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MATHEMATICAL MODEL OF THE MOLTEN METAL DROP'S MOTION ON THE SURFACE OF A STEEL ROTATING DISK

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Summary. *The objective of the article is to study the trajectory of motion of different alloys according to mass, rotation speed, force of friction resistance, force of viscous resistance that influence the properties and stability of weld metal. The obtained calculations allow to describe mathematically the molten metal particle under study, in particular its coordinates, trajectory and the relative speed of motion in different time from 0 to 5 s, at a given initial position and angular speed of disk rotation.*

Key words: *induction surfacing, disk, molten particle, trajectory, force, acceleration.*

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Statement of the problem. Thin steel disks are widely used in engineering, the working surfaces of which are welded with wear-resistant material; the thickness of the base and weld metal is 2 … 6 mm and 0.5 … 2 mm, respectively. Various methods of surfacing are used in order to ensure the optimal structure of the weld metal, the stability of the thickness of the weld layer, the efficiency of the surfacing process, etc. [1, 2].

Analyses of available investigations. The authors developed a technology of simultaneous induction surfacing over the entire working surface of toothed-shape thin disks with a surfacing width that is greater than the height of the tooth. A two-turn circular inductor, the turns of which are interconnected in parallel (in antiphase of current and magnetic flux) was applied [3]–[5].

In [4], the influence of technological schemes on the stability of the layer thickness of the weld metal with the help of powdered hard alloy ПГ-С1 is substantiated. One of the technological schemes, which applies thermal and electromagnetic screens [6, 7] is as follows: in the process of induction surfacing, at the beginning of melting the powder alloy and the formation of a single liquid bath, the disk is rotated around the axis of symmetry with some angular velocity. As a result of this movement, impurities in the molten metal layer move along some trajectories relative to the melt, and due to the action of centrifugal forces of inertia, the shape of the outer surface of the melt bath is changed. This leads to the formation of a suitable structure of the weld metal and provides better stability of the weld layer thickness.

Statement of the problem. The problem under study is of the interest of theoretical substantiation of the influence of part's rotation on the uniformity of the weld metal thickness. The studies concerning the problem of increasing the stability of the weld metal layer thickness with the help of centrifugal forces are not available in the scientific literature.

Objective of the paper. The objective of the paper is to find the trajectories, along which different impurities will move depending on their mass, speed, frictional resistance and viscous melt resistance that affect the properties and stability of the weld metal thickness.

Figure 1 shows a device for induction surfacing with the help of centrifugal rotation.

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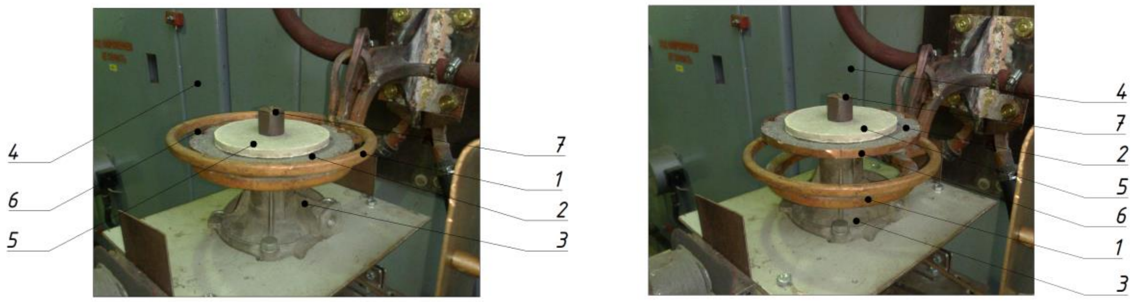

a b

Figure 1. Device for induction surfacing of a thin disk with the help of centrifugal rotation:
a – placement of the disk in the inductor together with thermal and electromagnetic screens during surfacing;
b – before surfacing with the release of an electromagnetic screen: 1 – two-turn circular inductor;
2 – disk; 3 – mechanism for rotating a disk; 4 – high frequency generator model VChG6-60/0.44;
5 – heat screen; 6 – electromagnetic screen; 7 – nut for clamping the disk to the mechanism of rotation

Figure 2 shows a diagram of the location of a point (molten particle) on the disk and the forces acting on it.

Suppose that the disk rotate around the vertical axis *Oz* with an angular velocity ω. A particle of mass *m* moves along the surface of the disk. It is necessary to find the trajectory of the relative motion of the particle relative to the disk.

Let us connect the *Oxyz* coordinate system to the disk, and the *Oxy* plane coincides with the upper surface of the disk (see Fig. 2). We obtain differential equations for the relative motion of a particle under the assumption that the forces of sliding friction resistance and viscous melt resistance act on it during the motion.

The equation of relative motion of a material particle of mass *m*:

$$m\vec{a}_r = \vec{F}_{\mathrm{e}} + \bar{F}_{cor} + \vec{F}_t + \vec{F}_{op}. \qquad (1)$$

Each of the forces included in the right part is defined (1).

$$\vec{F}_{\mathrm{e}} = -m\bar{a}_{\mathrm{e}}; a_{\mathrm{e}} = \omega^2 r = \omega^2\sqrt{x^2 + y^2}. \qquad (2)$$

$$\bar{F}_{cor} = -m\bar{a}_{cor}; \qquad (3)$$

$$\bar{a}_{cor} = 2(\bar{\omega}_{\mathrm{e}} \times \bar{V}_r) = 2\begin{vmatrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ 0 & 0 & \omega \\ \dot{x} & \dot{y} & 0 \end{vmatrix} = 2\vec{\imath}(-\omega\dot{y}) + 2\vec{\jmath}(\omega\dot{x}). \qquad (4)$$

If $\vec{\imath}$, $\vec{\jmath}$, $\vec{k}$ – unit vectors (orts) of coordinate axes *x, y, z*; $\dot{x}$, $\dot{y}$ – projection of the relative velocity of the point *M* on the coordinate axis *x* and *y*.

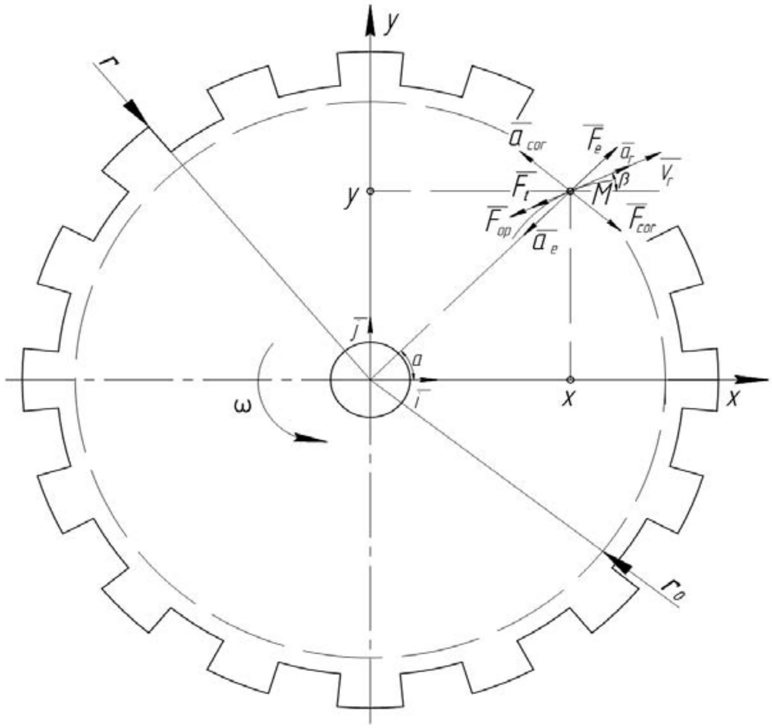

Figure 2. Diagram of the location of a point (molten particle) on the disk and the forces acting on it *M* – point (molten particle); $\bar{F}_t$ – sliding friction force; $\bar{F}_{op}$ – force of viscous resistance; $\bar{F}_e$ – transfer force of inertia; $\bar{F}_{cor}$ – Coriolis inertia force; $\bar{V}_r$ – relative velocity of the particle; β – angle between the vector $\bar{V}_r$ and axis *x*; $\bar{a}_{cor}$ – Coriolis acceleration; $\bar{a}_e$ – figurative acceleration; $\bar{a}_r$ – relative acceleration; ω – angular velocity; α – angle between the radius-vector of the point *M* and the axis *x*; r_0 – distance of the point *M* from the axis of rotation at the initial moment of time ($r_0 = 0.092\ m$); r – radius of the disk, the position of the point M at the final moment ($r = 0.105\ m$)

The vector of transfer acceleration is directed from the point *M* to the axis of rotation of the disk, and the transfer force of inertia is directed in the opposite direction. The projections of the inertia force of the transfer motion on the *x, y* coordinate axis are found:

$$F_{ex} = m\omega^2\sqrt{x^2+y^2}\cdot\cos\alpha = m\omega^2\sqrt{x^2+y^2}\cdot\frac{x}{\sqrt{x^2+y^2}} = m\omega^2 x,$$
$$F_{ey} = m\omega^2\sqrt{x^2+y^2}\cdot\sin\alpha = m\omega^2 y. \quad (5)$$

Projections of the Coriolis inertia force on the *x, y* axis based on (3), (4) are found:

$$F_{cor\ x} = 2m\omega\dot{y};$$
$$F_{cor\ y} = -2m\omega\dot{x}. \quad (6)$$

The force of sliding friction is determined by the Coulomb-Amonton law:

$$F_t = fN = fmg, \quad (7)$$

if f – sliding friction coefficient, g – acceleration of gravity. The sliding friction force is directed in the direction opposite to the vector of relative velocity $\vec{V}_r$. If we denote the angle that forms the vector $\vec{V}_r$ with the axis *x* through β, then:

$$F_{tx} = -fmg\cos\beta = -fmg\frac{\dot{x}}{V_r} = -fmg\frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}},$$
$$F_{ty} = -fmg\sin\beta = -fmg\frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}}. \quad (8)$$

Assume that the force of viscous melt resistance is proportional to the relative velocity and directed in the opposite direction to the relative velocity vector:

$$\bar{F}_{op} = -\mu\vec{V}_r. \quad (9)$$

$$\bar{F}_{op\,x} = -\mu\dot{x},$$
$$\bar{F}_{op\,y} = -\mu\dot{y}. \quad (10)$$

Based on the vector equation (1) on the coordinate axis x, y, the differential equations of relative motion of the particle *M* relative to the disk are deduced:

$$m\ddot{x} = m\omega^2 x + 2m\omega\dot{y} - fmg\frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} - \mu\dot{x},$$
$$m\ddot{y} = m\omega^2 y - 2m\omega\dot{x} - fmg\frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} - \mu\dot{y}. \quad (11)$$

$$\left.\begin{aligned} \ddot{x} &= \omega^2 x + 2\omega\dot{y} - fg\frac{\dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} - \frac{\mu}{m}\dot{x} \\ \ddot{y} &= \omega^2 y - 2\omega\dot{x} - fg\frac{\dot{y}}{\sqrt{\dot{x}^2 + \dot{y}^2}} - \frac{\mu}{m}\dot{y} \end{aligned}\right\} \quad (12)$$

The system (12) should be arranged to normal form:

$$x = x_1;\ y = x_2;$$
$$\dot{x} = \dot{x}_1 = x_3;\ \dot{y} = \dot{x}_2 = x_4. \quad (13)$$

Then, the system of equations (12) can be represented as:

$$\begin{cases} \dfrac{dx_1}{dt} = x_3, \\ \dfrac{dx_2}{dt} = x_4, \\ \dfrac{dx_3}{dt} = \omega^2 x_1 + 2\omega x_4 - fg\dfrac{x_3}{\sqrt{x_3^2 + x_4^2}} - \dfrac{\mu}{m}x_3, \\ \dfrac{dx_4}{dt} = \omega^2 x_2 + 2\omega x_3 - fg\dfrac{x_4}{\sqrt{x_3^2 + x_4^2}} - \dfrac{\mu}{m}x_4. \end{cases} \quad (14)$$

If at the initial moment of time the point is on the axis *Oy* at a distance r_0 to the axis of rotation and its relative speed equals to zero, then the initial conditions are deduced:
if t = 0:

$$x_1 = 0; x_2 = r_0; \\ x_3 = 0; x_4 = 0. \tag{15}$$

Thus, the Cauchy problem for the system of motion equations (14) and initial conditions is obtained (15).

The Runge-Kutta method was applied to find the solution of the obtained problem.

As an example, the trajectory of the relative motion of the particle with mass $m = 3 \cdot 10^{-5} kg$ (Fig. 3) on a disk with radius $r = 0.105\ m$ and width of the weld zone 13 mm is taken, considering the following indicators:

- coefficient of sliding friction $f = 0.01$;
- coefficient of viscous friction $\mu = 0.1$;
- angular speed of disk rotation $\omega = 3\pi$;
- radius of the particle position at the initial moment (when it has acquired a liquid state and its possible movement on the surface) $r_0 = 0.092\ m$.

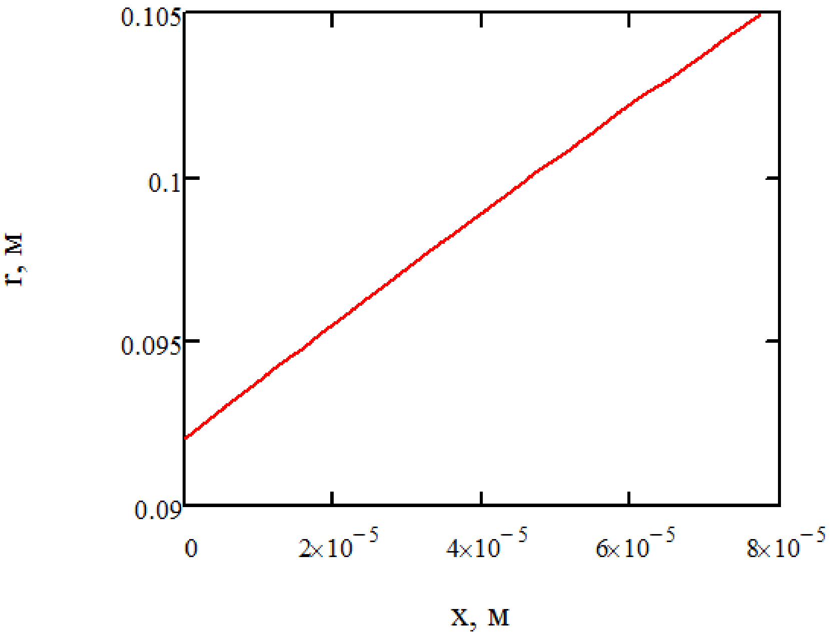

Figure 3. Diagram of the trajectory of molten particle motion on the disk surface relatively to the moving coordinate system

The above calculations allow to obtain complete information about the position and the speed of motion of the computational point (molten metal particles) at a given time on the test surface. The time interval is limited to 5 s (time of complete melting of powdered hard alloy ПГ-С1), and the whole process of induction surfacing is 22 s, which allows to obtain on the horizontal surface the best stability of the thickness of the weld metal layer.

For a better acquaintance with the position and movement of the particle (point) under study, a table is presented, which contains all the necessary data on the two-dimensional coordinate system *Oxy*.

Table 1

Calculation data of position and speed of the molten particle's motion relatively to the moving coordinate system

№	*Time, s*	*Coordinate x (m)*	*Coordinate y (m)*	*Projection of relative velocity on the axis x ($\dot{x}$, m/s)*	*Projection of relative velocity on the axis y ($\dot{y}$, m/s)*
-	0	0	0.092	0	0
1	0.227	$3.086 \cdot 10^{-6}$	0.093	$1.37 \cdot 10^{-5}$	$2.437 \cdot 10^{-3}$
2	0.455	$6.218 \cdot 10^{-6}$	0.093	$1.387 \cdot 10^{-5}$	$2.452 \cdot 10^{-3}$
3	0.682	$9.388 \cdot 10^{-6}$	0.094	$1.405 \cdot 10^{-5}$	$2.467 \cdot 10^{-3}$
4	0.909	$1.26 \cdot 10^{-5}$	0.094	$1.421 \cdot 10^{-5}$	$2.482 \cdot 10^{-3}$
5	1.136	$1.584 \cdot 10^{-5}$	0.095	$1.439 \cdot 10^{-5}$	$2.497 \cdot 10^{-3}$
6	1.364	$1.913 \cdot 10^{-5}$	0.095	$1.455 \cdot 10^{-5}$	$2.512 \cdot 10^{-3}$
7	1.591	$2.245 \cdot 10^{-5}$	0.096	$1.472 \cdot 10^{-5}$	$2.527 \cdot 10^{-3}$
8	1.818	$2.582 \cdot 10^{-5}$	0.097	$1.49 \cdot 10^{-5}$	$2.542 \cdot 10^{-3}$
9	2.045	$2.922 \cdot 10^{-5}$	0.097	$1.508 \cdot 10^{-5}$	$2.558 \cdot 10^{-3}$
10	2.273	$3.267 \cdot 10^{-5}$	0.098	$1.525 \cdot 10^{-5}$	$2.573 \cdot 10^{-3}$
11	2.5	$3.615 \cdot 10^{-5}$	0.098	$1.543 \cdot 10^{-5}$	$2.589 \cdot 10^{-3}$
12	2.727	$3.968 \cdot 10^{-5}$	0.099	$1.563 \cdot 10^{-5}$	$2.605 \cdot 10^{-3}$
13	2.955	$4.325 \cdot 10^{-5}$	0.099	$1.581 \cdot 10^{-5}$	$2.621 \cdot 10^{-3}$
14	3.182	$4.686 \cdot 10^{-5}$	0.1	$1.6 \cdot 10^{-5}$	$2.637 \cdot 10^{-3}$
15	3.409	$5.051 \cdot 10^{-5}$	0.101	$1.617 \cdot 10^{-5}$	$2.653 \cdot 10^{-3}$
16	3.636	$5.421 \cdot 10^{-5}$	0.101	$1.636 \cdot 10^{-5}$	$2.669 \cdot 10^{-3}$
17	3.864	$5.795 \cdot 10^{-5}$	0.102	$1.656 \cdot 10^{-5}$	$2.685 \cdot 10^{-3}$
18	4.091	$6.173 \cdot 10^{-5}$	0.102	$1.675 \cdot 10^{-5}$	$2.701 \cdot 10^{-3}$
19	4.318	$6.556 \cdot 10^{-5}$	0.103	$1.695 \cdot 10^{-5}$	$2.718 \cdot 10^{-3}$
20	4.545	$6.943 \cdot 10^{-5}$	0.104	$1.714 \cdot 10^{-5}$	$2.734 \cdot 10^{-3}$
21	4.773	$7.334 \cdot 10^{-5}$	0.104	$1.734 \cdot 10^{-5}$	$2.751 \cdot 10^{-3}$
22	5	$7.73 \cdot 10^{-5}$	0.105	$1.753 \cdot 10^{-5}$	$2.767 \cdot 10^{-3}$

In the table, 22 cases of finding the coordinates and relative velocity of the particle at one time or another for 5 s are presented with consideration of the width of the melt zone 13 mm.

Conclusions. The obtained calculations allow to describe mathematically the molten metal particle under study, in particular its coordinates, trajectory and the relative speed of motion in different time from 0 to 5 s, at a given initial position and angular speed of disk rotation. The obtained parameters of induction surfacing provide the increase of thickness layer stability of weld metal on the entire working surface to 6% as compared without the rotation of a detail. The fact is confirmed experimentally [6].

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МАТЕМАТИЧНА МОДЕЛЬ РУХУ КРАПЛІ РІДКОГО МЕТАЛУ НА ПОВЕРХНІ ОБЕРТОВОГО СТАЛЕВОГО ДИСКА

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Резюме. *На сьогодні в техніці широко застосовують тонкі сталеві диски, робочі поверхні яких наплавляють стійким до зношування матеріалом товщиною основного та наплавленого металу 2…6 mm і 0,5…2 mm відповідно. Використовують різні методи і способи наплавлення з метою забезпечення оптимальної структури наплавленого металу, стабільності товщини наплавленого шару, економічності процесу наплавлення. В різних роботах розроблена технологія одночасного індукційного наплавлення по всій робочій поверхні тонких дисків зубчастої форми з шириною наплавлення більшою за висоту зуба, з використанням двовиткового кільцевого індуктора. Також показано вплив технологічних схем на стабільність товщини шару наплавленого металу за допомогою порошкоподібного твердого сплаву ПГ-С1. Одна з технологічних схем із використанням теплових і електромагнітних екранів полягає в тому, що в процесі індукційного наплавлення в момент початку розплавлення порошкоподібного сплаву й утворення єдиної рідкої ванни диск піддають обертовому руху навколо осі симетрії з деякою кутовою швидкістю. В результаті цього руху домішки в розплавленому шарі металу будуть рухатися по деяких траєкторіях відносно розплаву, а також унаслідок дії відцентрових сил інерції буде змінюватися форма зовнішньої поверхні ванни розплаву. Це призводить до формування відповідної структури наплавленого металу й забезпечує кращу стабільність товщини наплавленого шару. Тому метою даної роботи є дослідження траєкторій руху різних домішок залежно від їх маси, швидкості обертання, сил опору тертя і в'язкого опору розплаву, які впливають на властивості і стабільність товщини наплавленого металу. Наведені розрахунки дозволяють математично описати досліджувану частинку розплавленого металу, а саме її координати, траєкторію та відносну швидкість переміщення в той чи інший момент часу в проміжку від 0 до 5 s при заданому початковому положенні й кутовій швидкості обертання диска.*

Ключові слова: *індукційне наплавлення, диск, розплавлена частинка, траєкторія, сила, прискорення.*

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