)\right)}\, e^{jarctg\frac{q'(A)}{q(A)}}. \quad (5)$$

These harmonic linearization coefficients are functions only of the amplitude of the input signal NOT.

In the block diagram of NCDS automatic control (Fig. 1, b), in which the IE immediately precedes the nonlinear with static characteristics $y = F(x)$, it is necessary to consider the harmonic linearization coefficients and the equivalent complex gain NOT, calculated by the formulas [11]:

$$q(A,N,\varphi) = \frac{2}{NA}\sum_{n=1}^{N-1} F\left[A\sin\left(n\frac{\pi}{N}+\varphi\right)\right]\sin n\frac{\pi}{N}; \quad (6)$$

$$q'(A,N,\varphi) = \frac{2}{NA}\sum_{n=1}^{N-1} F\left[A\sin\left(n\frac{\pi}{N}+\varphi\right)\right]\cos n\frac{\pi}{N}; \quad (7)$$

$$I^*(A,N,\varphi) = \frac{2}{NA}\sum_{n=1}^{N-1} F\left[A\sin\left(n\frac{\pi}{N}+\varphi\right)\right] e^{-j\left(\frac{\pi}{N}n+\varphi\right)}. \quad (8)$$

The application of these coefficients in this case correctly reflects the discrete nature of the processes at

the input and output of nonlinearity.

Harmonic linearization NOT $y = F(x, sx)$, included in the NCDS with a block diagram (Fig. 1, c), should be carried out using the usual coefficients of harmonic linearization

$$q(A,\omega) = \frac{1}{\pi A}\int_0^{2\pi} F(A\sin\omega t, A\omega\cos\omega t)\sin\omega t d\omega t; \quad (9)$$

$$q'(A,\omega) = \frac{1}{\pi A}\int_0^{2\pi} F(A\sin\omega t, A\omega\cos\omega t)\cos\omega t d\omega t. \quad (10)$$

The equivalent complex transfer factor NOT is determined by the expression

$$W(A, j\omega) = \sqrt{q^2(A,\omega) + q'^2(A,\omega)}\, e^{jarctg\frac{q'(A,\omega)}{q(A,\omega)}}. \quad (11)$$

Coefficients of harmonic linearization NOT, described by the same equation $y = F(x, sx)$, but which is included in the structural scheme (Fig. 1, d), are in this case function of the amplitude A, frequencies ω input signal NOT, as well as the relative half-life N [11]:

$$q(A,\omega,N,\varphi) =$$
$$= \frac{1}{\pi A}\int_0^{2\pi} F\left[A\sin\left(n\frac{\pi}{N}+\varphi\right), A\omega\cos\left(n\frac{\pi}{N}+\varphi\right)\right]\sin\omega t d\omega t; \quad (12)$$

$$q'(A,\omega,N,\varphi) =$$
$$= \frac{1}{\pi A}\int_0^{2\pi} F\left[A\sin\left(n\frac{\pi}{N}+\varphi\right), A\omega\cos\left(n\frac{\pi}{N}+\varphi\right)\right]\cos\omega t d\omega t; \quad (13)$$

$$I(A, j\omega, N, \varphi) =$$
$$= \sqrt{q^2(A,\omega,N,\varphi) + q'^2(A,\omega,N,\varphi)}\, e^{jarctg\frac{q'(A,\omega,N,\varphi)}{q(A,\omega,N,\varphi)}}. \quad (14)$$

We obtain the equations of involuntary periodic processes that occur in nonlinear NCDSs, the structural schemes of which are similar to those shown in Fig. 1 year A feature of the considered block diagram is the discreteness of the input signal NOT of the system and the dependence of the output signal NOT on the derived signal acting on its input. By replacing the NOT system with a corresponding equivalent complex transfer coefficient (14) and considering the processes in it with respect to Laplace images, NOT and the linear part of the system can be characterized by one transfer function [11]:

$$W(A,s,N,\varphi) = I(A,s,N,\varphi)W(s). \quad (15)$$

Transfer function $W(A,s,N,\varphi)$ is the product of the transfer coefficient NOT after linearization and the transfer function of the linear part of the system.

Based on the definition of the inverse D-transformation, we write the expression for the Laplace image of the original coordinate of the system

$$Z(A,s,N,\varphi) = \int_0^1 Z^*(A,s,N,\varphi,\varepsilon)e^{-sT\varepsilon}d\varepsilon =$$

$$= \int_0^1 G^*(s)\frac{W^*(A,s,N,\varphi,\varepsilon)}{1+W^*(A,s,N,\varphi)}e^{-sT\varepsilon}d\varepsilon =$$

$$= \frac{W(A,s,N,\varphi)}{1+W^*(A,s,N,\varphi)}G^*(s) =$$

$$= \Phi(A,s,N,\varphi)G^*(s). \quad (16)$$

From (16) it follows that the transfer function of a closed linearized NCDS, which is due to the ratio of the Laplace transform of the output coordinate of the system to the discrete Laplace transform of its input signal, is calculated by the formula

$$\Phi(A,s,N,\varphi) = \frac{Z(A,s,N,\varphi)}{G^*(s)} = \frac{W(A,s,N,\varphi)}{1+W^*(A,s,N,\varphi)}. \quad (17)$$

Expression (16) can be put in accordance with the block diagram shown in Fig. 2, a. From the analysis of this figure, it follows that as a result of the structural transformation it was possible to make IE from the closed circuit of the system. Therefore, the closed part of the block diagram shown in Fig. 2, b, can be considered as a continuous system, the transfer function of the open circuit which has the form

$$W_1(A,s,N,\varphi) = \frac{W(A,s,N,\varphi)}{W^*(A,s,N,\varphi) - W(A,s,N,\varphi) + 1}. \quad (18)$$

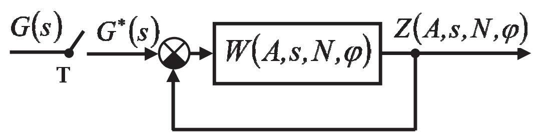
Spontaneous periodic processes that can occur in a closed loop of the considered system can be characterized by the equation

$$\frac{W(A,s,N,\varphi)}{W^*(A,s,N,\varphi) - W(A,s,N,\varphi) + 1} = -1. \quad (19)$$

Given (15), equation (19) can be reduced to the form

$$\{I(A,s,N,\varphi)W(s)\}^* = -1. \quad (20)$$

a)



b)

Fig. 2. Equivalent representation of the structural diagram of the system shown in Fig. 1

Equation (20), which determines the parameters of involuntary periodic motions in a nonlinear NCDS with a block diagram (Fig. 1, d), can be obtained without transforming the original block diagram. Indeed, by opening the circuit of the system at point "a" and using the definitions of the transfer functions of the continu-

ous and discrete parts of the automatic control system, the expression for the signal at the output of the IE can be written as

$$X^*(s) = G^*(s) - X_1^*(s)\{I(A,s,N,\varphi)W(s)\}^* .$$

Closing the circuit at the breakpoint, we obtain $X^*(s) = X_1^*(s)$ and, therefore,

$$X^*(s) = \frac{G^*(s)}{1+\{I(A,s,N,\varphi)W(s)\}^*} .$$

The expression for determining the signal at the output of the system will accordingly look like

$$Z(A,s,N,\varphi) = \frac{I(A,s,N,\varphi)W(s)}{1+\{I(A,s,N,\varphi)W(s)\}^*} G^*(s) =$$
$$= \Phi(A,s,N,\varphi)G^*(s),$$

where $\Phi(A,s,N,\varphi)$ – transfer function of closed linearized NCDS automatic control.

When finding involuntary periodic processes, the input signal of the system is assumed, therefore

$$\{I(A,s,N,\varphi)W(s)\}^* + 1 = 0 . \tag{21}$$

Equation (21) determines the conditions for the linearized nonlinear NCDS at the stability limit, completely coincides with equation (20) and, therefore, determines the parameters of possible involuntary periodic processes in the system. Since both functions under the sign of the discrete Laplace transform operation depend on the frequency, condition (21) can be rewritten as [17] on the basis of the D-transformation theorem on the multiplication of images of two functions.

$$\int_0^1 I^*(A, j\omega, N, \varphi, \varepsilon-\lambda) W^*(j\omega,\lambda) d\lambda = -1 . \tag{22}$$

In expressions (21) and (22):

$W^*(j\omega,\lambda)$, $W^*(j\omega,\varepsilon-\lambda)$ – offset discrete frequency characteristics of the linear part of the nonlinear NCDS;

$I^*(A, j\omega, N,\lambda)$, $I^*(A, j\omega, N, \varphi, \varepsilon-\lambda)$ – offset discrete amplitude-phase frequency characteristics of a harmonically linearized NOT system;

ε і λ – real numbers ranging from 0 to 1.

It is believed that the frequency of periodic regimes that occur in nonlinear pulse systems is strictly related to the frequency of IE short circuit and is in an integer ratio with it [14; 17]. However, the authors of some works do not deny the possibility of complex periodic processes in the systems of this class, the repetition periods of which are not multiples of the quantization period of the IE system [4; 15–17]. The above approach to the definition of periodic processes also confirms the theoretical possibility of such involuntary modes of operation of the system. This is evidenced by equations (21) and (22). Equations of periodic processes in notation (21) and (22) cover all the variety of close to harmonic "self-oscillating" processes that can occur in nonlinear NCDS with the considered structural scheme. However, given the great difficulties of practical use of these equations to calculate the parameters of periodic processes, the manifestation of nonlinear pulsed systems of resonant properties should be investigated only for frequencies $\omega = \frac{\pi}{T}$, if $N = 1,2,3,\ldots$. In this case, due to the equality of zero parameters ε and λ the equations of periodic regimes are significantly simplified and take the form

$$W^*\left(j\frac{\pi}{N}\right) = -\frac{1}{I^*\left(A, j\frac{\pi}{N}, N, \varphi\right)} . \tag{23}$$

Equation (23) in contrast to (21) and (22) characterizes only simple involuntary periodic regimes, the repetition period of which is a multiple of the closure period of the IE system $(T_1 = 2T \cdot N)$. However, to determine the parameters of these modes, equation (23) allows us to propose a relatively simple graph-analytical method [11; 15; 17].

To derive the equations of periodic processes of nonlinear NCDS, in the block diagrams of which the pulse and nonlinear elements are separated by a linear inertial link, and are NOT described by the equation $y = F(x, sx)$, harmonic linearization does NOT need to be performed by replacing it with an equivalent complex transfer factor calculated by formula (11).

By reasoning similar to the above, we can show that the equation of periodic processes for nonlinear NCDS with a block diagram (Fig. 1, c) has the form

$$\{W(A,s)W(s)\}^* = -1 . \tag{24}$$

Expanding the symbol of the conversion operation, equation (24) can be represented as each of the following records

$$\int_0^1 W^*(A, j\omega, \varepsilon-\lambda) W^*(j\omega,\varepsilon) d\varepsilon = -1; \tag{25}$$

$$\int_0^1 W^*(A, j\omega, \lambda) W^*(j\omega,\varepsilon-\lambda) d\varepsilon = -1. \tag{26}$$

According to this equation for determining periodic processes with a repetition period, a multiple of the IE closure period of a nonlinear NCDS with the considered block diagram, will look like

$$W^*\left(j\frac{\pi}{N}\right) = \frac{1}{W^*\left(A, j\frac{\pi}{N}\right)} . \tag{27}$$

It is not necessary to give a separate derivation of the equations of periodic processes for nonlinear NCDSs with structural schemes (Fig. 1, a, b), because these structural schemes differ from structural schemes

(Fig. 1, c, d) only by the properties NOT that are part of them. Therefore, the equations of periodic processes for automatic systems with structural schemes (Fig. 1, a, b) will differ from equations (20), (21) only by expressions for equivalent complex transfer coefficients NOT.

$$\left\{I^*\left(A,N,\varphi\right)W\left(j\omega\right)\right\}^* = -1\,; \qquad (28)$$

$$\left\{W^*\left(A\right)W\left(j\omega\right)\right\}^* = -1\,. \qquad (29)$$

Based on the transformation multiplication theorem and the linearity theorem, equations (28), (29) to determine the most probable involuntary periodic processes should be used as

$$W^*\left(j\frac{\pi}{N}\right) = -\frac{1}{I^*\left(A,N,\varphi\right)}\,; \qquad (30)$$

$$W^*\left(j\frac{\pi}{N}\right) = -\frac{1}{W\left(A\right)}\,. \qquad (31)$$

If a nonlinear NCDS automatic control contains several IE, for example two, it can be shown that depending on the location of the pulse elements in the block diagram of the system (Fig. 3, a, b, c, d) and the properties of its NOT equations of periodic modes will look like

$$\left\{I\left(A,j\omega,N,\varphi\right)\,W_1^*\left(j\omega\right)\,W_2\left(j\omega\right)\right\}^* = -1\,; \qquad (32)$$

$$\left\{W\left(A,j\omega\right)\,W_1^*\left(j\omega\right)\,W_2\left(j\omega\right)\right\}^* = -1\,; \qquad (33)$$

$$\left\{I^*\left(A,N,\varphi\right)\,W_1^*\left(j\omega\right)\,W_2\left(j\omega\right)\right\}^* = -1\,; \qquad (34)$$

$$\left\{W\left(A\right)\,W_1^*\left(j\omega\right)\,W_2\left(j\omega\right)\right\}^* = -1\,. \qquad (35)$$

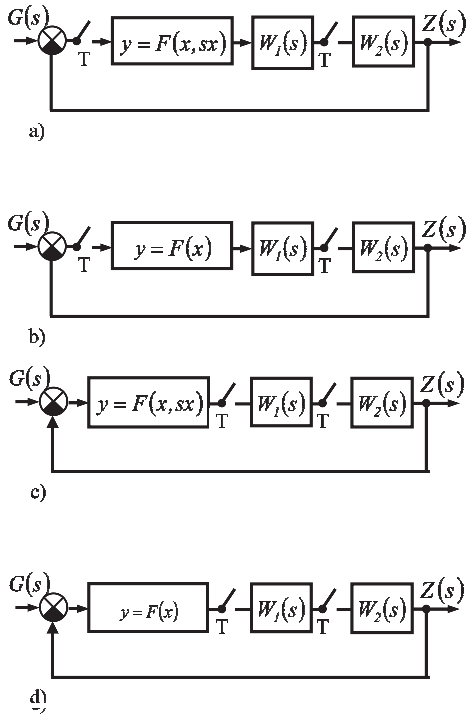


Fig. 3. Block diagrams of nonlinear NCDS with two IE

For the purposes of determining the parameters of the simplest involuntary periodic regimes, equations (32) – (35) can be used as

$$W_1^*\left(j\frac{\pi}{N}\right)W_2^*\left(j\frac{\pi}{N}\right) = -\frac{1}{I^*\left(A,j\dfrac{\pi}{N},\varphi\right)}\,; \qquad (36)$$

$$W_1^*\left(j\frac{\pi}{N}\right)W_2^*\left(j\frac{\pi}{N}\right) = -\frac{1}{W^*\left(A,j\omega\right)}\,; \qquad (37)$$

$$W_1^*\left(j\frac{\pi}{N}\right)W_2^*\left(j\frac{\pi}{N}\right) = -\frac{1}{I^*\left(A,N,\varphi\right)}\,; \qquad (38)$$

$$W_1^*\left(j\frac{\pi}{N}\right)W_2^*\left(j\frac{\pi}{N}\right) = -\frac{1}{W\left(A\right)}\,. \qquad (39)$$

Analysis of equations (20); (24); (28); (29) and (36) – (39) shows that in each of them under the sign of the operation of D-transformation is the expression of the transfer function of open linearized NCDS obtained from closed breaking the single feedback circuit. Thus, it can be argued that the common frequency condition for the occurrence of periodic processes in nonlinear NCDS is the equality of the negative unit of the D-transformation from the transfer function of an open harmonically linearized system.

The difference between the equations of periodic processes of nonlinear NCDSs with different relative positions of nonlinear and pulse elements in their block diagrams is the use of different coefficients of harmonic linearization NOT. This allows you to more fully take into account the nature of possible periodic modes in the system and increase the accuracy of determining their parameters.

Conclusions

Based on the discussed above, we can draw the following conclusions:

1. Comprehensive analysis of AAR control systems with PC as an object of study allows us to state that they belong to the class of nonlinear continuous-discrete automatic control systems.

Therefore, in the study and calculation of AAR control systems with PC in the frequency domain, it is advisable to use both the mathematical apparatus of Laplace transform and its discrete analogue of D-transform. This will allow the application of harmonic linearization NOT system to take into account the peculiarities of the periodic modes in systems of this class.

2. In solving the problem of the study to identify the specifics of harmonic linearization of NOT in systems of this class was justified the need to take into account its location in the structural scheme of the system relative to IE and the linear part of the system, as well as the presence or absence of inertial properties in NOT. In addition, it was noted that for NCDS with one nonlinear and one pulse element, there are four possible variants of block diagrams of single-circuit NCDS, for

each of which you need to apply different coefficients of harmonic linearization NOT.

3. In the process of solving the second problem of the study, mathematical dependences were obtained for the coefficients of harmonic linearization NOT of all possible variants of the structural construction of the system. In this case, the harmonic linearization of NOT, which are part of the NCDS with different structural structure, should be carried out by replacing it with coefficients having different formulaic expressions.

4. The result of solving the third problem of the study is to obtain using the proposed coefficients of harmonic linearization NOT equations that describe involuntary periodic processes that may occur in nonlinear NCDS automatic control of different structural construction.

The obtained equations determine the conditions of linearized nonlinear NCDS at the stability limit and, therefore, determine the parameters of possible involuntary periodic processes in the system.

5. When changing the structural scheme of NCDS due to the introduction of additional IE equations of periodic processes also change. The advantage of the proposed equations is not only their suitability for studying the parameters of involuntary periodic processes in any nonlinear NCDS automatic control, but also the ability to study using their characteristics of the emerging processes in each scheme of AAR control systems with onboard PC structural diagram of the system.

6. In the future for practical determination of conditions of occurrence and research of parameters of periodic processes in nonlinear NCDS, including in AAR control systems with onboard PC, it is planned to develop a technique which will allow to solve this problem by a graph-analytical method using logarithmic frequency characteristics of system scheme elements. or numerical methods of calculation on PC.

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РІВНЯННЯ ПЕРІОДИЧНИХ РЕЖИМІВ, ЯКІ ВРАХОВУЮТЬ ОСОБЛИВОСТІ ДИНАМІКИ ЇХ ПРОТІКАННЯ В НЕЛІНІЙНИХ АВТОМАТИЧНИХ СИСТЕМАХ З ЕОМ В КОНТУРІ УПРАВЛІННЯ

О.В. Войтко, В.Г. Солонніков, О.В. Полякова, А.М. Ткачов

У статті на основі застосування безперервно-дискретного підходу до опису періодичних режимів, що можливі в автоматичних системах з управляючою ЕОМ, отримані рівняння, які враховують особливості динаміки їх протікання в нелінійних системах зазначеного класу і забезпечують підвищення точності розрахунку їх параметрів. Це досягається шляхом врахуванням специфіки структурної схеми досліджуваної системи за рахунок заміни НЕ системи коефіцієнтами гармонічної лінеаризації, що мають відповідно різні формульні вирази. Застосування запропонованих рівнянь дозволить більш ефективно використовувати досліджувані режими при функціонуванні системи або навпаки гарантовано їх позбутися.

Ключові слова: *гармонічна лінеаризація, імпульсний елемент, нелінійний елемент, ЕОМ, безперервно-дискретна система.*

УРАВНЕНИЕ ПЕРИОДИЧЕСКИХ РЕЖИМОВ, УЧИТЫВАЮЩИХ ОСОБЕННОСТИ ДИНАМИКИ ИХ ПРОТЕКАНИЯ В НЕЛИНЕЙНЫХ АВТОМАТИЧЕСКИХ СИСТЕМАХ С ЭВМ В КОНТУРЕ УПРАВЛЕНИЯ

А.В. Войтко, В.Г. Солонников, Е.В. Полякова, А.М. Ткачев

В статье на основе применения непрерывно-дискретного подхода к описанию периодических режимов, возможных в автоматических системах с управляющей ЭВМ, получены уравнения, которые учитывают особенности динамики их протекания в нелинейных системах отмеченного класса и обеспечивают повышение точности расчета их параметров. Это достигается путем учета специфики структурной схемы исследуемой системы за счет замены НЕ системы коэффициентами гармонической линеаризации, которые имеют соответственно различные формульные выражения. Применение предлагаемых уравнений позволит более эффективно использовать исследуемые режимы при функционировании системы или наоборот гарантированно от них избавиться.

Ключевые слова: *гармоническая линеаризация, импульсный элемент, нелинейный элемент, ЭВМ, непрерывно-дискретная система.*